28. Quasiparticules etc...

8. n_k $\leftarrow$

29. Quelques résultats formels:

1. Comportement asymptotique $\mathcal{G}(k, ik_n)$, $\Sigma(k, ik_n)$
2. Causalité $\leftarrow$

30. Théorèmes généraux

✓ 1. Wick

✓ 2. Graphs connexes

✓ 3. Principe variationnel

févr. 22-09:31

$\boxed{\text{Partie V : Gaz de Coulomb}}$ Jellium

31. Méthode générale : champs source, dérivée fonctionnelle

1. Exemple de physique statistique simple
2. H et notation
3. Équations du mouvement
4. Dérivées fonctionnelles et structure du problème à N-corps

$$A(k,\omega) = -2\,\mathrm{Im}\,G^R = \frac{\boxed{-2\,\mathrm{Im}\,\Sigma^R(k,\omega)}}{\left(\omega - \xi_k - \mathrm{Re}\,\Sigma^R(k,\omega)\right)^2 + \left(\mathrm{Im}\,\Sigma^R\right)^2}$$

$$\boxed{\begin{aligned} \mathrm{Re}\, y &= y' \\ \mathrm{Im}\, y &= y'' \end{aligned}}$$

$$= Z_k\, 2\pi\, \frac{1}{\pi}\, \frac{\Gamma(k,\omega)}{(\omega - E_k + \mu)^2 + \Gamma(k,\omega)^2}$$

$$v_k \to v_k^*$$

$$\boxed{\begin{aligned} \Gamma &= -2\,\mathrm{Im}\,\Sigma^R \\ Z_k &= \frac{1}{1 - \left.\frac{\partial \Sigma'^R}{\partial \omega}\right|_{\omega = E_k - \mu}} \end{aligned}}$$

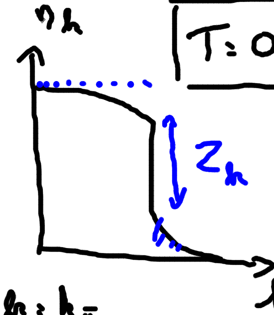
28.8 n_k

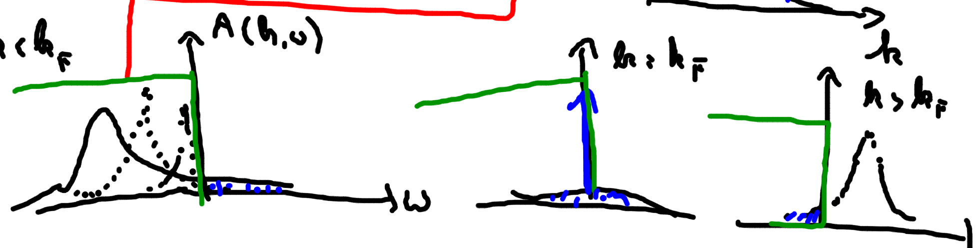
$Z \sim \omega^\alpha$

$$\int dt\, e^{i\omega t} \langle c_k^\dagger c_k(t) \rangle = \underline{f(\omega) A(k,\omega)}$$

$$\boxed{\int \frac{d\omega}{2\pi} f(\omega) A(k,\omega) = \langle \underbrace{c_k^\dagger c_k} \rangle}$$

$\boxed{T=0}$





29.1 Propriétés formelles

$$\mathcal{G}(ik_n) = \int \frac{d\omega'}{2\pi} \frac{A(k,\omega')}{ik_n - \omega'} \xrightarrow[ik_n \text{ grand}]{} \left[\int \frac{d\omega'}{2\pi} A(k,\omega')\right] \frac{1}{ik_n}$$

$$\longrightarrow \frac{1}{ik_n}$$

Si vrai $\Rightarrow$ anticom. : O.K.

$$\mathcal{G}(k,\tau) = -\langle T_\tau c_k(\tau) c_k^+ \rangle$$

$$\mathcal{G}(k,0^-) - \mathcal{G}(k,0^+) = \langle c_k^+ c_k \rangle + \langle c_k c_k^+ \rangle = 1$$

$$T \sum_n \left(e^{-ik_n 0^-} - e^{-ik_n 0^+}\right) \left[\mathcal{G}(k, ik_n) - \frac{1}{ik_n}\right]$$

$$0$$

$$+ T \sum_n \left(e^{-ik_n 0^-} - e^{-ik_n 0^+}\right) \frac{1}{ik_n} = 1$$

29.2 <u>Causalité</u> $\mathrm{Im}\,\Sigma^R(k,\omega) < 0$

Lehmann $\Rightarrow$ $A(k,\omega) = -2\,\mathrm{Im}\,G^R(k,\omega) > 0$

$$\mathrm{Re}\,G^R(r,r';\omega)^* = \mathrm{Re}\,G^R(r',r;\omega)$$

$$\left(\langle n|\psi(r)|m\rangle \langle m|\psi^+(r')|n\rangle \right)^*$$

$$G^R = \frac{1}{\omega - \xi_k - \Sigma^R}$$

févr. 22-10:30

31.1 Exemple méthode générale

$$Z = \mathrm{Tr}\left[e^{-\beta(H-\mu N-hM)} \right]$$

$$\frac{\partial \ln Z}{\beta \partial h} = \frac{\mathrm{Tr}\left(M e^{-\beta(H-\mu N-hM)}\right)}{Z} = \langle \underset{\sim}{M} \rangle_h$$

$$\frac{\partial^2 \ln Z}{\beta^2 \partial h^2} = \langle MM \rangle\Big|_{h=0} - \langle M \rangle \langle M \rangle\Big|_{h=0}$$

févr. 22-10:54

$$\langle M(r_1)\, M(r_2) \rangle \Big|_{h=0} - \langle M(r_1) \rangle \langle M(r_2) \rangle \Big|_{h=0}$$

$$Z = \mathrm{Tr}\left[e^{-\beta \left(H - \mu N - \int d^3r\, h(r)\, M(r) \right)} \right]$$

$$\frac{\delta \ln Z}{\beta\, \delta h(r_1)} = \frac{1}{Z}\, \mathrm{Tr}\left[e^{-\beta \left(H - \mu N - \int d^3r\, h(r)\, M(r) \right)} \frac{\delta}{\delta h(r_1)} \int d^3r\, h(r)\, M(r) \right]$$

$$= \langle M(r_1) \rangle$$

$$\boxed{\frac{\delta h(r_1)}{\delta h(r_2)} = \delta(r_1 - r_2)} \qquad \boxed{\frac{\partial y_1}{\partial y_2} = \delta_{1,2}}$$

$$\frac{\delta \ln Z[h]}{\beta\, \delta h(r_1)} = \langle M(r_1) \rangle$$

$$\frac{\delta^2 \ln Z[h]}{\beta^2\, \delta h(r_1)\, \delta h(r_2)} = \langle M(r_1)\, M(r_2) \rangle - \langle M(r_1) \rangle \langle M(r_2) \rangle$$

Notre cas:

$$Z[\varphi] = \mathrm{Tr}\left[e^{-\beta K} T_\tau \, e^{-\int_{1'}\int_{2'} \sum_\sigma \psi^+_\sigma(1')\,\varphi_\sigma(1',2')\,\psi_\sigma(2')} \right]$$

$$1' \equiv (r_1', \tau_1')$$

$$\int_{1'} \equiv \int d^3 r_1' \int_0^\beta d\tau_1'$$

$$\langle T_\tau\, U[\varphi]\, \psi^+(1)\, \psi(2) \rangle \cancel{\;} \; \pm T_\tau$$

$$-\frac{\delta \ln Z[\varphi]}{\delta \varphi(1,2)} = \mathcal{G}(2,1)_\varphi = -\frac{\langle T_\tau\, U[\varphi]\, \psi(2)\, \psi^+(1) \rangle}{\langle T_\tau\, U[\varphi] \rangle}$$

$$H = h_0 \psi^+\psi + V\, \underbrace{\psi^+\psi^+\psi\psi}$$

$$\frac{\partial \psi}{\partial \tau} = \frac{\partial}{\partial \tau}\left[e^{K\tau}\psi e^{-K\tau}\right] = [K,\psi] = -h_0\psi - V\,\psi^+\psi\psi$$

$$\frac{\partial}{\partial \tau}\mathcal{G}(r,\tau;r',\tau') = h_0\,\mathcal{G} + \varphi\,\mathcal{G} - V\langle \psi^+\psi\psi\;\psi^+\rangle$$

$$\left(\mathcal{G}_0^{-1} - \varphi\right)\mathcal{G} = 1 - V\,\langle\psi^+\psi\psi\psi^+\rangle_{\varphi=0} \qquad \cancel{\Sigma\mathcal{G}}$$

$$\Sigma(1,\bar{1})\mathcal{G}(\bar{1},2)$$

$$\boxed{\frac{\delta \mathcal{G}_\varphi}{\delta\varphi}}\Bigg|_{\varphi=0}$$

$$\underbrace{\left(\mathcal{G}_0^{-1} - \varphi - \Sigma\right)}_{\mathcal{G}^{-1}}\mathcal{G} = 1$$

$$\frac{\delta(\mathcal{G}\mathcal{G}^{-1})}{\delta\varphi} = 0$$

$$\frac{\delta\mathcal{G}}{\delta\varphi}\mathcal{G}^{-1} + \mathcal{G}\frac{\delta\mathcal{G}^{-1}}{\delta\varphi} = 0$$

$$\frac{\delta\mathcal{G}}{\delta\varphi} = -\mathcal{G}\frac{\delta\mathcal{G}^{-1}}{\delta\varphi}\mathcal{G} = +\mathcal{G}\frac{\delta\varphi}{\delta\varphi}\mathcal{G} + \mathcal{G}\frac{\delta\Sigma}{\delta\varphi}\mathcal{G}$$

$$\boxed{\frac{\delta\mathcal{G}}{\delta\varphi} = \mathcal{G}\frac{\delta\varphi}{\delta\varphi}\mathcal{G} + \mathcal{G}\int\frac{\delta\Sigma}{\delta\mathcal{G}}\mathcal{G}\,\frac{\delta\mathcal{G}}{\delta\varphi}}$$

$$\frac{\delta\Sigma}{\delta\varphi} = \frac{\delta\Sigma}{\delta\mathcal{G}}\frac{\delta\mathcal{G}}{\delta\varphi}$$

Vertex irréductible (canal particule-trou)

$$\left[1 - \mathcal{G}\cancel{\frac{\delta\Sigma}{\delta\mathcal{G}}}\mathcal{G}\right]\frac{\delta\mathcal{G}}{\delta\varphi} = \mathcal{G}\frac{\delta\varphi}{\delta\varphi}\mathcal{G}$$

$$\delta(1-\bar{1})\delta(2-\bar{2}) \qquad \delta\mathcal{G}(\bar{1},\bar{2})$$

$$\frac{\mathcal{G}(1,\bar{2})\,\delta\varphi(\bar{2},\bar{3})\,\mathcal{G}(\bar{3},1')}{\delta\varphi(3,4)}$$

$$\mathcal{G}(1,\bar{1})\,\frac{\delta\Sigma(\bar{1},\bar{2})}{\delta\varphi(3,4)}\,\mathcal{G}(\bar{2},1')$$

$$=\mathcal{G}(1,\bar{1})\left[\frac{\delta\Sigma(\bar{1},\bar{2})}{\delta\mathcal{G}(\bar{3},\bar{4})}\,\frac{\delta\mathcal{G}(\bar{3},\bar{4})}{\delta\varphi(3,4)}\right]\mathcal{G}(\bar{2},1')$$

31.2 H $\qquad K = H - \mu N$

$$H = \sum_{\sigma_1} \int d^3x_1\, d^3x_2\, \psi^+_{\sigma_1}(x_1) \langle x_1 | H_0 | x_2 \rangle \psi_{\sigma_1}(x_2)$$

$$\psi^+(x)\left[-\frac{\nabla^2}{2m}\right]\psi(x)$$

$$[\psi^+ \nabla^2 \psi, \psi]$$

$$\boxed{\langle x_1 | H_0 | x_2 \rangle} = -\frac{\nabla^2_{x_2}}{2m}\,\delta(x_2 - x_1) \qquad p^2/2m$$

$$+\frac{1}{2}\sum_{\sigma\sigma'} \int d^3x_1\, d^3x_2\, V(x_1 - x_2)\, \psi^+_{\sigma}(x_1)\, \psi^+_{\sigma'}(x_2)\, \psi_{\sigma'}(x_2)\, \psi_{\sigma}(x_1)$$

$$\frac{\partial \psi_\sigma(x,t)}{\partial t} = \cancel{\sum_{\sigma_1}} \int \cancel{d^3x_1}\, d^3x_2\, \langle x_1 | H_0 | x_2 \rangle \left[\psi^+_{\sigma_1}(x_1)\, \psi_{\sigma_1}(x_2), \psi_\sigma(x)\right]$$

$$= -\int d^3x_2\, \langle x | H_0 | x_2 \rangle\, \psi_\sigma(x_2)$$

$$[AB, C] = -\{A, C\}B + A\{B, C\}$$

$$\boxed{\langle x_1 | \frac{P^2}{2m} | x_2 \rangle} = \int \frac{dp}{2\pi} \langle x_1 | p \rangle \frac{p^2}{2m} \langle p | x_2 \rangle$$

$$= \int \frac{dp}{2\pi} e^{ip(x_1 - x_2)} \frac{p^2}{2m} = -\frac{\nabla^2_{x_1}}{2m} \delta(x_1 - x_2)$$

$$\int d^3x \, \psi^*(x) \left[-\frac{\nabla^2}{2m} \right] \psi(x)$$

$$= \int d^3x \, (\nabla \psi^\dagger) \frac{1}{2m} (\nabla \psi)$$

$$\int d^3x_1 \, d^3x_2 \, \psi^\dagger(x_1) \left[-\frac{\nabla^2_{x_1}}{2m} \delta(x_2 - x_1) \right] \psi(x_2)$$