35.3.1 Fonction de Lindhard, continuum p-t ✓

36. Les interactions, simplement ✓

1. Paramètre de développement
2. Écrantage Thomas-Fermi
3. Oscillations plasma

37. Réponse de la densité en présence d'interactions

1. RPA
2. Cte diélectrique + cas particuliers

37.2 ϵ + cas particuliers

1. Continuum p-t
2. Écrantage
3. Oscillations de Friedel
4. Plasmons
5. Règle de somme-f

36.1 Paramètre de développement

$$\frac{\hbar^2}{m a_0^2} = \frac{e^2}{4\pi\epsilon_0 a_0} \qquad \frac{1}{a_0} = \frac{me^2}{4\pi\epsilon_0 \hbar^2} = \left[0.5\,\mathring{A}\right]^{-1}$$

r_s

$$n_0 = \frac{1}{\frac{4\pi}{3}\left(r_s a_0\right)^3} = \frac{k_F^3}{3\pi^2} \qquad n = 2\int_0^{k_F} \frac{d^3k}{(2\pi)^3} = \frac{2}{(2\pi)^3}\,\frac{4\pi}{3}\,k_F^3 = \frac{k_F^3}{3\pi^2}$$

$$\boxed{r_s^3 = \frac{9\pi}{4}\,\frac{1}{k_F^3 a_0^3}}$$

Gaz d'électron

$$\frac{\text{E, Potentielle}}{\text{E, cinétique}} = \frac{\frac{e^2}{4\pi\epsilon_0}\,k_F}{\frac{k_F^2}{2m}} = \frac{1}{k_F a_0} \sim r_s$$

36.2 T.F. (écrantage)

ρ_i

$$-\nabla^2\phi = \frac{\rho_i + \delta\rho}{\epsilon_0}$$

$$n(r) = \frac{k_F^3(r)}{3\pi^2}$$

$$\delta\rho(r) = -e\left[n(r) - n_0\right] = -e\frac{\partial n}{\partial\mu}e\phi$$

$$\frac{k_F^2(r)}{2m} - e\phi(r) = \mu = \frac{k_F^2}{2m}$$

$$-\nabla^2\phi = \frac{1}{\epsilon_0}\left[\rho_i - e^2\frac{\partial n}{\partial\mu}\phi(r)\right]$$

$$\frac{k_F^2(r)}{2m} = \mu + e\phi(r) \quad \delta\mu$$

$$-\nabla^2\phi + \frac{e^2\partial n}{\epsilon_0\partial\mu}\phi = \frac{1}{\epsilon_0}\rho_i$$

$$\left(q^2 + q_{TF}^2\right)\phi = \frac{1}{\epsilon_0}\rho_i$$

$$q_{TF}^2 = \frac{e^2\partial n}{\epsilon_0\partial\mu}$$

$$\phi(q) = \frac{1}{q^2 + q_{TF}^2}\frac{1}{\epsilon_0}\rho_i \longrightarrow \phi(r) \sim \frac{1}{r}e^{-r/\lambda_{TF}}$$

$$\lambda_{TF} = \frac{1}{q_{TF}}$$

$$\lambda_{TF}^2 = \frac{1}{q_{TF}^2} \qquad q_{TF}^2 = \frac{e^2}{\epsilon_0}\frac{\partial n}{\partial\mu}$$

$$n \propto \frac{1}{\frac{4\pi}{3}(r_s a_0)^3}$$

$$\propto \frac{e^2 m}{\epsilon_0}\frac{k_F^3}{k_F^2} \cong \frac{k_F}{a_0}$$

$$n = \frac{k_F^3}{3\pi^2}$$

$$q_{TF}^2 \propto k_F^2 r_s$$

$$q_{TF} \propto k_F\sqrt{r_s} \qquad \lambda_{TF} \sim \frac{1}{k_F}\frac{1}{\sqrt{r_s}}$$

36.3 Oscillation plasma

$$\vec{j} = -en\vec{v} \qquad \frac{\partial j}{\partial t} = -en\frac{\partial \vec{v}}{\partial t} = -en\frac{(-eE)}{m}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0 \qquad \boxed{\frac{\partial \nabla \cdot j}{\partial t} = \frac{ne^2}{m}\nabla \cdot E} \qquad -\frac{\partial^2 \rho}{\partial t^2} = \frac{ne^2}{m}\left[\frac{\rho}{\epsilon_0}\right]$$

$$\boxed{\omega_p^2 = \frac{ne^2}{\epsilon_0 m}}$$

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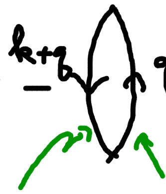
37. Fluctuations de densité en présence d'interactions

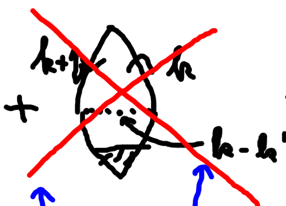
37.1 RPA

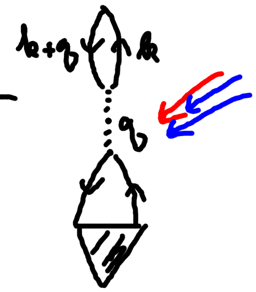
$$\frac{\delta \mathcal{G}(1,2)}{\delta \phi(3,4)} = \langle T_\tau \psi^+(3)\psi(4)\psi(1)\psi^+(2)\rangle + \mathcal{G}(1,2)\,\mathcal{G}(4,3)$$

$$-\sum_{\sigma_1\sigma_2} \frac{\delta \mathcal{G}(1,1^+)}{\delta \phi(2^+,2)} = \sum_{\sigma_1\sigma_2} \langle T\, \psi^+(2^+)\psi(2)\, \psi^+(1^+)\psi(1)\rangle$$

$$- \sum_{\sigma_1\sigma_2} \mathcal{G}_{\sigma_1}(1,1^+)\,\mathcal{G}_{\sigma_2}(2,2^+)$$

$-\sum_{\sigma_1\sigma_2} \frac{\delta \mathcal{G}}{\delta \varphi} = -$ 

$+$ 

$-$ 

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RPA

$$\chi_{nn}(q) = \chi^0_{nn}(q) - V(q)\,\chi^0_{nn}(q)\,\chi_{nn}(q)$$

$$\frac{\delta\Sigma}{\delta\mathcal{G}}$$

$\Sigma = -$  $+$ 

$$\boxed{\chi_{nn}(q) = \frac{\chi^0_{nn}(q)}{1+V_q\,\chi^0_{nn}(q)}}$$

$$\chi_{nn}(q) = \frac{1}{\chi_0^{-1}(q)+V_q} \qquad \mathcal{G} = \frac{1}{\mathcal{G}_0^{-1}-\Sigma}$$

Relation entre constante diélectrique longitudinale et χ

$$\nabla \cdot E(r) = \int d^3r' \frac{1}{\epsilon_L(r-r')} \rho(r')$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0}(\rho_e + \delta\rho) \qquad \frac{1}{\epsilon_L}\rho_e = \frac{1}{\epsilon_0}(\rho_e + \delta\rho)$$

$$q \cdot \underbrace{\epsilon_L \vec{E}}_{\vec{D}} = \rho_e \qquad \frac{1}{\epsilon_L} = \frac{1}{\epsilon_0}\left(\frac{\rho_e + \delta\rho}{\rho_e}\right)$$

$$\boxed{\delta\rho = -\chi_{\rho\rho}\,\phi_e \quad \Big| \quad \phi_e = \frac{1}{\epsilon_0}\frac{\rho_e}{q^2}}$$

$$\frac{1}{\epsilon_L} = \frac{1}{\epsilon_0}\left(1 - \frac{\chi_{\rho\rho}}{\epsilon_0 q^2}\right) = \frac{1}{\epsilon_0}\left(1 - \frac{e^2}{\epsilon_0 q^2}\chi_{nn}(q,\omega)\right)$$

$$1 - V_q \chi_{nn}(q,\omega) = 1 - \frac{V_q \chi^0_{nn}(q,\omega)}{1 + V_q \chi^0_{nn}(q,\omega)} = \frac{1}{1 + V_q \chi^0_{nn}(q,\omega)}$$

$$\boxed{\epsilon_L(q,\omega) = \epsilon_0\left(1 + V_q \chi^0_{nn}(q,\omega)\right)}$$