24. Motivation pour G^R

1. ARPES
2. Définition
3. Exemples ← ✓

25. Représentation d'interaction et produit chronologique ←

26. K.B. + K.S. (contours) ←

37. $\mathcal{G}$ Matsubara et relation à G^R

1. Déf.
2. Antipériodicité
3. Représentation spectrale ← X'' A
4. Poids spectral + règles pour prolongement analytique
5. Calculs cas sans int. ←

24.3

$$\boxed{\psi(r,t) = e^{iHt}\psi_s e^{-iHt}}$$

$$G^R(r,t;r',t') = -\frac{i}{\hbar}\langle\{\psi(r,t),\psi^+(r',t')\}\rangle\,\theta(t-t')$$

Équations du mvt:

$$i\frac{\partial}{\partial t}G^R(r,t;r',t') = \delta(r-r')\delta(t-t')$$

$$+\frac{i}{\hbar}\langle\{[H,\psi(r,t)],\psi^+(r',t')\}\rangle\theta(t-t')$$

$$H = \int d^3r_1\, d^3r_2\, \langle r_2|H|r_1\rangle \psi^+(r_2,t)\,\psi(r_1,t)$$

$$[\psi^+(r_2,t)\,\psi(r_1,t),\psi(r,t)] = \psi^+\{\psi,\psi\} - \{\psi^+_1,\psi\}\psi$$

$$[H,\psi(r,t)] = -\int d^3r_1\,\langle r|H|r_1\rangle\psi(r_1,t)$$

$$i\frac{\partial}{\partial t}G^R(r,t;r',t') = \delta(r-r')\delta(t-t') + \int d^3r_1\,\langle r|H|r_1\rangle G^R(r_1,t;r',t')$$

$$\int d^3r_1\left(i\frac{\partial}{\partial t}\langle r|r_1\rangle - \langle r|H|r_1\rangle\right)G^R(r_1,t;r',t') = \delta(r-r')\delta(t-t')$$

$$\int d^3r_1\,dt_1 \boxed{\langle r|\,i\frac{\partial}{\partial t} - H\,|r_1\rangle\delta(t-t_1)}\; G^R(r_1,t_1;r',t') = \delta(r-r')\delta(t-t')$$

$$\equiv G^{R^{-1}}(r,t;r_1,t_1)$$

$$G^{R^{-1}}(1,\bar{1})\,G^R(\bar{1},1') = \delta(1-1')$$

$$1 \equiv (r,t) \qquad \bar{1} \text{ barre} \equiv \int d^3r_1\,dt_1$$

25. Repr. d'interaction et produit chronologique

$$G^R(r,t;r',t') = -i \langle \{ \psi_H(r,t), \psi_H^+(r',t') \} \rangle \, \theta(t-t')$$

$H = H_0 + V \qquad [H_0, V] \neq 0$

$\hat{\psi}(r,t) = e^{iK_0 t} \psi_s(r) e^{-iK_0 t}$ calculable

$\psi_H(r,t) = e^{iKt} \psi_s(r) e^{-iKt}$ pas calculable

$$\boxed{U(t,0) = e^{-iKt}}$$

$K = H - \mu N$

$U^+(t,0) = e^{iKt} = U(0,t)$

$U^+(t,0)U(t,0) = I$

$$\boxed{U(t,t_1)U(t_1,0) = U(t,0)}$$

$$\langle i| e^{-\beta K} \; U(0,t)\, \psi_s\, U(t,0)\; U(0,t')\, \psi_s^{+}\, U(t',0)\, |i\rangle$$

ici

$H_0 \to K_0 = H_0 - \mu N$

$H \longrightarrow K = H - \mu N$

$$e^{-iH_0 t} \quad e^{iH_0 t} \quad e^{-iH_0 t} \quad e^{iH_0 t} \quad e^{-iH_0 t'} \quad e^{iH_0 t'} \quad e^{-iH_0 t'} \quad e^{iH_0 t'}$$

$$\hat{U}(0,t)\,\hat{\psi}(t)\,\hat{U}(t,t')\quad \hat{\psi}^{+}(t')\,\hat{U}(t',0)$$

$$\hat{U}(t,0) = \underline{e^{iH_0 t}\; U(t,0)}$$

$$i\frac{\partial}{\partial t}\hat{U}(t,0) = -H_0\,\boxed{\hat{U}(t,0)} + e^{iH_0 t}\, i\frac{\partial U(t,0)}{\partial t}$$

$$= e^{iH_0 t}\left[-H_0 + H \right] U(t,0)$$

$$= e^{iH_0 t}\, V\, e^{-iH_0 t}\, e^{iH_0 t}\, U(t,0)$$

$$i\frac{\partial \hat{U}(t,0)}{\partial t} = \hat{V}(t)\,\hat{U}(t,0)$$

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$$i \frac{\partial \hat{U}}{\partial t}(t,0) = \hat{V}(t)\,\hat{U}(t,0) \longrightarrow \cancel{\hat{U}(t,0) = e^{-i\int_0^t dt' \hat{V}(t')}}$$

$$i\left[\hat{U}(t,0) - \hat{U}(0,0)\right] = \int_0^t dt'\, \hat{V}(t')\,\hat{U}(t',0)$$

$$\hat{U}(t,0) = 1 - i\int_0^t dt'\, \hat{V}(t')\,\underbrace{\hat{U}(t',0)}$$

$$= 1 - i\int_0^t dt'\,\hat{V}(t') + (-i)^2 \int_0^t dt'\,\hat{V}(t') \int_0^{t'} dt''\,\hat{V}(t'')$$

$$+ (-i)^3 \int_0^t dt'\,\hat{V}(t') \int_0^{t'} dt''\,\hat{V}(t'') \int_0^{t''} dt'''\,\hat{V}(t''') + \dots$$

définition T_c = produit chronologique sur contour C

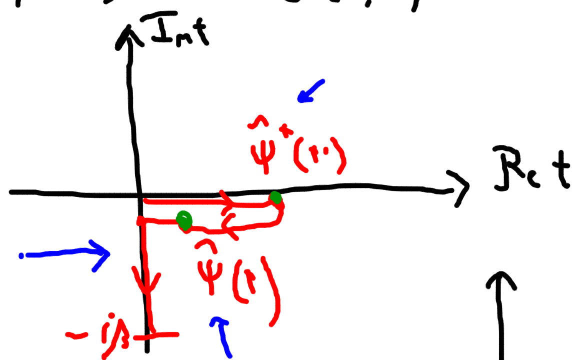
$$= \frac{(-i)^3}{3!}\, T_c \left[\int_0^t dt'\,\hat{V}(t')\right]^3 \qquad 0 \longrightarrow t$$

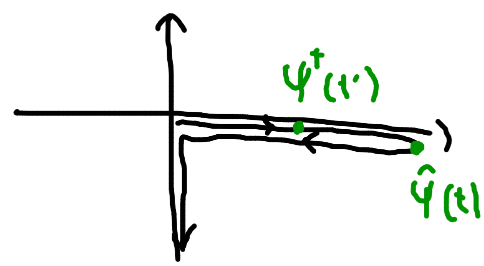
$$\hat{U}(t,0) = \underbrace{T_c\left[e^{-i\int_0^t \hat{V}(t')\,dt'}\right]}$$

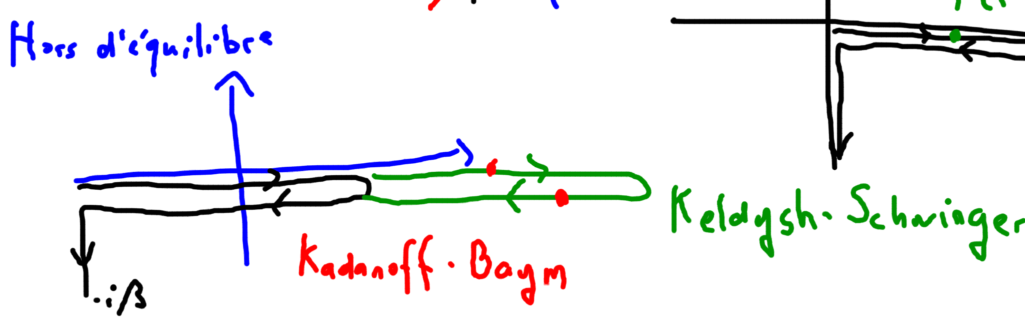
26. Contours de K.B. et K.S.

$$\langle i | e^{-\beta K_0} \boxed{e^{\beta K_0} e^{-\beta K}} \hat{U}(0,t)\, \hat{\psi}(t)\, \hat{U}(t,t')\, \hat{\psi}^{+}(t')\, \hat{U}(t',0) | i \rangle$$

$$\hat{U}(-i\beta,0) \qquad \hat{U}(t,0) = e^{iK_0 t}\, e^{-iKt}$$







27. Fonction $\mathcal{G}$ de Matsubara et relation avec G^R

Déf: $\tau = it$

$0 \longrightarrow \beta$ plus tard

$$\mathcal{G}(r,r';\tau) = -\langle T_\tau\, \psi(r,\tau)\,\psi^+(r',0)\rangle$$

$$= -\langle \psi(r,\tau)\psi^+(r',0)\rangle\,\theta(\tau) + \langle \psi^+(r',0)\psi(r,\tau)\rangle\,\theta(-\tau)$$

$$\langle T_\tau\, \psi^+(\tau_1)\,\psi(\tau_2)\,\psi^+(\tau_3)\,\psi(\tau_4)\rangle = \frac{\downarrow}{\uparrow}\langle T_\tau\, \psi(\tau_2)\,\psi^+(\tau_1)\,\psi^+(\tau_3)\,\psi(\tau_4)\rangle$$

$\tau_1 = 0$, $\tau_2 = \beta/2$, $\tau_3 = \beta$, $\tau_4 = \beta/3$

$$= \langle \psi^+(\beta)\,\psi(\beta/2)\,\psi(\beta/3)\,\psi^+(0)\rangle \;(-)\;(-)$$

pas adjoint

$$\psi(\tau) = e^{K\tau}\,\psi_s\,e^{-K\tau}$$

$$\psi^+(\tau) = e^{K\tau}\,\psi_s^+\,e^{-K\tau}$$

$\tau \equiv it$

$$\boxed{\text{Domaine: } -\beta < \tau < \beta}$$

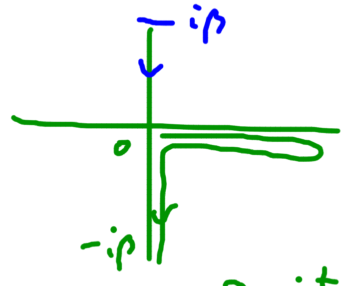
Bosons $\{[a_1, a_2^+]\} = 0$

$a_1 a_2^+ \overset{-}{+} a_2^+ a_1 = 0$

$a_1 a_2^+ = -a_2^+ a_1$

$[a_1(t), a_2^+(0)]$

$[e^{ikt} a_1(0), a_2^+(0)]$



$\tau = it$

$\beta = i(-i\beta)$

27.2 Antipériodicité

$\boxed{\tau < 0}$ $\quad \boxed{\mathcal{G}(\tau) = -\langle T_\tau\, \psi(\tau)\psi^\dagger(0)\rangle}$

$$e^{K\tau}\psi_S e^{-K\tau}$$

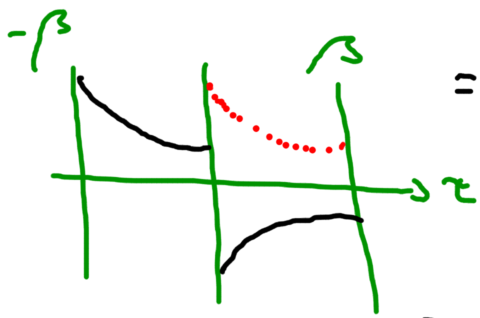
$$\mathcal{G}(\tau) = +\langle \psi^\dagger(0)\psi(\tau)\rangle$$

$$\frac{1}{Z}\mathrm{Tr}\left[e^{-\beta K}\psi^\dagger(0)\psi(\tau)\right] = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta K}e^{\beta K}\psi(\tau)e^{-\beta K}\psi^\dagger(0)\right]$$

$$= \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta K}\psi(\tau+\beta)\psi^\dagger(0)\right]$$

$$= -\mathcal{G}(\tau+\beta)$$

$$\boxed{\langle \psi(\tau+\beta)\psi^\dagger(0)\rangle}$$



$$\mathcal{G}(\tau) = T\sum_{n=-\infty}^{\infty} e^{-ik_n\tau}\,\mathcal{G}(ik_n)$$

où $k_n = (2n+1)\pi T$

$= (2n+1)\dfrac{\pi}{\beta}$

$k_n\beta = (2n+1)\pi$

$$\mathcal{G}(\tau+\beta) = -\mathcal{G}(\tau)$$

$$\mathcal{G}(ik_n) = \int_0^\beta d\tau\, e^{+ik_n\tau}\,\mathcal{G}(\tau)$$

Fréquences de Matsubara (Fermions)