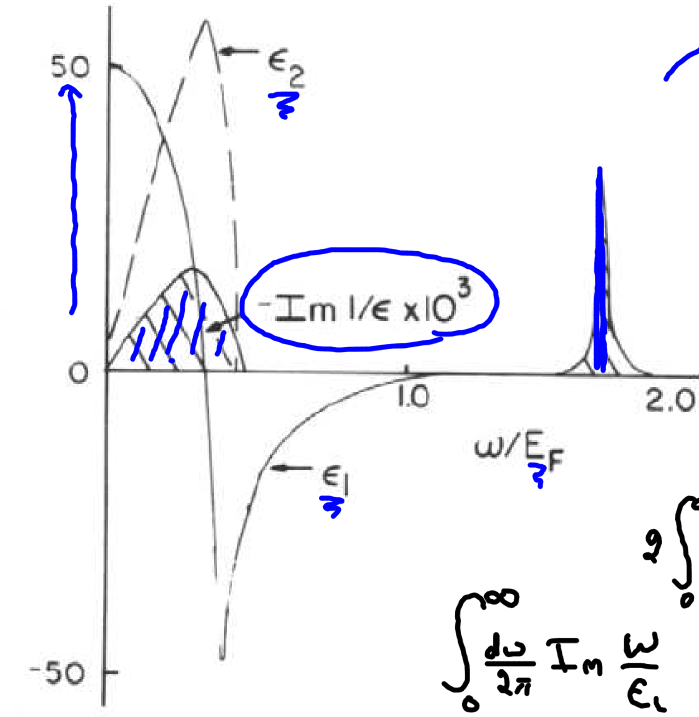
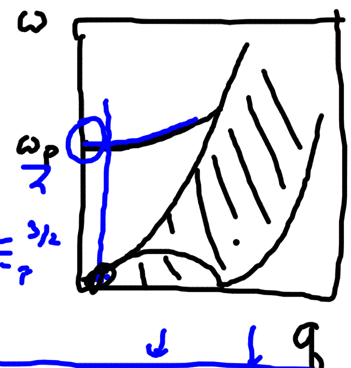




$\frac{ne^2}{\epsilon_0 m}$

$k_F^3 \propto E_F^{3/2}$

$$\boxed{\frac{1}{\epsilon_L(q,\omega)} = 1 - V_q\,\chi_{nn}(q,\omega)}$$

$$2\int_0^\infty \frac{d\omega}{\pi}\,\omega\,\chi''_{nn}(q,\omega) = \frac{nq^2}{m}$$

$$\int_0^\infty \frac{d\omega}{2\pi}\,\mathrm{Im}\,\frac{\omega}{\epsilon_L} = \frac{1}{4}(-V_q)\frac{nq^2}{m} = -\frac{1}{4}\frac{e^2}{\epsilon_0 \cancel{q^2}}\frac{n\cancel{q^2}}{m}$$

$$= -\frac{\omega_p^2}{4}$$

l and imaginary parts of the dielectric constant and Im $(1/\varepsilon)$ as a
ency, calculated for $r_s = 3$ and $q = 0.2k_F$. Shaded plots correspond
en from Mahan *op. cit.* p.430

q petit $\quad \epsilon_L = 1 - \frac{\omega_p^2}{\omega^2} = \frac{\omega^2 - \omega_p^2}{\omega^2} = \frac{(\omega-\omega_p)(\omega+\omega_p)}{\omega^2}$

$$\sim \frac{2}{\omega_p}(\omega - \omega_p)$$

$$\int_0^\infty \frac{d\omega}{2\pi}\,\omega\,\mathrm{Im}\,\frac{1}{\frac{2}{\omega_p}(\omega + i\eta - \omega_p)} = \int \frac{d\omega}{2\pi}\,\omega\left(-\pi\,\delta\left(\frac{2}{\omega_p}(\omega - \omega_p)\right)\right)$$

$$= -\pi\,\frac{\omega_p^2}{4\pi} = -\frac{\omega_p^2}{4}$$

39. Propriétés à 1 ptcle et H.F.

- 1. Approche variationnelle
- 2. H.F. du point de vue milieu effectif (théorie des pert. renormalisée)
- 3. Pathologie

40. Deuxième étape GW (sauver H.F.)

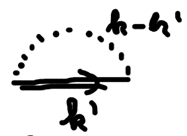
    40.2 Self vs écrantage (Schwinger)

41. Physique des prop. à 1 particule

    1. A(k,ω)
    2. Interp. physique

39.1 Approche variationnelle

Direct

$\Sigma^{(1)}(k) = -$ 

$$\frac{1}{ik_n-(\epsilon_k-\mu)-\Sigma^{(1)}(k)}$$

$$\Sigma^{(1)}(k) = -\int\frac{d^3k'}{(2\pi)^3}\,T\sum_{ik'_n}\frac{e^2}{\epsilon_0\,|k-k'|^2}\;\frac{e^{-ik'_n0^-}}{ik'_n-(\epsilon_{k'}-\mu)-\Sigma^{(1)}(k')}$$

$$= -\int\frac{d^3k'}{(2\pi)^3}\frac{e^2}{\epsilon_0|k-k'|}\,f(\hat{\zeta}_{k'})$$

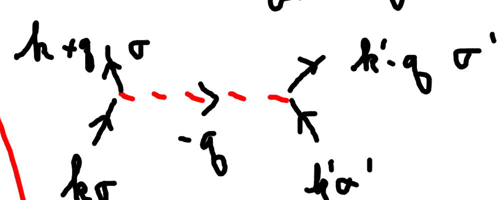
$$F \leq F_0 + \langle H - \tilde{H}_0 \rangle_0$$

$$\tilde{H}_0 = \sum_{k\sigma} \tilde{\epsilon}_k \, c^\dagger_{k\sigma} c_{k\sigma} \qquad Z = \prod_{k\sigma} Z_{k\sigma} = \prod_{k\sigma}\left(1 + e^{-\beta(\tilde{\epsilon}_k - \mu)}\right)$$

$$F_0 = -T \sum_{k\sigma} \ln\left(1 + e^{-\beta(\tilde{\epsilon}_k - \mu)}\right)$$

$$\langle H - \tilde{H}_0 \rangle_0 = \sum_{k\sigma} (\epsilon_k - \tilde{\epsilon}_k) \langle c^\dagger_{k\sigma} c_{k\sigma} \rangle_0 + \frac{1}{2V} \sum_{\substack{kk'q \\ \sigma\sigma'}} V_q \langle c^\dagger_{k\sigma} c^\dagger_{k'\sigma'} c_{k'-q\sigma'} c_{k+q\sigma} \rangle_0$$

$$\sum_{\sigma\sigma'} \frac{1}{2} \int d^3r\, d^3r' \; \psi^\dagger_\sigma(r) \psi^\dagger_{\sigma'}(r') V(r-r') \psi_{\sigma'}(r') \psi_\sigma(r)$$



$$\boxed{\begin{aligned} \frac{\delta \mathcal{G}}{\delta \varphi} &= \langle \psi^\dagger \psi \psi \psi^\dagger \rangle - \mathcal{G}\mathcal{G} \\ &= -\mathcal{G} \frac{\delta \mathcal{G}^{-1}}{\delta \varphi} \mathcal{G} = -\mathcal{G}\mathcal{G} \end{aligned}}$$

mars 11-09:59

$$\langle c^{+}_{k\sigma}\, c^{+}_{k'\sigma'}\, c_{k'-q\,\sigma'}\, c_{k+q\,\sigma} \rangle_0$$

$$= \langle c^{+}_{k'\sigma'}\, c_{k'-q\,\sigma'} \rangle_0 \langle c^{+}_{k\sigma}\, c_{k+q\,\sigma} \rangle_0$$

$$- \langle c^{+}_{k\sigma}\, c_{k'-q\,\sigma'} \rangle_0 \langle c^{+}_{k'\sigma'}\, c_{k+q\,\sigma} \rangle_0 \qquad k' = k+q \qquad q = k'-k$$

$$\longrightarrow - \langle c^{+}_{k\sigma}\, c_{k\sigma'} \rangle_0 \langle c^{+}_{k'\sigma'}\, c_{k'\sigma} \rangle_0\, \delta_{q,k'-k}$$

devient

car $V_0 = 0$

$$= - f(\tilde{\xi}_k)\, f(\tilde{\xi}_{k'})\, \delta_{q,k'-k}\, \delta_{\sigma\sigma'}$$

$$\langle H - \tilde{H}_0 \rangle_0 = \sum_{k\sigma} (\epsilon_k - \tilde{\epsilon}_k)\, f(\tilde{\xi}_k) \qquad \Big| \qquad F_0 = -\sum_{k\sigma} T \ln\left(1 + e^{-\beta(\tilde{\epsilon}_k - \mu)}\right)$$

$$- \frac{g}{2V} \sum_{k k'} V_{k-k'}\, f(\tilde{\xi}_k)\, f(\tilde{\xi}_{k'})$$

$$\frac{\partial F_0}{\partial \tilde{\epsilon}_k} = + 2 \frac{e^{-\beta(\tilde{\epsilon}_k - \mu)}}{1 + e^{-\beta(\tilde{\epsilon}_k - \mu)}} = \cancel{2 f(\tilde{\xi}_k)}$$

$$\frac{\partial}{\partial \tilde{\epsilon}_k} \langle H - \tilde{H}_0 \rangle_0 = \cancel{-2 f(\tilde{\xi}_k)} + 2(\epsilon_k - \tilde{\epsilon}_k)\left(\frac{\partial f(\tilde{\xi}_k)}{\partial \tilde{\epsilon}_k}\right)$$

$$- \frac{g}{V} \sum_{k'} V_{k-k'}\, f(\tilde{\xi}_{k'}) \left(\frac{\partial f(\tilde{\xi}_k)}{\partial \tilde{\epsilon}_k}\right)$$

$$\boxed{\tilde{\epsilon}_k = \epsilon_k - \frac{1}{V} \sum_{k'} V_{k,k'}\, f(\tilde{\xi}_{k'})} \qquad \boxed{n = 2 \frac{1}{V} \sum_{k} f(\tilde{\xi}_k)}$$

$$\epsilon_{k_F} - \mu_0 = 0$$

Théorème de Luttinger

$$\tilde{\epsilon}_{k_F} - \mu = 0 = \left(\epsilon_{k_F} - \overbrace{\Sigma^{(1)}(k_F) - \mu}\right)$$

k_F : indép. des int.

2. Autre approche: Théorie des perturbations renormalisée

$$H = \tilde{H}_0 + \overbrace{(H - \tilde{H}_0)}^{\epsilon_k - \tilde{\epsilon}_k}$$

$\Sigma_{TPR} =$ 

$$\langle \Sigma \rangle_{TPR} = 0$$

3- Pathologies

$$\Sigma^{(1)}(k) = -\int \frac{d^3k'}{(2\pi)^3} V_{k-k'} f(\tilde{\xi}_{k'}) = -\frac{1}{8\pi^3} 2\pi \int_0^{k_F} k'^2 dk' \int_{-1}^{1} d(\cos\theta)$$

$$\times \frac{e^2}{\epsilon_0 (k^2 + k'^2 - 2kk'\cos\theta)}$$

$$= -\frac{1}{4\pi^2} \int_0^{k_F} k'^2 dk' \frac{e^2}{\epsilon_0} \ln\left|\frac{k^2+k'^2-2kk'}{k^2+k'^2+2kk'}\right| \left(-\frac{1}{2kk'}\right)$$

$$= \frac{2}{8\pi^2} \int_0^{k_F} \frac{k'}{k} dk' \frac{e^2}{\epsilon_0} \ln\left|\frac{k-k'}{k+k'}\right|$$

$$\mathcal{P}\int_b^c \frac{1}{x-a} = \ln\left|\frac{c-a}{b-a}\right|$$

$$\Sigma^{(1)}(k) = -\frac{e^2}{4\pi^2\epsilon_0} k_F \left[1 + \frac{1-y^2}{2y} \ln\left|\frac{1+y}{1-y}\right| \right] \quad y = k/k_F$$

$$\frac{\Sigma^{(1)}(k_F)}{\frac{k_F^2}{2m}} \propto \frac{me^2}{\epsilon_0 k_F} \sim \frac{1}{k_F a_0} \sim r_s$$

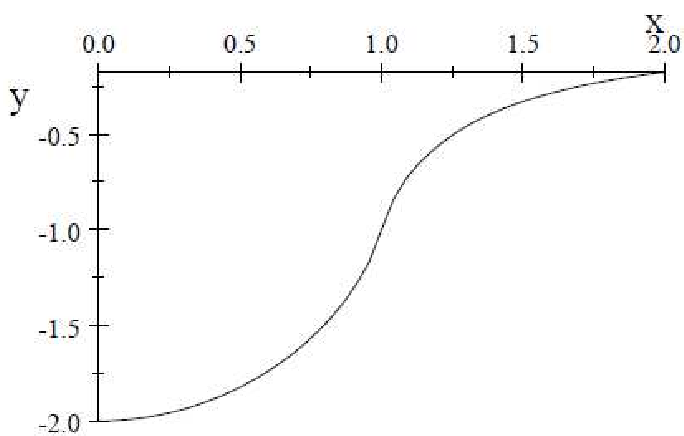
$$1-y^2 = (1+y)(1-y)$$

$$\lim_{x\to 0} x \ln x = 0$$

mars 11-10:55

$$\frac{m}{m^*} = \frac{1 + \frac{\partial k}{\partial \xi_k}\frac{\partial}{\partial k}\Sigma^{(1)}(k)}{1 - \underbrace{\cancel{\frac{\partial}{\partial \omega}\Sigma^{(1)}(k)}}_{0}} \xrightarrow[a k_F]{\infty} m^* = 0$$

$$\frac{\partial}{\partial y}\left[\frac{1-y^2}{2y}\ln(1-y)\right] = \left(-\frac{1}{2y^2} - \frac{1}{2}\right)\ln(1-y) + \frac{1-y^2}{2y}\,\frac{(-1)}{1-y}$$



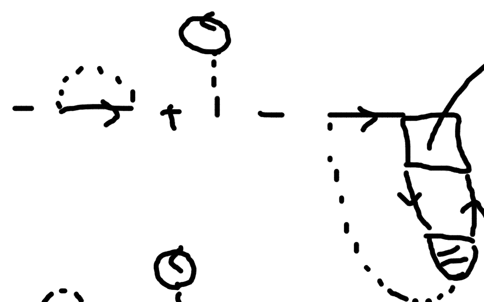
Plot of the Hartree-Fock self-energy at zero temperature.

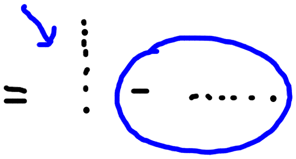
40. "Réparer" H.F. (approx. GW)

$$\Sigma \mathcal{G} = \langle V \rangle$$

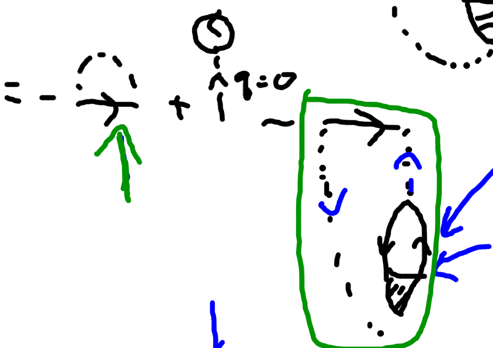
$$T_r(\Sigma \mathcal{G}) \approx \langle V \rangle$$

$$\Sigma = - \quad + \quad - \quad \frac{\delta \Sigma}{\delta \mathcal{G}} =$$



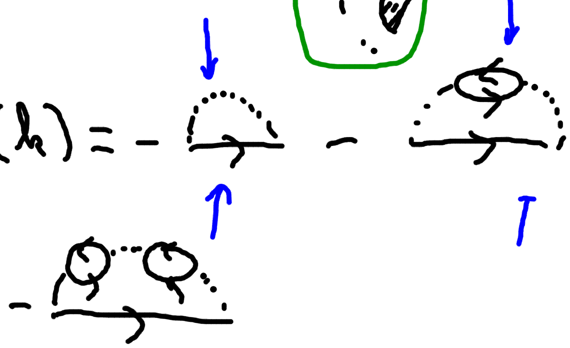


$$1 - V_q \chi^{nn}$$



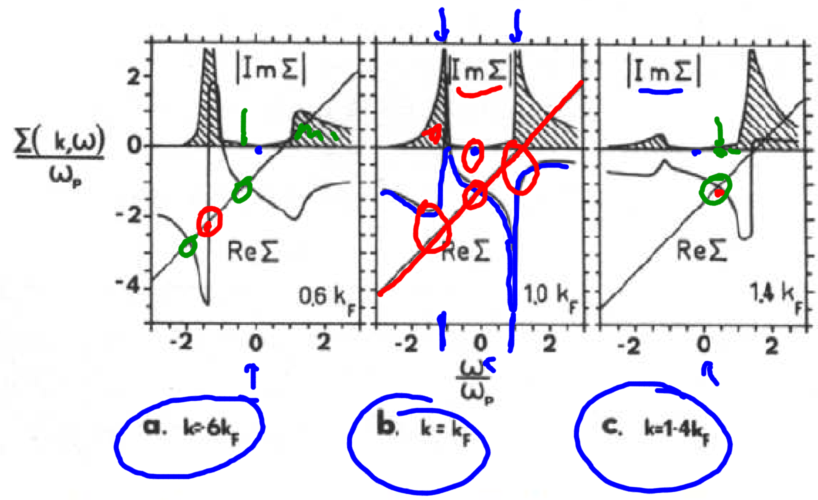
$$= -\int \frac{d^3q}{(2\pi)^3} V_q \left[1 - \frac{V_q \chi^0_{nn}}{1 + V_q \chi^0_{nn}} \right] T \sum_{iq_n} \mathcal{G}(k+q)$$

$$\Sigma^{(2)}(k) = - \quad - \quad -$$



$$-\int \frac{d^3q}{(2\pi)^3} T \sum_{iq_n} \frac{\mathcal{G}(k+q) V_q}{1 + V_q \chi^0_{nn}}$$

41 Physique des excitations à 1 particule



$$\mathrm{Im}\,\Sigma(k,0)=0$$

$$Z=\frac{1}{1-\frac{\partial\Sigma'}{\partial\omega}}$$

Figure 41-1 Real and imaginary part of the RPA self-energy for three wave vectors, in units of the plasma frequency. The chemical potential is included in ReΣ. The straight line that appears on the plots is $\omega-\varepsilon_{\mathbf{k}}$. Taken from B.I. Lundqvist, Phys. Kondens. Mater. **7,** 117 (1968). $r_s=5$?

$$A(k,\omega) = \frac{-2\Sigma''}{(\omega - \epsilon_k - \Sigma')^2 + (\Sigma'')^2}$$

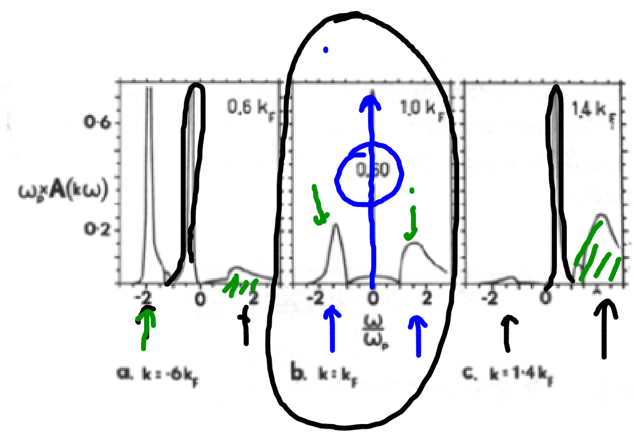


Figure 41-2 RPA spectral weight, in units of the inverse plasma frequency. Taken from B.I. Lundqvist, Phys. Kondens. Mater. **7**, 117 (1968).

mars 11-11:22