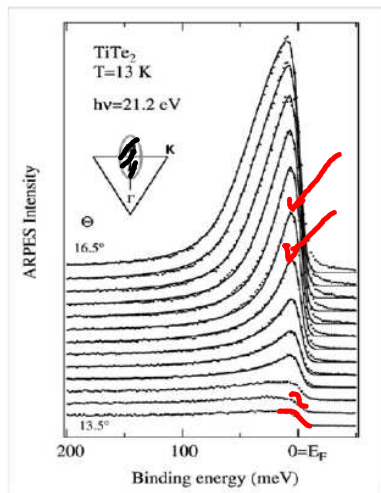


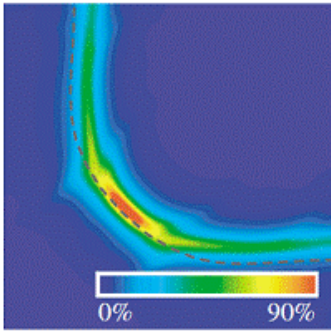
Figure 28-1 ARPES spectrum of $1 - T - TiTe_2$, after R. Claessen, R.O. Anderson, J.W. Allen, C.G. Olson, C. Janowitz, W.P. Ellis, S. Harm, M. Kalning, R. Manzke, and M. Skibowski, Phys. Rev. Lett **69**, 808 (1992).



$$\begin{aligned}
 \Gamma &= -Z_n \text{Im} \Sigma^R \\
 &= Z_n (\gamma') \omega^2 = \gamma' \omega^2
 \end{aligned}$$

↓
↑

Figure 28-2 Figure 1 from Ref.[19] for the ARPES spectrum of 1T-TiTe₂ measured near the Fermi surface crossing along the high-symmetry TM direction ($\theta = 0$ is normal emission). The lines are results of Fermi liquid fits and the inset shows a portion of the Brillouin zone with the relevant ellipsoidal electron pocket.



28. Signification physique du poids spectral: Q.P., m^* , Z_k , η_k 

- 1. $A(k, \omega)$ sans interaction
- 2. Repr. de Lehmann
- 3. Interprétation probabiliste
- 4. Analogie du théorème de F.-D.
- 5. Expériences ARPES
- 6. Quasiparticules
- 7. Liquides de Fermi
- 8. η_k système en interaction

29. Résultats formels

- 1. Comportement $\omega \rightarrow \infty$
- 2. Causalité

28.1 A sans interaction

$$A(k, \omega) = i \left(G^R(k, \omega) - G^A(k, \omega) \right)$$

$$G^R(k, \omega) = \frac{1}{\omega + i\eta - \epsilon_k}$$

$$A(k, \omega) = 2\pi \delta(\omega - \epsilon_k)$$

$$\epsilon_k = \epsilon_k - \mu$$

$$G(k, z) = \int \frac{d\omega'}{2\pi} \frac{A(k, \omega')}{z - \omega'}$$

G^R

$$z = \omega + i\eta$$

G^A

$$z = \omega - i\eta$$

28.2 Repr. de Lehmann

$$\rho = \frac{e^{-\beta K}}{Z}$$

$$\Omega = -T \ln Z$$

$$Z = e^{-\beta \Omega}$$

$$A(r, r', \omega) = \int dt e^{i\omega t} \langle \{ \Psi(r, t), \Psi^\dagger(r') \} \rangle$$

$$= e^{\beta \Omega} \int dt e^{i\omega t} \sum_{m, n} \left[e^{-\beta K_n} \langle n | e^{iKt} \Psi_S(r) e^{-iKt} | m \rangle \langle m | \Psi_S^\dagger(r') | n \rangle \right. \\ \left. + e^{-\beta K_n} \langle n | \Psi_S^\dagger(r') | m \rangle \langle n | e^{iKt} \Psi_S(r) e^{-iKt} | n \rangle \right]$$

$$= e^{\beta \Omega} \sum_{m, n} \left[e^{-\beta K_n} \langle n | \Psi_S(r) | m \rangle \langle m | \Psi_S^\dagger(r') | n \rangle 2\pi \delta(\omega + K_n - K_m) \right.$$

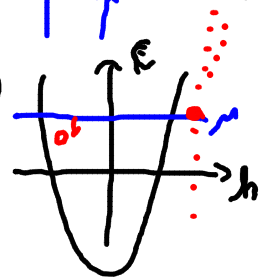
$$\left. + e^{-\beta K_n} \langle n | \Psi_S^\dagger(r') | m \rangle \langle m | \Psi_S(r) | n \rangle 2\pi \delta(\omega + K_m - K_n) \right]$$

(T=0)

BIS: $\omega + (E_0 - N_0 \mu) - (E_m - (N_0 + 1) \mu)$

$$= \omega + \mu + E_0 - E_m$$

ARPES: $\omega + \mu + E_m - E_0$ $E_m - E_0 = \mu$



28.3 Interpretation probabiliste

$$\begin{aligned}
 A(h, \omega) &= e^{\beta \Omega} \sum_{m, n} \left(e^{-\beta k_n} \langle n | c_h | m \rangle \langle m | c_h^\dagger | n \rangle 2\pi \delta(\omega + k_n - k_m) \right. \\
 &\quad \left. + e^{-\beta k_m} \langle n | c_h^\dagger | m \rangle \langle m | c_h | n \rangle 2\pi \delta(\omega + k_m - k_n) \right) \\
 &= e^{\beta \Omega} \sum_{m, n} \left(e^{-\beta k_n} + e^{-\beta k_m} \right) |\langle n | c_h | m \rangle|^2 2\pi \delta(\omega + k_n - k_m)
 \end{aligned}$$

$$A(h, t=0) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A(h, \omega) = \langle \{c_h, c_h^\dagger\} \rangle = 1$$

$\int_{-\infty}^{\infty} e^{i\omega 0} d\omega$



23.4 Analogie du théorème F.-D.

$$\frac{d^2\sigma}{d\Omega d\omega} \propto \int dt e^{i\omega t} \langle c_h^\dagger c_h(t) \rangle = f(\omega) A(k, \omega)$$

$$\begin{aligned} S_{A_i A_j}(\omega) &= \int dt e^{i\omega t} \langle A_i(t) A_j \rangle = \frac{2}{e^{-\beta\hbar\omega} - 1} \chi''_{ij}(\omega) \\ &= (1 + n_B(\omega)) \int dt e^{i\omega t} \langle [A_i(t), A_j(0)] \rangle \end{aligned}$$

Preuve: $\int dt e^{i\omega t} \langle \{c_k(t), c_k^\dagger\} \rangle$

$$\int dt e^{i\omega t} \left[\langle c_k(t) c_k^\dagger \rangle + \langle c_k^\dagger c_k(t) \rangle \right]$$

$$\left\{ \begin{aligned} \langle c_k(t) c_k^\dagger \rangle &= \frac{1}{2} \text{Tr} \left[e^{-\beta H} c_k(t) c_k^\dagger \right] \\ &= \frac{1}{2} \text{Tr} \left[e^{\beta \hbar \omega} e^{-\beta \hbar \omega} c_k^\dagger e^{-\beta \hbar \omega} c_k(t) \right] \\ &= \langle c_k^\dagger c_k(t+i\beta) \rangle \end{aligned} \right.$$

Handwritten notes in red:
 - $e^{-i\omega(t+i\beta)}$ and $e^{-iH(t+i\beta)}$ with arrows pointing to the exponential factors in the trace.
 - A red circle around $e^{\beta \hbar \omega}$ and arrows pointing to c_k^\dagger and $c_k(t)$ in the second line.
 - A red arrow pointing from $c_k(t+i\beta)$ in the third line to $c_k(t)$ in the second line.

$$= \int dt e^{i\omega(t+i\beta-i\beta)} \left[\langle c_k^\dagger c_k(t+i\beta) \rangle + \langle c_k^\dagger c_k(t) \rangle \right]$$

$$A(k, \omega) = (e^{\beta \hbar \omega} + 1) \int dt e^{i\omega t} \langle c_k^\dagger c_k(t) \rangle$$

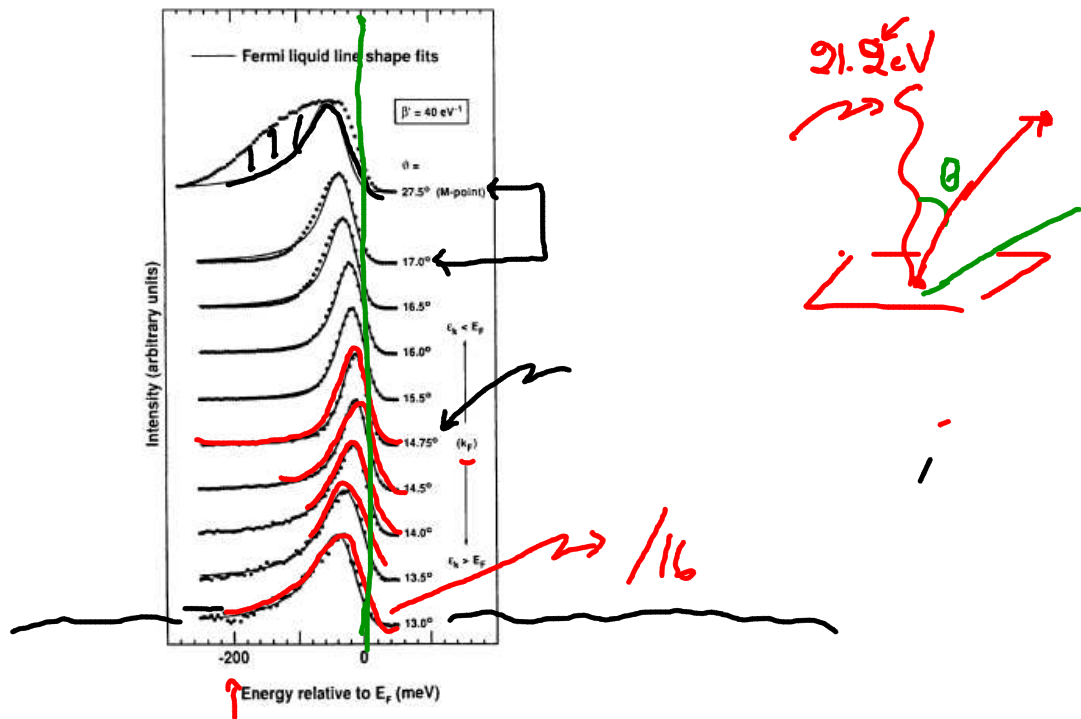


Figure 28-1 ARPES spectrum of $1 - T - TiTe_2$, after R. Claessen, R.O. Anderson, J.W. Allen, C.G. Olson, C. Janowitz, W.P. Ellis, S. Harm, M. Kalning, R. Manzke, and M. Skibowski, Phys. Rev. Lett **69**, 808 (1992).

28.6 Quasiparticules

$$N_k = \nabla_k \int_k$$

$$G^R(k, \omega) = \frac{1}{\omega - \epsilon_k - \Sigma^R(k, \omega)}$$

$$A(k, \omega) = -2 \text{Im} G^R(k, \omega) = \frac{-2 \text{Im} \Sigma^R(k, \omega)}{(\omega - \epsilon_k - \text{Re} \Sigma^R(k, \omega))^2 + (\text{Im} \Sigma^R(k, \omega))^2}$$

$\omega = E_k - \mu$ de la Q.P.
Satisfait

$$(\omega - \epsilon_k - \text{Re} \Sigma^R(k, \omega)) = \left(\begin{matrix} 0 \\ \omega = E_k - \mu \end{matrix} \right) + \frac{\partial}{\partial \omega} (\omega - \epsilon_k - \text{Re} \Sigma^R) \Big|_{\omega = E_k - \mu}$$

$$= \left[1 - \frac{\partial \text{Re} \Sigma^R}{\partial \omega} \right]_{\omega = E_k - \mu} (\omega - E_k + \mu)$$

$$A(k, \omega) = 2\pi \sum_k \frac{Z_k}{\pi} \frac{-(\text{Im} \Sigma^R) Z_k}{(\omega - E_k + \mu)^2 + \text{Im} \Sigma^R{}^2 Z_k^2}$$

$$\Gamma(k, \omega) \equiv -Z_k \text{Im} \Sigma^R(k, \omega)$$

$$A(k, \omega) = 2\pi \sum_k \left(\frac{1}{\pi} \frac{\Gamma(k, \omega)}{(\omega - E_k + \mu)^2 + \Gamma^2(k, \omega)} \right) \quad \text{cohérente!}$$

$$\int \frac{d\omega}{2\pi} A(k, \omega) = \sum_k = \frac{1}{\left| 1 - \frac{\partial \text{Re} \Sigma^R(k, \omega)}{\partial \omega} \right|_{\omega = E_k - \mu}} < 1$$

$$\omega = E_{k-m} - \mu$$

$$E_{k-m} - \mu - \text{Re} \Sigma^R(k, E_{k-m}) = 0$$

$$\nabla_k E_{k-m} = N_k^*$$

$$\left[\begin{array}{l} N_k^* - N_k - \nabla_k \text{Re} \Sigma^R(k, E_{k-m}) \\ - \frac{\partial \text{Re} \Sigma^R(k, \omega)}{\partial \omega} \end{array} \right] \Bigg|_{\omega = E_{k-m}} N_k^* = 0$$

S_c sphérique.

$$N_k^* - N_k - \frac{\partial \text{Re} \Sigma^R(k, E_{k-m})}{\partial E_{k-m}} v_k - \frac{\partial \text{Re} \Sigma}{\partial \omega} N_k^* = 0$$

$$\frac{1}{m^*} N_k^* = \frac{N_k}{m} \left[\frac{1 + \frac{\partial \text{Re} \Sigma^R}{\partial E_{k-m}}}{1 - \frac{\partial \text{Re} \Sigma^R}{\partial \omega}} \right] \Bigg|_{\omega = E_{k-m}}$$

$$\boxed{mN = \hbar k_F}$$

$$\boxed{m^* N^* = mN}$$

Liquide de Fermi:

Théorème Luttinger: Volume dans zone de Brillouin de la surface de Fermi ne dépend pas des int.

28.7 Le liquide de Fermi

À $T=0$ $\text{Im} \Sigma^R(k_F, 0) = 0$

hypothèse que $\text{Im} \Sigma^R$ est analytique en ω

$G^R = G^A + G^R \Sigma^R G^R + \dots$
 $\text{Im} \Sigma^R = \cancel{\alpha \omega} - \gamma \omega^2 + \dots$

$\left[\text{Re} \Sigma^R(k, \omega) - \text{Re} \Sigma^R(k, \infty) \right] = \mathcal{P} \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \frac{-\gamma \omega'^2}{\omega' - \omega}$

$= \mathcal{P} \int \frac{d\omega'}{\pi} (-\gamma) \frac{(\omega'^2 - \omega^2 + \omega^2)}{\omega' - \omega} = \mathcal{P} \int \frac{d\omega'}{\pi} (-\gamma) \frac{(\omega' - \omega)(\omega' + \omega)}{\omega' - \omega}$

$= \mathcal{P} \int \frac{d\omega'}{\pi} \omega' (-\gamma) - \gamma \omega \int \frac{d\omega'}{\pi} + \mathcal{O}(\omega^2)$

$\Rightarrow Z_h < 1$