32. Equations du mouvement pour $\mathcal{G}$ en présence de sources

1. H et equs. pour $\Psi(1)$
✓ 2. Equs. du mvt pour $\underline{\mathcal{G}}_\varphi$ et déf. de Σ_φ
✓ 3. Fonctions à 4 points de dérivées fonctionnelle
4. Σ à partir de dérivée fonctionnelle

33. Première etape: H.F. et R.P.A.

$$Z[h] = \mathrm{Tr}\left[e^{-\beta K + \int d^3r\, \beta h(r) M(r)} \right]$$

$$\frac{\delta^2 \ln Z[h]}{\beta^2\, \delta h(r_1)\, \delta h(r_2)} = \langle M(r_1) M(r_2) \rangle_h - \langle M(r_1) \rangle_h \langle M(r_2) \rangle_h$$

$$\mathcal{G}(1,2)_\varphi = - \frac{\langle T_\tau\, S[\varphi]\, \psi(1)\psi^+(2) \rangle}{\langle T_\tau S[\varphi] \rangle}$$

$$\boxed{\langle T_\tau S[\varphi] \rangle = \mathrm{Tr}\left[e^{-\beta K}\, T_\tau\, S[\varphi] \right] / \mathrm{Tr}\left[e^{-\beta K} \right]}$$

$$S[\varphi] = e^{-\psi^+(\bar{1})\,\varphi(\bar{1},\bar{2})\,\psi(\bar{2})}$$

$$1 = (x_1, \tau_1 ; \sigma_1) \qquad \bar{1} \to \int d^3x_1 \int_0^\beta d\tau_1 \sum_{\sigma_1}$$

39.1 $\mathcal{H}$ et $\frac{\partial \psi_\sigma(x,\tau)}{\partial \tau} = [K, \psi_\sigma(x,\tau)]$ $\quad \psi(\tau) = e^{K\tau}\psi_s e^{-K\tau}$

$$K = \sum_{\sigma_1}\int d^3x_1\, \psi^+_{\sigma_1}(x_1)\left[-\frac{\nabla^2}{2m}\right]\psi_{\sigma_1}(x_1) - \mu\sum_{\sigma_1}\int d^3x_1\, \psi^+_{\sigma_1}(x_1)\psi_{\sigma_1}(x_1)$$

$$+\frac{1}{2}\sum_{\sigma_1\sigma_2}\int d^3x_1 d^3x_2\, \psi^+_{\sigma_1}(x_1)\psi^+_{\sigma_2}(x_2)V(x_1-x_2)\psi_{\sigma_2}(x_2)\psi_{\sigma_1}(x_1)$$

$$[\psi^+_{\sigma_1}(x_1)\psi_{\sigma_1}(x_1), \psi_\sigma(x)] = -\delta_{\sigma_1\sigma}\delta(x_1-x)\psi_{\sigma_1}(x_1)$$

$$\boxed{\frac{\partial\psi_\sigma(x,\tau)}{\partial\tau} = -\frac{\nabla^2}{2m}\psi_\sigma(x,\tau) - \mu\psi_\sigma(x,\tau) - \int d^3x_2\sum_{\sigma_2} V(x-x_2)\psi^+_{\sigma_2}(x_2)\psi_{\sigma_2}(x_2)\psi_\sigma(x,\tau)}$$

$$[\psi^+\psi^+\psi\psi, \psi] = [\psi^+\psi^+, \psi]\psi\psi$$

$$[\psi^+_{\sigma_1}(x_1)\psi^+_{\sigma_2}(x_2), \psi_\sigma(x)]\psi_{\sigma_2}(x_2)\psi_{\sigma_1}(x_1)$$

$$= -\{\psi^+_{\sigma_1}(x_1), \psi_\sigma(x)\}\psi^+_{\sigma_2}(x_2)\psi_{\sigma_2}(x_2)\psi_{\sigma_1}(x_1)$$

$$+\psi^+_{\sigma_1}\{\psi^+_{\sigma_2}(x_2), \psi_\sigma(x)\}\psi_{\sigma_2}(x_2)\psi_{\sigma_1}(x_1)$$

$$= -\delta_{\sigma_1\sigma}\delta(x-x_1)\psi^+_{\sigma_2}(x_2)\psi_{\sigma_2}(x_2)\psi_{\sigma_1}(x_1)$$

$$+\delta_{\sigma_2\sigma}\delta(x-x_2)\psi^+_{\sigma_1}(x_1)\psi_{\sigma_2}(x_2)\psi_{\sigma_1}(x_1)$$

$$\boxed{\frac{\partial \psi(1)}{\partial \tau_1} = -\frac{\nabla^2}{2m}\psi(1) - \mu\psi(1) - V(1-\bar{2})\,\psi^{+}(\bar{2})\,\psi(\bar{2})\,\psi(1)}$$

$$V(1-2) = \frac{e^2}{4\pi\epsilon_0 |x_1 - x_2|}\,\delta(\tau_1 - \tau_2)$$

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32.3. Eqs. du mvt pour $\mathcal{G}$

$$\frac{\partial}{\partial\tau_1}\mathcal{G}(1,2)_\varphi = -\delta(1-2)+\left(-\frac{\nabla^2}{2m}-\mu\right)\mathcal{G}(1,2)_\varphi - \varphi(1,\bar{2})\mathcal{G}(\bar{2},2)$$

$$+V(1-\bar{2})\,\langle T_\tau \psi^+(\bar{2}^+)\,\psi(\bar{2})\,\psi(1)\,\psi^+(2)\rangle_\varphi$$

$$\left[-\frac{\partial}{\partial\tau_1}-\frac{\nabla^2}{2m}-\mu\right]\mathcal{G}(1,2)_\varphi - \varphi(1,\bar{2})\mathcal{G}(\bar{2},2)$$

$$= \delta(1-2)\boxed{-V(1-\bar{2})\langle T_\tau\, \psi^+(\bar{2}^+)\psi(\bar{2})\,\psi(1)\psi^+(2)\rangle_\varphi}$$

$$\boxed{\left[\mathcal{G}_0^{-1}(1,\bar{2})-\varphi(1,\bar{2})\right]\mathcal{G}(\bar{2},2)_\varphi = \delta(1-2)+\boxed{\Sigma(1,\bar{2})_\varphi\mathcal{G}(\bar{2},2)_\varphi}}$$

$$\mathcal{G}_0^{-1}(1,2)=\left[-\frac{\partial}{\partial\tau_1}-\frac{\nabla^2}{2m}-\mu\right]\delta(1-2)$$

$$\langle T_\tau S[\varphi]\, \psi(1)\, \psi^+(2) \rangle$$

$$= \langle T_\tau\, e^{-\int_{\tau_1}^{\beta} \psi^+(\bar{1})\varphi(\bar{1},\bar{2})\psi(\bar{2})}\, \psi(1)\, e^{-\int_0^{\tau_1} \psi^+(\bar{1})\varphi(\bar{1},\bar{2})\psi(\bar{2})}\, \psi^+(2) \rangle$$

$$S[\varphi] = e^{-\psi^+(\bar{1})\varphi(\bar{1},\bar{2})\psi(\bar{2})}$$

$$\frac{\partial}{\partial \tau_1} e^{-\int_0^{\tau_1} \psi^+(\bar{1})\varphi(\bar{1},\bar{2})\psi(\bar{2})}$$

$$= -\sum_{\sigma_1}\int d^3x_1\, \psi^+_{\sigma_1}(x_1,\tau_1)\, \varphi_{\sigma_1}(x_1,\tau_1;\bar{2})\psi(\bar{2})\, e^{-\int_0^{\tau_1} \psi^+(\bar{1})\varphi(\bar{1},\bar{2})\psi(\bar{2})}$$

$$\frac{\partial}{\partial y} e^{-\int_0^y dx\, f(x)} = e^{-\int_0^y dx\, f(x)} \left[\frac{\partial}{\partial y}\Big[-\int_0^y dx\, \underbrace{f(x)}_{F(y)}\Big] \right]$$

$$= e^{-\int_0^y dx\, f(x)} \left[-f(y) \right]$$

$$\frac{\partial}{\partial y}\left[\int_0^y dx\, x^2\right] = \frac{\partial}{\partial y} \left.\frac{x^3}{3}\right|_0^y = \frac{\partial}{\partial y}\frac{y^3}{3} = y^2$$

39.4 Dérivée fonctionelle pour fct. à 4 points :

$$\Sigma_\varphi(1,\bar{2})\,\mathcal{G}_\varphi(\bar{2},2) = -\mathcal{V}(1-\bar{2})\,\langle T_\tau\, \psi^\dagger(\bar{2}^+)\psi(\bar{2})\psi(1)\psi^\dagger(2)\rangle_\varphi$$

$$\mathcal{G}(1,2)_\varphi = -\frac{\langle T_\tau\, S[\varphi]\,\psi(1)\psi^\dagger(2)\rangle}{\langle T_\tau\, S[\varphi]\rangle}$$

$$\boxed{\frac{\delta}{\delta\varphi(3,4)}\, e^{-\psi^\dagger(\bar{1})\varphi(\bar{1}\,\bar{2})\psi(\bar{2})} = -\psi^\dagger(3)\psi(4)\,S[\varphi]}$$

$$\frac{\delta\mathcal{G}(1,2)_\varphi}{\delta\varphi(\bar{2}^+,\bar{2})} = \langle T_\tau\, \psi^\dagger(\bar{2}^+)\psi(\bar{2})\psi(1)\psi^\dagger(2)\rangle_\varphi$$

$$-\frac{1}{\langle T_\tau S[\varphi]\rangle^2}\langle T_\tau\, S[\varphi]\psi(1)\psi^\dagger(2)\rangle\langle T_\tau\, S[\varphi]\psi^\dagger(\bar{2}^+)\psi(\bar{2})\rangle$$

$$= \boxed{\langle T_\tau\, \psi^\dagger(\bar{2}^+)\psi(\bar{2})\psi(1)\psi^\dagger(2)\rangle} + \mathcal{G}(1,2)\,\mathcal{G}(\bar{2},\bar{2}^+)$$

$$\boxed{\Sigma(1,\bar{3})_\varphi\, \mathcal{G}(\bar{3},2)_\varphi = -V(1-\bar{3})\left[\frac{\delta \mathcal{G}(1,2)}{\delta\varphi(\bar{3}^+,\bar{3})} - \mathcal{G}(1,2)\,\mathcal{G}(\bar{3},\bar{3}^+)\right]}$$

$$\mathcal{G}_\varphi \mathcal{G}_\varphi^{-1} = 1 \; ; \quad \frac{\delta}{\delta\varphi}\left[\mathcal{G}_\varphi \mathcal{G}_\varphi^{-1}\right] = 0$$

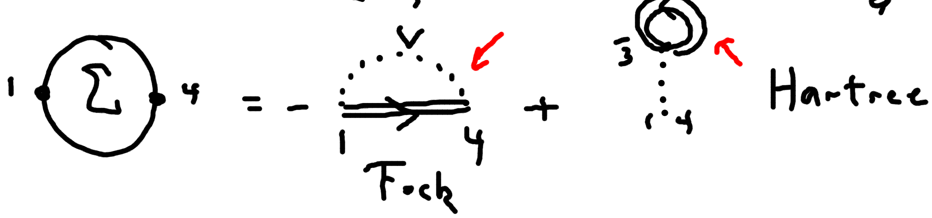
$$\boxed{\frac{\delta \mathcal{G}_\varphi}{\delta\varphi}\mathcal{G}_\varphi^{-1}} = -\mathcal{G}_\varphi \frac{\delta \mathcal{G}_\varphi^{-1}}{\delta\varphi} = -\mathcal{G}_\varphi \frac{\delta}{\delta\varphi}\left[\mathcal{G}_0^{-1} - \varphi - \Sigma\right] = \boxed{+\mathcal{G}_\varphi \frac{\delta\varphi}{\delta\varphi} + \mathcal{G}_\varphi \frac{\delta\Sigma}{\delta\varphi}}$$

$$\left(\Sigma\, \mathcal{G}(\bar{3},\bar{2})\right)\mathcal{G}^{-1}(\bar{2},4)$$

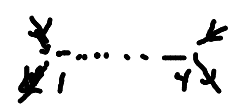
$$\Sigma(1,4) = -V(1-\bar{3})\left[\frac{\delta \mathcal{G}_\varphi(1,\bar{2})}{\delta\varphi(\bar{3}^+,\bar{3})}\mathcal{G}_\varphi^{-1}(\bar{2},4) - \mathcal{G}_\varphi(\bar{3},\bar{3}^+)\,\delta(4-1)\right]$$

$$= -V(1-\bar{3})\left[\mathcal{G}(1,\bar{2})\frac{\delta\varphi(\bar{2},4)}{\delta\varphi(\bar{3}^+,\bar{3})} - \mathcal{G}_\varphi(\bar{3},\bar{3}^+)\,\delta(4-1)\right]$$

$$= -V(1-4)\left[\mathcal{G}_\varphi(1,4)\right] + V(1-\bar{3})\,\mathcal{G}_\varphi(\bar{3},\bar{3}^+)\,\delta(4-1)$$



$\mathcal{G}(1,4) =$ 



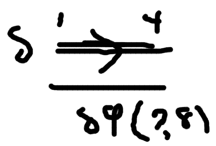
$= V(1-4)$

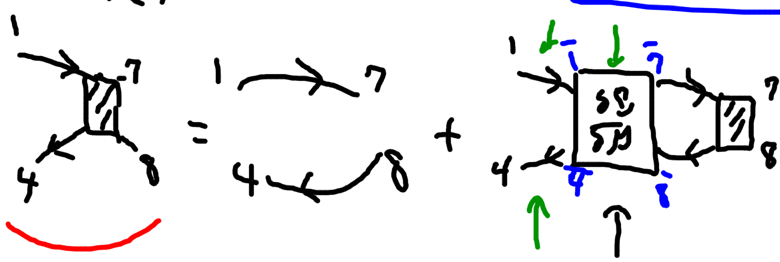
$$\frac{\delta \mathcal{G}}{\delta\varphi} = -\mathcal{G}\,\frac{\delta \mathcal{G}^{-1}}{\delta\varphi}\,\mathcal{G} = \mathcal{G}\,\frac{\delta\varphi}{\delta\varphi}\,\mathcal{G} + \mathcal{G}\,\frac{\delta\Sigma}{\delta\varphi}\,\mathcal{G}$$

$$\boxed{\frac{\delta\Sigma}{\delta\varphi} = \frac{\delta\Sigma}{\delta\mathcal{G}}\,\frac{\delta\mathcal{G}}{\delta\varphi}}$$

$$\Sigma_\varphi(1,4) = -V(1-4)\,\mathcal{G}_\varphi(1,4) + V(1-\bar{3})\,\mathcal{G}_\varphi(\bar{3},\bar{3}^+)\,\delta(1-4)$$

$$\boxed{\frac{\delta\Sigma_\varphi(1,4)}{\delta\mathcal{G}_\varphi(5,6)} = -V(1-4)\,\delta(5-1)\,\delta(6-4) + V(1-\bar{3})\,\delta(1-4)\,\delta(\bar{3}-5)\,\delta(\bar{3}-6)}$$





$$\boxed{\mathcal{G}(1,\bar{2})\,\frac{\delta\varphi(\bar{2},\bar{3})}{\delta\varphi(7,8)}\,\mathcal{G}(\bar{3},4)}$$

Vertex irréductible