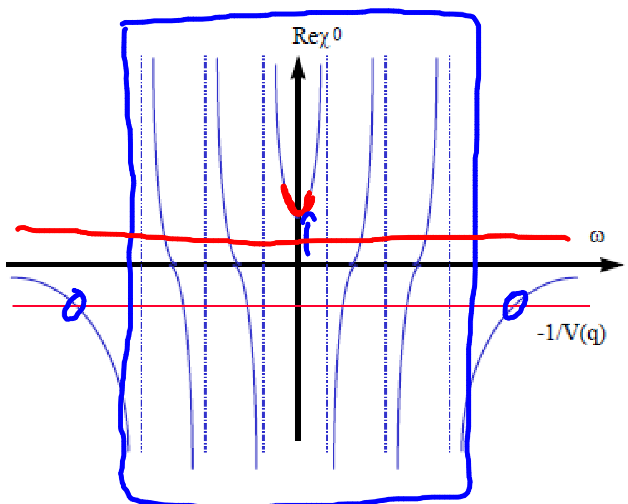

Figure 37-4 Graphical solution for the poles of the charge susceptibility in the interacting system.

$$\sum_i \frac{A_i}{u_i^2 - (\omega + i\eta)^2}$$

$\eta = 0$

$$\boxed{1 + V_q \, \mathrm{Re}\,\chi^0 = 0}$$

$$\mathrm{Re}\,\chi^0 = -\frac{1}{V_q}$$

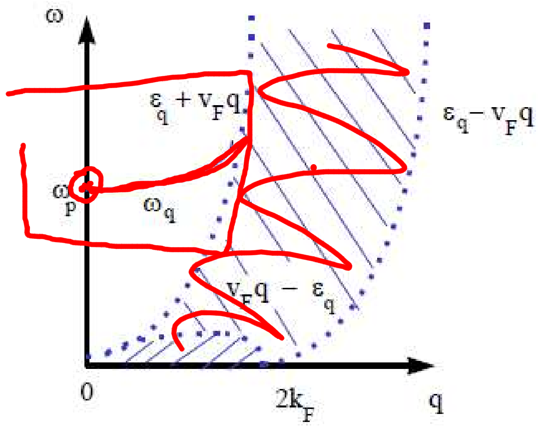

Figure 37-5 Schematic representation of the domain of frequency and wave vector where there are poles in the charge susceptibility, or zeros in the longitudinal dielectric function. In addition to the particle-hole continuum, there is a plasma pole.

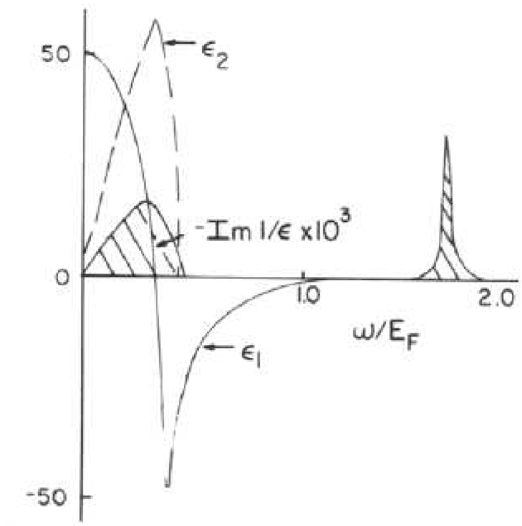

Figure 37-6 Real and imaginary parts of the dielectric constant and $\mathrm{Im}\,(1/\varepsilon)$ as a function of frequency, calculated for $r_s = 3$ and $q = 0.2k_F$. Shaded plots correspond to $\mathrm{Im}\,(1/\varepsilon)$. Taken from Mahan *op. cit.* p.430

37. Fluctuations de densité dans l'approx. RPA

14.2 Cte diélectrique longitudinale ✓

37.2 Forme explicite pour ϵ et cas particuliers

✓ 1. Continuum p.t

✓ 2. Ecrantage

3. Oscillations de Friedel

4. Plasmons

5. Règle de somme f

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✓ 38. Relation entre Σ, $\langle v \rangle$ et F

38.1.2 Cohérence entre self et densité

38.2 Energie libre

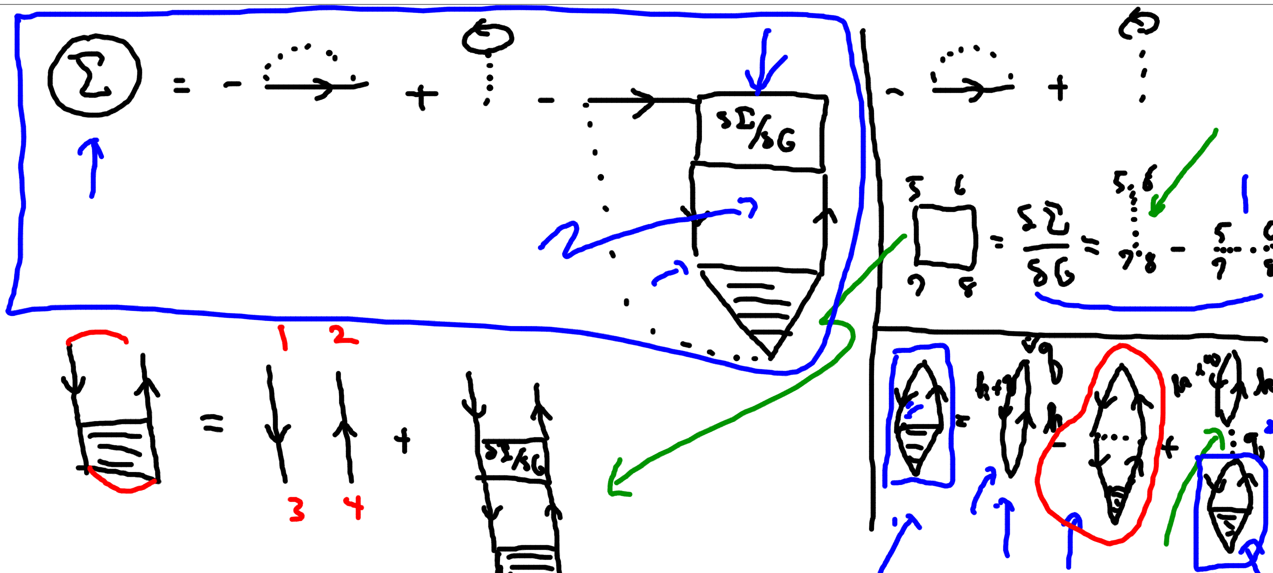
39. Σ une particule et Hartree-Fock ✓

→ 30.3.2 Principe variationnel

39.1 H.F.-Variationnel

39.2 H.F. fct de Green

39.3 Pathologies

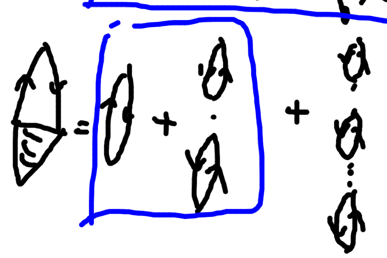


$$\chi = \chi_0 - V\chi_0\,\chi$$

$$\boxed{\chi^{(1)} = \chi_0 - V\chi_0^2}$$

$$\chi^{(2)} = \chi_0 - V\chi_0^2 + V^2\chi_0^3 + \dots$$

$$\chi_{nn} = \frac{\chi^0_{nn}}{1 + V\chi^0_{nn}}$$

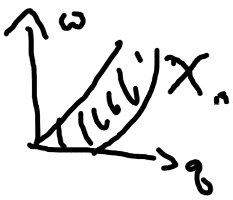


$\chi_0 = -$ 

$$\boxed{1 - x + x^2 - x^3 \dots = \frac{1}{1+x}}$$

$$-\chi = -\chi_0 + V\chi_0\chi$$

37.2 Cas particuliers



$$\chi_{nn} = \frac{\chi^0_{nn}}{1+V\chi^0_{nn}}$$

$$\boxed{\operatorname{Im}\chi_{nn} = \frac{\operatorname{Im}\chi^0_{nn}}{\left(1+V_q \operatorname{Re}\chi^0_{nn}\right)^2 + \left(V_q \operatorname{Im}\chi^0_{nn}\right)^2}}$$

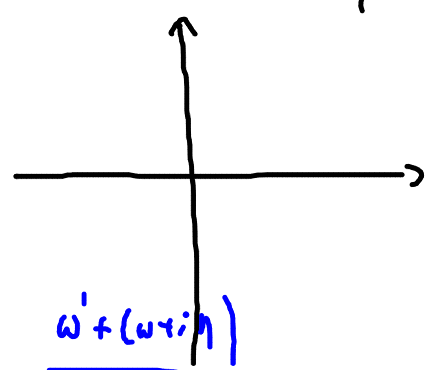
$$\chi^0_{nn} = x + iy$$

$$\chi = \frac{x+iy}{1+V(x+iy)} = \frac{x+iy}{1+Vx+Viy} = \frac{(x+iy)(1+Vx-iVy)}{(1+Vx)^2+V^2y^2}$$

$$\operatorname{Im}\chi = \frac{y(1+\cancel{Vx}) - \cancel{Vxy}}{(1+Vx)^2+V^2y^2}$$

$$\chi^0_{nn}(q,\omega) = \int \frac{d\omega'}{\pi} \frac{\chi^{0''}_{nn}(\omega')}{\omega' - (\omega + i\eta)} = \int \frac{d\omega'}{\pi} \frac{\omega' \chi^{0''}_{nn}(q,\omega')}{\omega'^2 - (\omega + i\eta)^2}$$

$$= \sum_i \frac{A_i}{\omega_i^2 - (\omega + i\eta)^2}$$



$$\frac{\omega' + (\omega + i\eta)}{\omega' + (\omega + i\eta)}$$

37.2 écrantage

$$1+V_q\chi^0_{nn}(q,\omega) = \frac{\epsilon_L(q,\omega)}{\epsilon_0}$$

$$\lim_{\omega\to 0}\frac{\epsilon_L(q,\omega)}{\epsilon_0} = 1+V_q\left[-2\sum_k \frac{f(\xi_k)-f(\xi_{k+q})}{\xi_k-\xi_{k+q}}\right]$$

pour q petit

$$= 1+V_q\left[\frac{\partial}{\partial\mu}\left(+2\sum_k f(\xi_k)\right)\right]$$

$$= 1+V_q\frac{\partial n}{\partial\mu} = 1+\frac{e^2}{\epsilon_0 q^2}\frac{\partial n}{\partial\mu}$$

$$= 1+\frac{q_{TF}^2}{q^2}$$ où $\boxed{q_{TF}^2 = \frac{e^2}{\epsilon_0}\frac{\partial n}{\partial\mu}}$

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3? 3 Oscillations de Friedel

$$V(r) \propto \frac{\cos 2k_F r}{r^3} \longrightarrow \text{RKKY}$$

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37.4 Plasmon:

$$1+V_q\chi^0_{nn}(q,\omega) = 1+V_q\left[-2\sum_k \frac{f(\xi_k)-f(\xi_{k+q})}{\omega+\xi_k-\xi_{k+q}}\right]$$

$$= 1+V_q\left[-2\sum_k \left(f(\xi_k)-f(\xi_{k+q})\right)\left[\frac{1}{\omega}-\frac{\xi_k-\xi_{k+q}}{\omega^2}+\cdots\right]\right]$$

1 2

$$= 1+2V_q\left[2\sum_k \frac{f(\xi_k)(\xi_k-\xi_{k+q})}{\omega^2}\right].$$

$$+f(\xi_{-k})\left[-\xi_{-k-q}+\xi\right.$$

$$= 1+4V_q\sum_k \frac{f(\xi_k)}{\omega^2}\left(-\frac{q^2}{2m}\right)$$

$$k \to -k-q$$

$$= 1-\frac{2nV_q}{\omega^2}\frac{q^2}{2m} = 1-\frac{e^2}{\epsilon_0 q^2}\frac{n q^2}{\omega^2} = 1-\frac{ne^2}{\epsilon_0\omega^2}$$

$$\frac{\epsilon_c(q,\omega)}{\epsilon_0} = 1-\frac{\omega_p^2}{\omega^2}-\frac{3}{5}\frac{(v_F q)^2}{\omega^2}\frac{\omega_p^2}{\omega^2}+\cdots \qquad = 1-\frac{\omega_p^2}{\omega^2}$$

$$\xi_k - \xi_{k+q} = \frac{k^2}{2m} - \frac{(k+q)^2}{2m} = -\frac{k\cdot q}{m} - \frac{q^2}{2m}$$

37.5. Règle de somme f.

$$\int \frac{d\omega}{\pi}\, \omega\, \chi''_{nn}(q,\omega) = \frac{nq^2}{m}$$

$$\chi = \frac{\chi^0}{1+V_q\chi_0} = \frac{\chi^0}{\frac{\epsilon_L}{\epsilon_0}}$$

$$\epsilon_L(q,\omega) = \epsilon_0\left[1 + V_q\, \chi^0_{nn}(q,\omega)\right]$$

$$\operatorname{Im}\chi = \operatorname{Im}\left[\frac{\chi^0}{\frac{\epsilon_L}{\epsilon_0}}\right]$$

$$\operatorname{Im}\left(\frac{\epsilon_0}{\epsilon_L}\right) = \operatorname{Im}\left[\frac{1}{1+V_q\chi^0_{nn}(q,\omega)}\right]$$

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38 Relation $\Sigma, \langle V \rangle, F$.

38.1.2

$$\Sigma(1,\bar{1})\mathcal{G}(\bar{1},3) = -V(1-\bar{2})\langle T_\tau \psi^\dagger(\bar{2}^+)\psi(\bar{2})\psi(1)\psi^\dagger(3)\rangle$$

$3 = 1^+$

$$\Sigma(\bar{1},\bar{2})\mathcal{G}(\bar{2},\bar{1}^+) = +V(\bar{1}-\bar{2})\langle T_\tau \psi^\dagger(\bar{2}^+)\psi(\bar{2})\psi^\dagger(\bar{1}^+)\psi(\bar{1})\rangle$$

38.2 Énergie libre

$$-T\ln Z_\lambda = -T\ln \mathrm{Tr}\left[e^{-\beta(H_0+\lambda V-\mu N)}\right]$$

$$-T\frac{\partial}{\partial\lambda}\ln Z_\lambda = -T\,\frac{1}{Z}\,\mathrm{Tr}\left[e^{-\beta(H_0+\lambda V-\mu N)}V\right](-\beta)$$

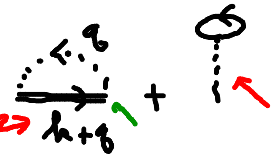
$$= \langle V\rangle_\lambda$$

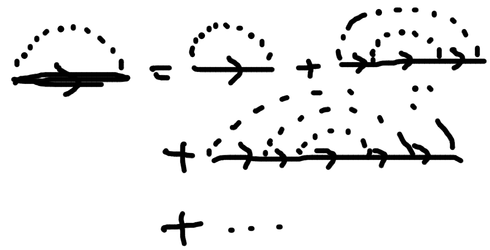
$$-T\frac{\partial}{\partial\lambda}\ln Z_\lambda = \frac{1}{\lambda}\langle\lambda V\rangle_\lambda$$

Pauli

$$-T\ln Z_{\lambda=1} = -T\ln Z_{\lambda=0} + \int_0^1 d\lambda\,\frac{1}{\lambda}\langle\lambda V\rangle_\lambda$$

39. Hartree-Fock:

$\Sigma(k) = -$ 



$$\Sigma^{(1)} = -\int \frac{d^3q}{(2\pi)^3} \sum_{iq_n} V_q \frac{e^{-iq_n 0^-}}{ik_n + iq_n - (\epsilon_{k+q} - \mu) - \Sigma^{(1)}}$$

Principe variationnel (Feynmann)

$$\boxed{F \leq F_0 + \langle H - H_0 \rangle_0}$$

$$H = H_0 + (H - H_0)$$

Dérivation:

$$\boxed{-\mathrm{Tr}[\rho \ln \rho] \leq -\mathrm{Tr}[\rho \ln \rho']}$$

$$\rho|m\rangle = p_m|m\rangle$$

$$\rho'|m'\rangle = p'_{m'}|m'\rangle$$

$$-\mathrm{Tr}[\rho \ln \rho - \rho \ln \rho'] = -\sum_m p_m \ln p_m$$

$$+ \sum_m \sum_{m'} \langle m|\rho|m'\rangle\langle m'|\ln \rho'|m\rangle$$

$$-\sum_m p_m \ln p_m + \sum_{mm'} p_m \ln p'_{m'} \langle m|m'\rangle\langle m'|m\rangle$$

$$\langle m|m\rangle = \sum_{m'} \langle m|m'\rangle\langle m'|m\rangle$$

$$-\sum_{mm'} p_m \ln\left(\frac{p_m}{p'_{m'}}\right) \langle m|m'\rangle\langle m'|m\rangle \leq \sum_{mm'} p_m \left[\frac{p'_{m'}}{p_m} - 1\right] \langle m|m'\rangle\langle m'|m\rangle$$

$$\left(y = \frac{p'_{m'}}{p_m}\right) \quad \boxed{\ln y - y \leq -1}$$

$$\boxed{\ln y \leq y - 1}$$

$$= \sum_{mm'} (p'_{m'} - p_m) |\langle m|m'\rangle|^2$$

$$\leq \sum_{m'} p'_{m'} - \sum_m p_m = 0$$

$$-\mathrm{Tr}[\rho \ln \rho] \leq -\mathrm{Tr}[\rho \ln \rho']$$

$$\rho = \frac{e^{-\beta(H-\mu N)}}{Z}$$

$$-\mathrm{Tr}[\rho_0 \ln \rho_0] \leq -\mathrm{Tr}[\rho_0 \ln \rho]$$

$$\ln \rho = -\beta(H-\mu N) - \ln Z$$

$$+S_0 \leq -\mathrm{Tr}[\rho_0[-\beta(H-\mu N) - \ln Z]$$

$$\leq \beta \langle H-\mu N\rangle_0 + \ln Z$$

$$TS_0 \leq \langle H-\mu N\rangle_0 + T\ln Z$$

$$-T\ln Z \leq [\langle H_0-\mu N\rangle_0 - TS_0] + \langle H-H_0\rangle_0$$

$$\leq -T\ln Z_0 + \langle H-H_0\rangle_0$$

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