32. Equations du mouvement pour $\mathcal{G}_\varphi$

✓ 1. Equs du mvt pour $\psi(1)$

✓ 2. " " " pour $\mathcal{G}_\varphi$ et Σ_φ

✓ 3. $\langle \psi^\dagger \psi^\dagger \psi \psi^\dagger \rangle$ de $\delta\mathcal{G}/\delta\varphi$

✓ 4. Σ_φ de $\delta\mathcal{G}/\delta\varphi$

33. 1ère étape : H.F. et R.P.A.

1. Dans l'espace-temps ✓ ✓

2. Dans l'espace de Fourier $(\varphi = 0)$

35. Modes collectifs et fonction diélectrique

1. Défs. + prolongement analytique
2. Fonction de Lindhard
    1. $T = 0$

32.2 Equs. du mvt pour $\mathcal{G}_\varphi$ et Σ_φ

$$\left(\mathcal{G}_0^{-1}(1,\bar{2}) - \varphi(1,\bar{2}) - \Sigma_\varphi(1,\bar{2})\right)\mathcal{G}_\varphi(\bar{2},2) = \delta(1-2)$$

$$\Sigma_\varphi(1,\bar{2})\mathcal{G}_\varphi(\bar{2},2) = -V(1-\bar{2})\langle T_\tau\, \psi^+(\bar{2})\psi(\bar{2})\psi(1)\psi^+(2)\rangle$$

$$\mathcal{G}_0^{-1}(1,2) = \left(-\frac{\partial}{\partial\tau_1} - \frac{\nabla_1^2}{2m} - \mu\right)\delta(1-2)$$

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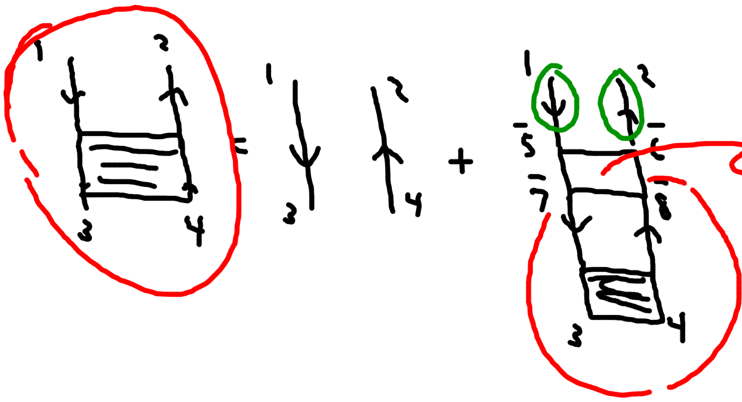
$$\mathcal{G}\mathcal{G}^{-1} = 1$$

$$\frac{\delta \mathcal{G}_\varphi(1,2)}{\delta \varphi(3,4)} = -\mathcal{G}\,\frac{\delta \mathcal{G}^{-1}}{\delta \varphi}\,\mathcal{G}$$

$$= +\mathcal{G}\,\frac{\delta \varphi}{\delta \varphi}\,\mathcal{G} + \mathcal{G}\,\frac{\delta \Sigma}{\delta \varphi}\,\mathcal{G}$$

$$\boxed{\frac{\delta \mathcal{G}_\varphi(1,2)}{\delta \varphi(3,4)}} = \mathcal{G}_\varphi(1,3)\,\mathcal{G}_\varphi(4,2) + \mathcal{G}(1,\bar{5})\,\boxed{\frac{\delta \Sigma(\bar{5},\bar{6})}{\delta \mathcal{G}(\bar{7},\bar{8})}}\,\mathcal{G}(\bar{6},2)\ \boxed{\frac{\delta \mathcal{G}(\bar{7},\bar{8})}{\delta \varphi(3,4)}}$$

$$\mathcal{G}(1,2) = 1 \longrightarrow 2$$



$$\frac{\delta \Sigma}{\delta \mathcal{G}}$$

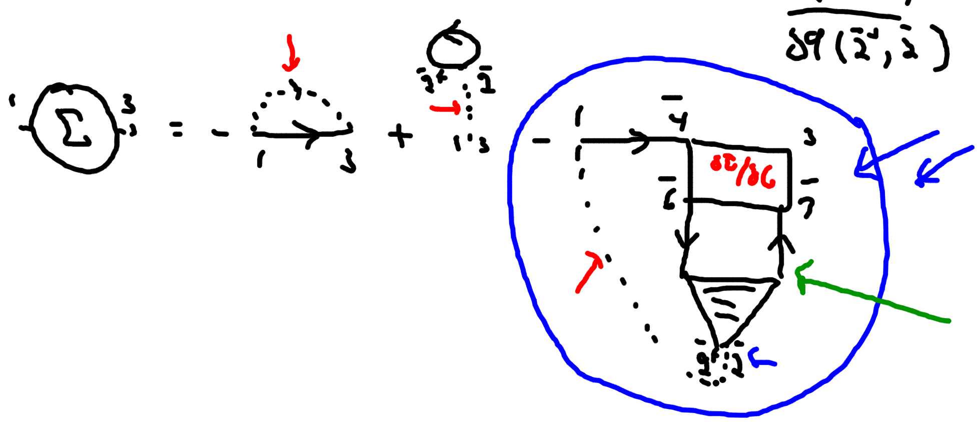
Vertex irréductible à 2 particules (canal p-t)

$$\Sigma_\varphi(1,\bar 2)\,\mathcal{G}(\bar 2,\bar 3) = -V(1-\bar 2)\left[\frac{\delta \mathcal{G}(1,\bar 3)}{\delta\varphi(\bar 2^+,\bar 2)} - \mathcal{G}(1,\bar 3)\,\mathcal{G}(\bar 2,\bar 2')\right]\mathcal{G}^{-1}(\bar 3,3)$$

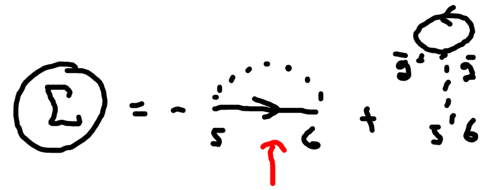
$$\frac{\delta \mathcal{G}}{\delta\varphi}\mathcal{G}^{-1} = -\mathcal{G}\frac{\delta \mathcal{G}^{-1}}{\delta\varphi} = \mathcal{G}\frac{\delta\varphi}{\delta\varphi} + \mathcal{G}\frac{\delta\Sigma}{\delta\varphi}$$

$$\Sigma(1,3) = -V(1-\bar 2)\left[\mathcal{G}(1,\bar 3)\,\delta(\bar 2-3) + \mathcal{G}(1,\bar 4)\frac{\delta\Sigma(\bar 4,3)}{\delta\varphi(\bar 2',\bar 2)} - \mathcal{G}(\bar 2,\bar 2')\,\delta(1-3)\right]$$

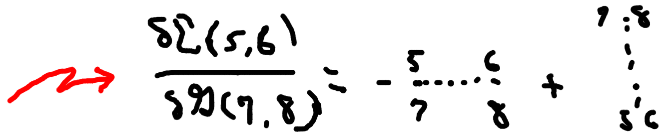
$$\Sigma_\varphi(1,3) = -V(1-3)\,\mathcal{G}(1,3) + V(1-\bar 2)\,\mathcal{G}(\bar 2,\bar 2')\,\delta(1-3) - V(1-\bar 2)\,\mathcal{G}(1,\bar 4)\frac{\delta\Sigma(\bar 4,3)}{\delta\varphi(\bar 2',\bar 2)}$$

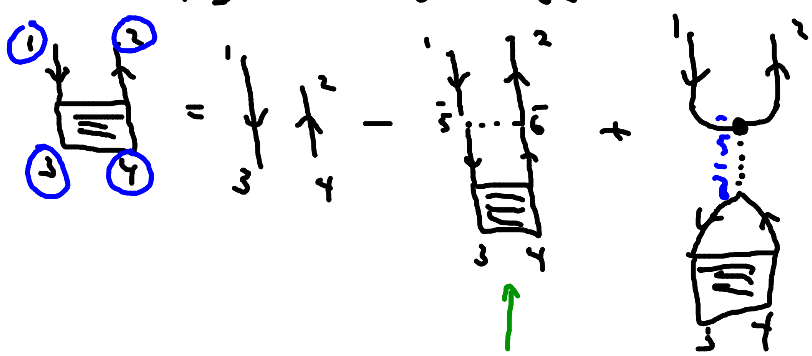


33. H.F. et RPA

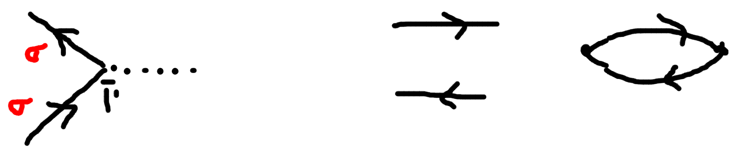


$$\frac{\delta \mathcal{G}(5,6)}{\delta \mathcal{G}(7,8)} = \delta(5-7)\,\delta(6-8)$$





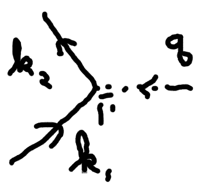
33.2 $q=0$, Transformées de Fourier :



Vertex

$$\mathcal{G}(\vec{k}, ik_n) = \int d^3(x_1-x_2)\int_0^\beta d(\tau_1-\tau_2)\, e^{-ik\cdot(x_1-x_2)+ik_n(\tau_1-\tau_2)}\, \mathcal{G}_\sigma(x_1\tau_1; x_2\tau_2)$$

$$\mathcal{G}_\sigma(x_1\tau_1; x_2\tau_2) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{\beta}\sum_n e^{ik\cdot(x_1-x_2)-ik_n(\tau_1-\tau_2)}\, \mathcal{G}_\sigma(\vec{k}, ik_n)$$

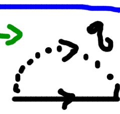


$$\int d^3x_1' \int_0^\beta d\tau_1' \; e^{-i\vec{k}_1\cdot\vec{x}_1' + i\vec{k}_2\cdot\vec{x}_1' - i\vec{q}\cdot\vec{x}_1'}$$

$$e^{i k_{1n}\tau_1' - i k_{2n}\tau_1' + i q_n \tau_1'}$$

$$= (2\pi)^3 \delta(\vec{k}_1 - \vec{k}_2 + \vec{q})\, \beta\, \delta_{(k_{1n}-k_{2n}),\, -q_n}$$

$k = (\vec{k}, ik_n)$

$\Sigma_\sigma(k) = -$  $\mathcal{G}_\sigma(k-q)$ $+$ 

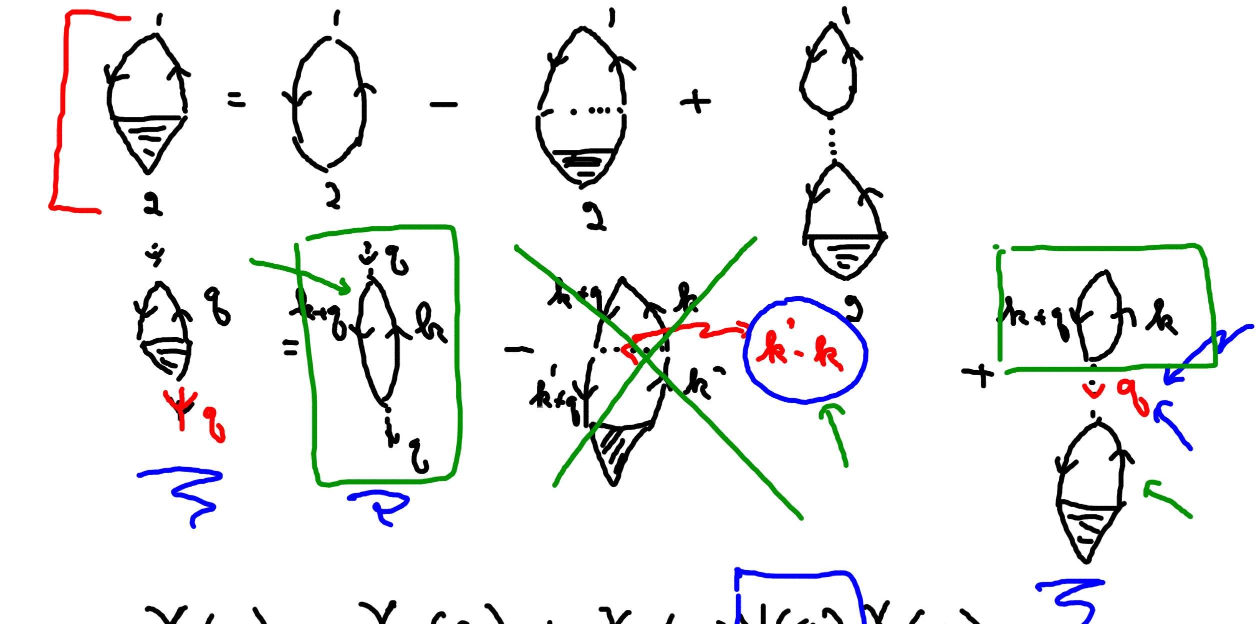
$q=0$

$\boxed{V(1-2) = v(\vec{r}_1 - \vec{r}_2)\,\delta(\tau_1 - \tau_2)}$

 $\Rightarrow 2 \sum_{k} T \sum_{ik_n} \mathcal{G}(k)\, e^{-ik_n 0^-}$

$\mathcal{G}(\bar{2}, \bar{2}^+)$

$\mathcal{G}(\bar{2}^-, \bar{2})$



$$\underline{\chi(q)} = \chi_0(q) + \chi_0(q)\boxed{V(q)}\underline{\chi(q)}$$

$$\left[1-\chi_0 V\right]\chi(q) = \chi_0 \qquad \chi = \frac{\chi_0}{1-V\chi_0} = \frac{1}{\chi_0^{-1}-V}$$

35.1 Défs. + prolongement.

$$\frac{\delta \mathcal{G}(1,2)}{\delta \varphi(3^+,3)} = \langle T_\tau \underbrace{\psi^+(3^+)\psi(3)} \underbrace{\psi(1)\psi^+(2)} \rangle + \underbrace{\mathcal{G}(3,3^+)} \underbrace{\mathcal{G}(1,2)}$$

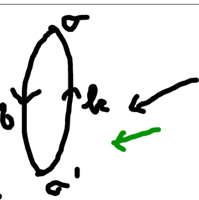
$2 \to 1^+$

$$=$$

$$\langle T_\tau \psi^+(3^+)\psi(3)\psi(1)\psi^+(1^+)\rangle = -\underbrace{\langle T_\tau \psi^+(3^+)\psi(3)\psi^+(1^+)\psi(1)\rangle}_{n(3) \quad n(1)}$$

χ^R

$$\langle T_\tau (n(1) - \langle n \rangle)(n(2) - \langle n \rangle) \rangle = -\frac{\delta \mathcal{G}(1,1^+)}{\delta \phi(2,2^+)} =$$



T.F.

$$\boxed{-2\sum_{k} T \sum_{k_n} \mathcal{G}^0(k, ik_n)\,\mathcal{G}^0(k+q, ik_n+iq_n) = \chi_0(q, iq_n)}$$

$$\boxed{\chi^R(q,\omega) = \chi_0(q, iq_n \to \omega + i\eta)}$$

$\downarrow$

$\langle [n_q, n_{-q}] \rangle$

$n(1) \to n(1) + cte$

$$\left\langle T_\tau \left(n(1) - \langle n \rangle\right)\left(n(2) - \langle n \rangle\right) \right\rangle$$

$$= \left\langle T_\tau\, n(1)\, n(2) \right\rangle - \langle n \rangle \langle n \rangle = -\frac{\delta \mathcal{G}}{\delta \varphi} =$$ 

35.2 Fct. de Lindhard

$$\chi_0(q, iq_n) = -\frac{2}{V}\sum_k T\sum_{ik_n} \mathcal{G}^0(k, ik_n)\,\mathcal{G}^0(k+q, ik_n+iq_n)$$

$$= -2\int\frac{d^3k}{(2\pi)^3}\, T\sum_{ik_n}\frac{1}{ik_n-\xi_k}\;\frac{1}{ik_n+iq_n-\xi_{k+q}}$$

$$= -2\int\frac{d^3k}{(2\pi)^3}\, T\sum_{ik_n}\left[\frac{1}{ik_n-\xi_k}-\frac{1}{ik_n+iq_n-\xi_{k+q}}\right]\frac{1}{iq_n+\xi_k-\xi_{k+q}}$$

$$\chi^R(q,\omega) = -2\int\frac{d^3k}{(2\pi)^3}\,\frac{f(\xi_k)-f(\xi_{k+q})}{\omega+i\eta-\xi_{k+q}+\xi_k}$$

$$\boxed{\operatorname{Im}\chi^R_0(q,\omega) = 2\pi\int\frac{d^3k}{(2\pi)^3}\left(f(\xi_k)-f(\xi_{k+q})\right)\delta(\omega-\xi_{k+q}+\xi_k)}$$

$\boxed{T=0}$

$$2\pi\int\frac{d^3k}{(2\pi)^3}\, f(\xi_k)\left[\delta(\omega-\xi_{k+q}+\xi_k)-\delta(\omega-\xi_k+\xi_{k-q})\right]$$

$$= \frac{1}{2\pi}\int_0^{k_F} k^2\,dk\int_{-1}^{1} d(\cos\theta)\left[\delta\left(\omega-\frac{q^2}{2m}-\frac{kq}{m}\cos\theta\right)-\delta\left(\omega-\frac{q^2}{2m}+\frac{kq}{m}\cos\theta\right)\right]$$

$$\xi_{k+q}-\xi_k = \frac{(k+q)^2}{2m}-\frac{k^2}{2m}$$

$$= \frac{q^2}{2m}+\frac{kq\cos\theta}{m}$$

Doing the replacement $f(\zeta_{\mathbf{k}}) = \theta(k_F - k)$, going to polar coordinates with $\mathbf{q}$ along the polar axis and doing the replacement $\varepsilon_{\mathbf{k}} = k^2/2m$, we have

$$\operatorname{Im}\chi^{0R}_{nn}(\mathbf{q},\omega) = \frac{1}{2\pi}\int_0^{k_F} k^2 dk \int_{-1}^{1} d(\cos\theta)\,\frac{m}{kq}\left[\delta\left(\frac{\omega-\varepsilon_{\mathbf{q}}}{kq/m}-\cos\theta\right)-\delta\left(\frac{\omega+\varepsilon_{\mathbf{q}}}{kq/m}-\cos\theta\right)\right] \tag{35.20}$$

It is clear that this strategy in fact allows one to do the integrals in any spatial dimension. One finds, for an arbitrary ellipsoidal dispersion [12]

$$\varepsilon_{\mathbf{k}} = \sum_{i=1}^{d}\frac{k_i^2}{2m_i} \tag{35.21}$$

$$\operatorname{Im}\chi^{0R}_{nn}(\mathbf{q},\omega) = \frac{\prod_{i=1}^{d}\left(\sqrt{2m_i}\right)}{2^d\pi^{(d-1)/2}\Gamma\left(\frac{d+1}{2}\right)\sqrt{\varepsilon_{\mathbf{q}}}}\times$$

$$\left\{\theta\left(\mu-\frac{(\omega-\varepsilon_{\mathbf{q}})^2}{4\varepsilon_{\mathbf{q}}}\right)\left[\mu-\frac{(\omega-\varepsilon_{\mathbf{q}})^2}{4\varepsilon_{\mathbf{q}}}\right]^{\frac{d-1}{2}}-\theta\left(\mu-\frac{(\omega+\varepsilon_{\mathbf{q}})^2}{4\varepsilon_{\mathbf{q}}}\right)\left[\mu-\frac{(\omega+\varepsilon_{\mathbf{q}})^2}{4\varepsilon_{\mathbf{q}}}\right]^{\frac{d-1}{2}}\right\}$$

The real part is also calculable [12] but we do not quote it here.

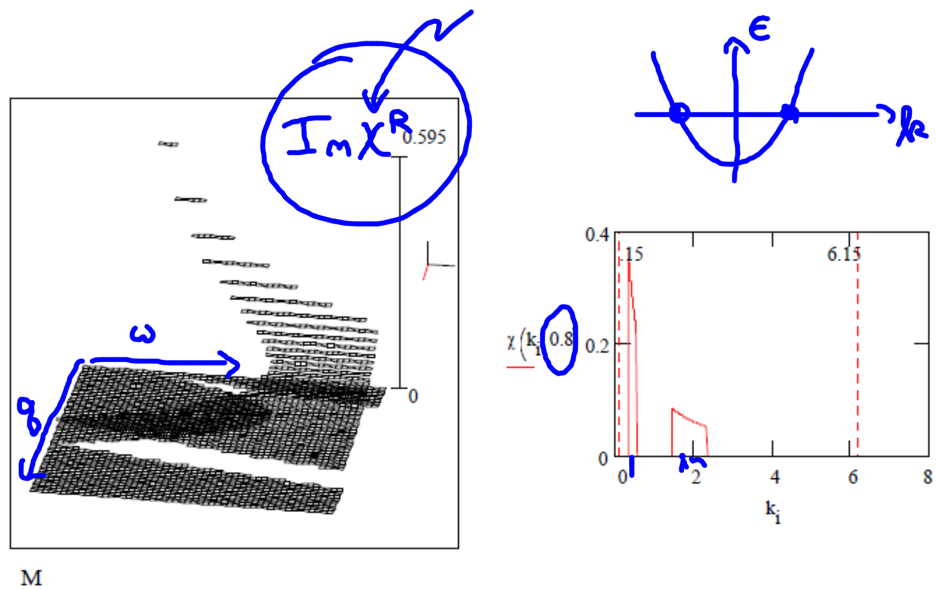


Figure 35-2 Imaginary part of the Lindhard function in $d = 1$ on the vertical axis. Frequency increases from left to right and wave vector from back to front.

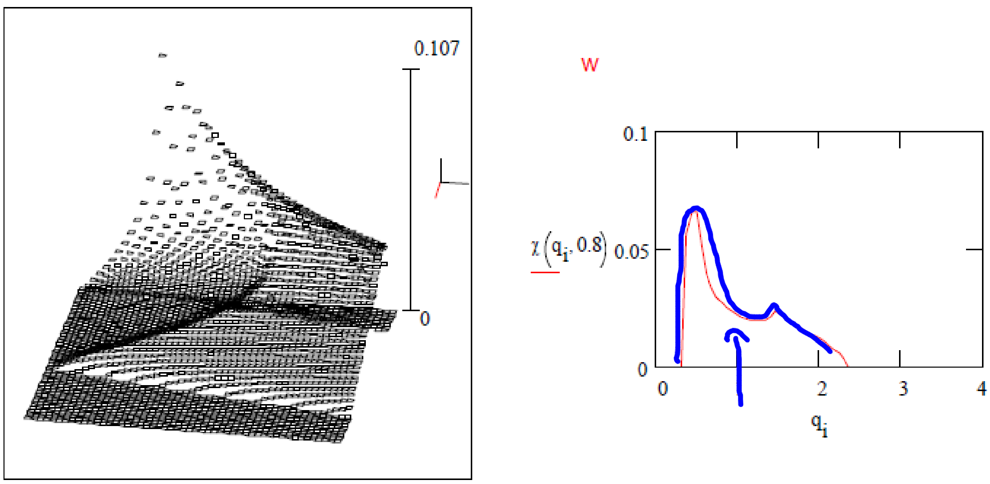


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Figure 35-3 Imaginary part of the Lindhard function in $d = 2$. Axes like in the $d = 1$ case.

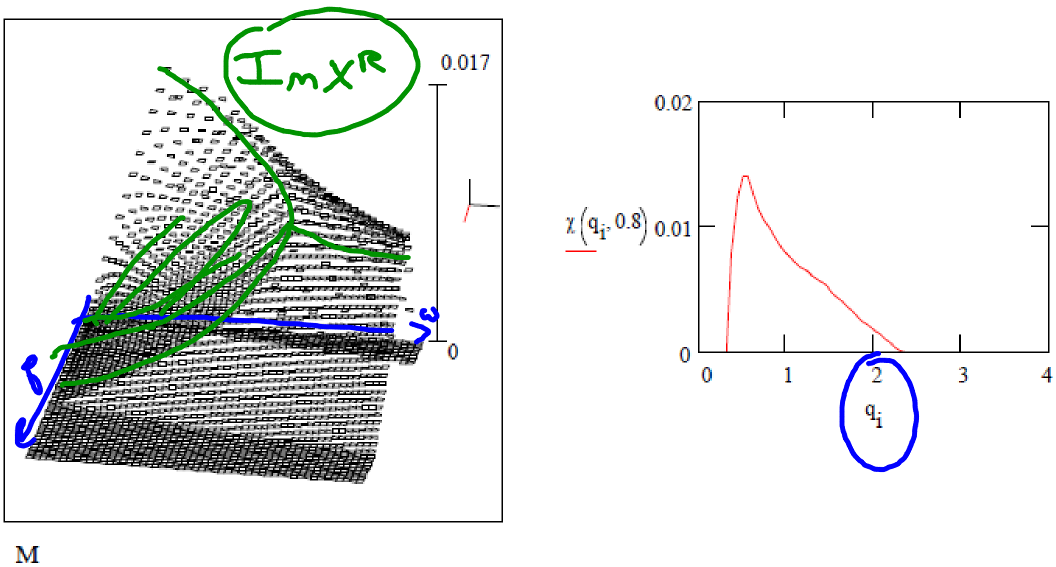


Figure 35-4 Imaginary part of the Lindhard function in $d = 3$. Axes like in the $d = 1$ case.

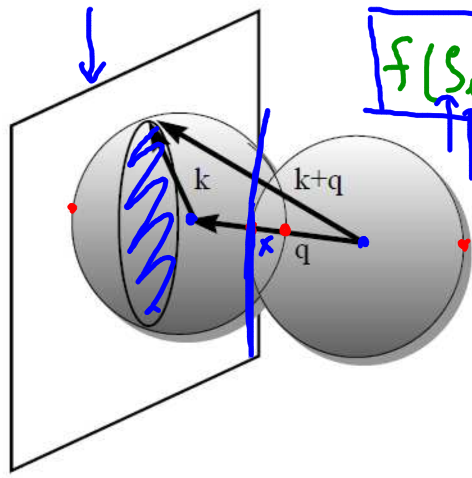
mars 1-09:41

$\delta(\omega - \xi_{k+q} + \xi_k)$

$$\omega - \frac{(\vec{k}+\vec{q})^2}{2m} + \frac{\vec{k}^2}{2m} = 0$$

$$\omega - \frac{\vec{k}\cdot\vec{q}}{m} - \frac{q^2}{2m} = 0$$

$$\frac{\vec{k}\cdot\vec{q}}{m} = \omega - \frac{q^2}{2m}$$



$\boxed{f(\xi_k) - f(\xi_{k+q})}$

Figure 35-5 Geometry for the integral giving the imaginary part of the $d = 3$ Lindhard function. The wave vectors in the plane satisfy energy conservation as well as the restrictions imposed by the Pauli principle. The plane located symmetrically with respect to the miror plane of the spheres corresponds to energies of opposite sign.

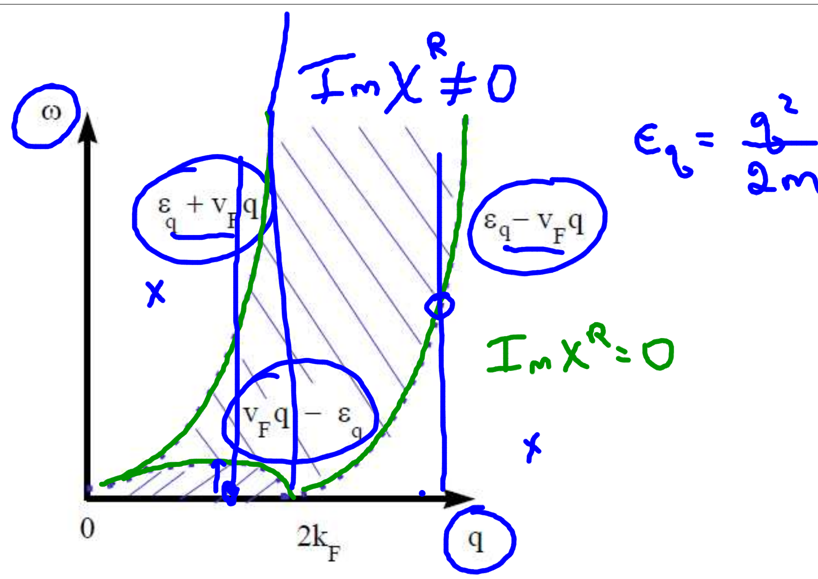


Figure 35-6 Schematic representation of the domain of frequency and wave vector where there is a particle-hole continuum.

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