37. $\mathcal{G}$ de Matsubara et lien avec G^R

✓ 1. Définition

✓ 2. Produit chronologique

✓ 3. Antipériodicité

→ 4. Représentation spectrale et lien entre G^R et $\mathcal{G}$

→ 5. Poids spectral A et règles pour le prolongement analytique

→ 6. $\mathcal{G}$ sans interaction

1. À partir de la déf
2. " des équs du mouvement

→ 7. Somme sur fréquences de Matsubara

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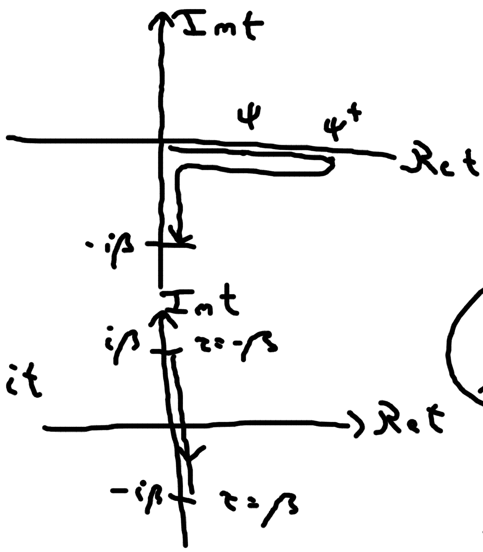
28. Signification physique : quasiparticules, m^*, Z, n_k

1. A sans interaction
2. Représentation de Lehmann ✓
3. Interprétation probabiliste de A ✓
4. Analogue du théorème F.D.
5. Expériences et ARPES
6. Quasiparticules
7. Liquide de Fermi
8. n_k et Z_k



$\tau = it$

$$G^R(r,r';t) = -i\left\langle \boxed{\left\{\psi(r,t),\psi^\dagger(r')\right\}}\right\rangle \quad \theta(t)$$

$$\mathcal{G}(r,r';\tau) = -\left\langle T_\tau\, \psi(r,\tau)\,\psi^\dagger(r')\right\rangle$$

$$\mathcal{G}(r,r';\tau) = -\mathcal{G}(r,r';\tau+\beta) \quad (\tau<0)$$

$$\left\{\begin{aligned} \mathcal{G}(r,r';\tau) &= T\sum_n e^{-i\omega_n\tau}\,\mathcal{G}(r,r';i\omega_n) \quad \text{où } \boxed{\omega_n = (2n+1)\pi T} \\ \mathcal{G}(r,r';i\omega_n) &= \int_0^\beta d\tau\, e^{i\omega_n\tau}\,\mathcal{G}(r,r';\tau) \end{aligned}\right.$$

27.4 $\mathcal{A}$ vs G^R

$$G^R(r,r';t) = -i\langle\{\psi(r,t),\psi^\dagger(r')\}\rangle\theta(t)$$

$$\chi^R_{ij}(t) = +i\langle[A_i(t),A_j]\rangle\,\theta(t)$$

$$A(r,r';t) = \langle\{\psi^\dagger(r,t),\psi^\dagger(r')\}\rangle$$

$$\boxed{G^R(r,r';\omega) = \int\frac{d\omega'}{2\pi}\frac{A(r,r';\omega')}{\omega+i\eta-\omega'}}$$

$$\chi''_{ij} = \frac{1}{2}\langle[A_i(t),A_j]\rangle$$

$$\chi^R_{ij}(\omega) = \int\frac{d\omega'}{\pi}\frac{\chi''_{ij}(\omega')}{\omega'-(\omega+i\eta)}$$

$$\int\frac{d\omega}{2\pi}A(r,r';\omega') = \delta(r-r')$$

$$\boxed{\mathcal{G}(r,r';ik_n) = \int \frac{d\omega'}{2\pi}\,\frac{A(r,r';\omega')}{ik_n-\omega'}}$$

à montrer

$$\mathcal{G}(r,r';ik_n) = \int_0^\beta d\tau\, e^{ik_n\tau}\left[-\langle \psi(r,\tau)\,\psi^+(r')\rangle\right]$$

$ik_n > 0$

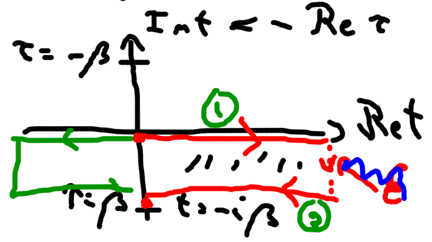
$\omega + i\eta$

$e^{ik_n\, i(t+i\eta)}$

$\psi(\tau) = e^{K\tau}\psi_s e^{-K\tau}$

$\psi(t)$

$\tau = it$



$$\int_0^{i\infty} i\,dt\; e^{ik_n(it)} \;\; ① \left[-\langle \psi(r,t)\,\psi^+(r',0)\rangle\right]$$

$$+\int_0^{\infty} i\,dt\; e^{ik_n(it)} \left[-\langle \psi(r,t-i\beta)\,\psi^+(r',0)\rangle\right]$$

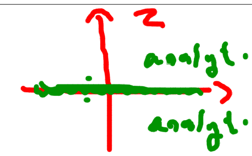
$$\boxed{\langle \psi(r,t-i\beta)\,\psi^+(r',0)\rangle} = \langle e^{iK(t-i\beta)}\psi_s(r)\, e^{-iK(t-i\beta)}\psi_s^+(r')\rangle$$

$$= \langle e^{\beta K}\,\psi_s(r,t)\, \boxed{e^{-\beta K}\,\psi_s^+(r')}\rangle$$

$$= \boxed{\langle \psi_s^+(r')\,\psi_s(r,t)\rangle}$$

$$= -i\int_{-\infty}^{\infty} dt\; e^{ik_n(it)}\,\langle\{\psi(r,t),\psi^+(r')\}\rangle\,\theta(t)$$

II.5 A et prolongement analytique.



$$\boxed{G(r,r';z) = \int \frac{d\omega'}{2\pi}\,\frac{A(r,r';\omega')}{z-\omega'}}$$

où z est var. complexe

$$\frac{1}{\omega+i\eta-\omega'} = \mathcal{P}\left(\frac{1}{\omega-\omega'}\right) - i\pi\delta(\omega-\omega')$$

$$\mathcal{G}(r,r';ik_n) = G(r,r';z=ik_n)$$

$$G^{R}_{A}(r,r';\omega) = \lim_{\eta\to 0} G(r,r';\underset{\omega-i\eta}{\omega+i\eta})$$

$$A(r,r';\omega) = i\left[G^R(r,r';\omega) - G^A(r,r';\omega)\right]$$

Prolongement unique ? Non e.g.

$$G(z)\left[1+\left(e^{\beta z}+1\right)\right]$$

ik_n

Si :

- $G(z)$ est analytique dans demi-plan sup.
- $G(z) = \mathcal{G}(ik_n)$ sur $z = ik_n$ $\forall k_n$
- $\lim_{z\to\infty} z\,G(z) = \text{constante}$

$\Rightarrow$ prolongement est unique

27.6 Cas sans interaction

Si invariance sous translation

$$\boxed{\frac{1}{V}\int d^3\frac{(r+r')}{2}}\int d^3\underline{(r-r')}\, e^{-ik\cdot(r-r')}\left[-\langle T_\tau \psi(r,\tau)\psi^+(r')\rangle\right]$$

$$\frac{1}{\sqrt{V}}\sum_{k'} e^{ik'\cdot r}\, c_{k'}(\tau)\,\frac{1}{\sqrt{V}}\sum_{k''} e^{-ik''\cdot r'}\, c^+_{k''}(0)$$

$$\boxed{\mathcal{G}(k,\tau) = -\langle T_\tau\, c_k(\tau)\, c_k^+\rangle}$$

Cas sans interaction:

Déf.: $\mathcal{G}(k,\tau) = -\langle c_k(\tau)\, c_k^+ \rangle \theta(\tau) + \langle c_k^+\, c_k(\tau) \rangle \theta(-\tau)$

$$K_0 = \sum_k (\epsilon_k - \mu) c_k^+ c_k \qquad \boxed{\xi_k \equiv \epsilon_k - \mu}$$

$$\frac{\partial c_k}{\partial \tau} = [K_0, c_k] = -(\epsilon_k - \mu) c_k = -\xi_k c_k$$

$$c_k(\tau) = c_k e^{-\xi_k \tau}$$

$$\mathcal{G}(k,\tau) = \underbrace{-\langle c_k c_k^+ \rangle}_{-1+f(\xi_k)} e^{-\xi_k \tau} \theta(\tau) + \underbrace{\langle c_k^+ c_k \rangle}_{f(\xi_k)} e^{-\xi_k \tau} \theta(-\tau)$$

$$\boxed{\mathcal{G}(k, ik_n) = \frac{1}{ik_n - \xi_k}} = \int_0^\beta d\tau\, e^{ik_n \tau} \mathcal{G}(k,\tau) \quad \text{pas fait}$$

Avec Eqs. du mvt :

$$\frac{\partial}{\partial\tau}\mathcal{G}(k,\tau) = \frac{\partial}{\partial\tau}\left[-\langle c_k(\tau)\, c_k^+\rangle\theta(\tau) + \langle c_k^+\, c_k(\tau)\rangle\theta(-\tau)\right]$$

$$= -\delta(\tau) - \langle T_\tau\left(\frac{\partial}{\partial\tau} c_k(\tau)\right)c_k^+\rangle$$

$$= -\delta(\tau) + \underbrace{\xi_k\langle T_\tau c_k(\tau)\, c_k^+\rangle}_{}\quad -\xi_k\mathcal{G}(k,\tau)$$

$$\left(\frac{\partial}{\partial\tau}+\xi_k\right)\mathcal{G}(k,\tau) = -\delta(\tau)$$

$$\int_{0^-}^{\beta^-} d\tau\, e^{ik_n\tau}\left[\frac{\partial}{\partial\tau}+\xi_k\right]\mathcal{G}(\tau,\tau) = -1$$

$$e^{ik_n\tau}\mathcal{G}(k,\tau)\Big|_{0^-}^{\beta^-} - ik_n\left[\int_1^{\beta} d\tau\, e^{ik_n\tau}\mathcal{G}(k,\tau)\right] + \xi_k\mathcal{G}(k,ik_n) = -1$$

$$\underbrace{\boxed{-\mathcal{G}(k,\beta^-) - \mathcal{G}(k,0^-)}}_{=0}$$

$$\boxed{\mathcal{G}(k,ik_n) = \frac{1}{ik_n-\xi_k}}$$

$$G^R(k,\omega) = \frac{1}{\omega+i\eta-\xi_k}$$

$$\mathcal{G}(k,\tau) = -\mathcal{G}(k,\tau+\beta) \quad \text{si } \tau<0$$

$$\mathcal{G}(k,0^-) = -\mathcal{G}(k,\beta^-)$$

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22.7 Somme sur ik_n

$$\mathcal{G}(k,\tau) = \frac{1}{\beta}\sum_n e^{-ik_n\tau}\frac{1}{ik_n-\xi_k} = -\int_C \frac{dz}{2\pi i}\frac{1}{e^{\beta z}+1}\frac{1}{z-\xi_k}$$

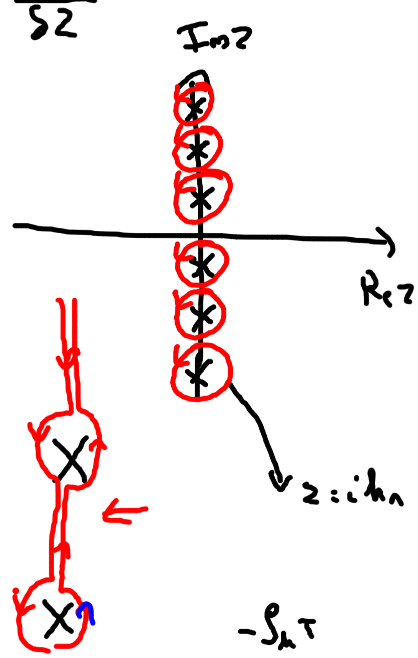
$$= e^{-\xi_k\tau}\left[(-1+f(\xi_k))\theta(\tau) + f(\xi_k)\theta(-\tau)\right].$$

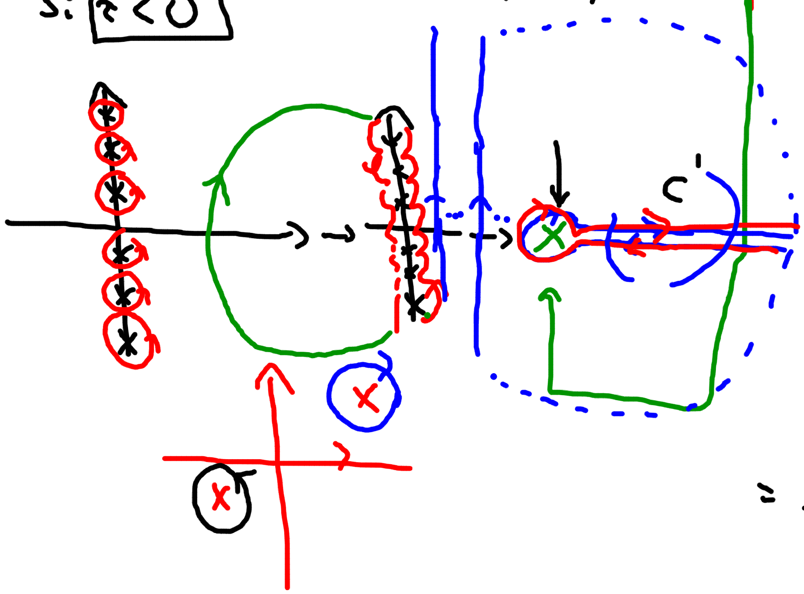
$$-\beta\frac{1}{e^{\beta z}+1}$$ a des pôles simples à $\begin{cases} z = ik_n \\ z = ik_n+\delta z\end{cases}$ $\boxed{\text{Résidu} = 1}$

$$-\beta\frac{1}{e^{\beta ik_n+\beta\delta z}+1} \approx -\beta\frac{1}{-1[1+\beta\delta z]+1} = \frac{1}{\delta z}$$

$$\frac{1}{\beta}\sum_n \frac{e^{-ik_n\tau}}{ik_n-\xi_k} = (-)\int_C \frac{dz}{2\pi i}\boxed{\frac{e^{-z\tau}}{e^{\beta z}+1}}\frac{1}{z-\xi_k}$$

Si $\boxed{\tau<0}$





$$-(-2\pi i)\frac{e^{-\xi_k\tau}}{2\pi i}\frac{1}{e^{\beta\xi_k}+1}$$

$$= \frac{e^{-\xi_k\tau}}{e^{\beta\xi_k}+1} = e^{-\xi_k\tau}f(\xi_k)$$

Si $z > 0$ $\qquad \beta \dfrac{1}{e^{-\beta z}+1} \longrightarrow \beta \dfrac{1}{e^{-\beta \xi_k}+1}$

$$\beta \left[\underbrace{+1 - \frac{1}{e^{\beta \xi_k}+1}} \right]$$

$$\frac{1}{e^{-\beta \xi_k}+1} = \frac{e^{\beta \xi_k}}{e^{\beta \xi_k}+1}$$