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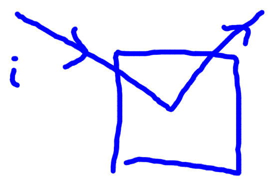


$$\left[\frac{g^2 |\langle p_f | \overset{j}{\downarrow}{A_p} | p_i \rangle|^2}{\hbar^2}\right]$$

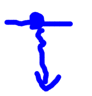
$$0 \neq \langle s | j | p \rangle$$

$$\times \int dt\, e^{i\omega t} \langle A_s(t) A_s \rangle$$

$$\langle A^2 \rangle \neq 0$$

$$\langle A \rangle = 0$$

$A \quad A$



$$\frac{\vec{j} \cdot \vec{A}}{c}$$

$$A \to (a^\dagger + a)$$

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9- Perturbations :

1. Sch. – Heisenberg.

$$\boxed{i\hbar \frac{\partial \Psi_s(t)}{\partial t} = \mathcal{H}(t)\,\Psi_s(t)}$$

$$\frac{\partial}{\partial t}\left(\langle \Psi_s(t) | \Psi_s(t) \rangle\right) = 0$$

$$\langle \Psi_s(0) | \underbrace{U^{+}(t,0)\,U(t,0)} | \Psi_s(0) \rangle$$

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$$\boxed{\Psi_s(t) = U(t,0)\,\Psi_s(0)}$$

$$U(t',t)\,U(t,0) = U(t',0)$$

$$U^+(t,0) = U^{-1}(t,0)$$

S: sym. sous $t \to -t$

$$U(0,t) = U^+(t,0) = U^{-1}(t,0)$$

$$\langle \Psi_s(t) | O_s | \Psi_s(t) \rangle$$

$$= \langle \Psi_s(0) | \boxed{U^+(t,0)\, O_s\, U(t,0)} | \Psi_s(0) \rangle$$

$$= \langle \Psi_H(0) | \boxed{O_H(t)} | \Psi_H(0) \rangle$$

2. Représentation d'interaction

$$\mathcal{H} = \mathcal{H}_0 + \delta\mathcal{H}(t)$$

① → $$\boxed{U(t,0) = e^{-iH_0 t/\hbar}\, U_I(t,0)}$$

$$U(t,t_0) = U(t,0)\,U(0,t_0)$$

→ $$\boxed{U(0,t_0) = U_I(0,t_0)\, e^{+iH_0 t_0/\hbar}}$$

② $$i\hbar \frac{\partial}{\partial t} U(t,0) = \left[H_0 + \delta\mathcal{H}(t)\right] U(t,0)$$

$$\cancel{H_0 e^{-iH_0 t/\hbar}} U_I(t,0) + e^{-iH_0 t} i\hbar \frac{\partial}{\partial t} U_I(t,0)$$

$$= \cancel{H_0 e^{-iH_0 t/\hbar}} U_I(t,0)$$

$$+\delta\mathcal{H}(t)\, e^{-iH_0 t/\hbar}\, U_I(t,0)$$

$$\boxed{\begin{gathered} i\hbar \frac{\partial}{\partial t} U_I(t,0) = \delta H_I(t)\, U_I(t,0) \\ \delta H_I(t) = e^{iH_0 t/\hbar}\, \delta\mathcal{H}(t)\, e^{-iH_0 t/\hbar} \end{gathered}}$$

$$\langle \psi_S(t) | O_S | \psi_S(t) \rangle = \langle \psi_S | U_I^{\dagger}(t,0)\, \boxed{e^{iH_0 t/\hbar}\, O_S\, e^{-iH_0 t/\hbar}}\, U_I(t,0)\, |\psi_S\rangle$$

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10. Réponse linéaire

$O_I(t) = e^{iH_0 t/\hbar}\, O\, e^{-iH_0 t/\hbar}$

$\mathcal{H}_{tot} = H_0 + \delta\mathcal{H}(t)$

$O_H(t) = e^{iHt/\hbar}\, O\, e^{-iHt/\hbar}$

$$\delta\mathcal{H}(t) = -\int d^3r\ A_i(\vec r,t)\, a_i(\vec r,t)$$

A_i	a_i
$M_z(\vec r,t)$	$B_z(\vec r,t)$
S_z	F_z
ρ	φ
$\vec j$	$\vec A$
A_s	A_φ

$\langle B(r,t)\rangle = Tr[\rho\, B(r,t)]$

opérateur

$$\rho = \frac{e^{-\beta(H-\mu N)}}{Tr\left[e^{-\beta(H-\mu N)}\right]}$$

à t_0 syst. à l'équilibre

$\langle B(r,t)\rangle = \langle U^\dagger(t,t_0)\, B(r)\, U(t,t_0)\rangle$

$U(t,t_0) = e^{-iHt/\hbar}\, U_I(t,t_0)\, e^{+iHt_0/\hbar}$

③ $$\boxed{\langle \cancel{e^{-iHt_0/\hbar}}\, U_I^\dagger(t,t_0)\, B_I(t)\, U_I(t,t_0)\, \cancel{e^{iHt_0/\hbar}}\rangle}$$

$B_I(r,t) = e^{iHt/\hbar}\, B(r,t)\, e^{-iHt/\hbar}$

Cyclique: $Tr[AB] = Tr[BA]$

$$\sum_{ij} A_{ij} B_{ji} = \sum_{ij} B_{ij} A_{ji}$$

$\langle O\rangle = Tr[\rho\, O]$

$$i\hbar \frac{\partial}{\partial t} U_I(t,t_0) = \delta\mathcal{H}_I(t)\, U_I(t,t_0)$$

$$\int_{t_0}^{t}:\quad U_I(t,t_0) - \underbrace{U_I(t_0,t_0)}_{1} = -\frac{i}{\hbar}\int_{t_0}^{t} dt\ \delta H_I(t)\ \underbrace{U_I(t,t_0)}_{1}$$

$$U_I(t,t_0) = 1 - \frac{i}{\hbar}\int_{t_0}^{t} dt\ \delta H_I(t) + \dots$$

$$U_I^\dagger(t,t_0) = 1 + \frac{i}{\hbar}\int_{t_0}^{t} dt\ \delta H_I(t) + \dots$$

$$\delta\langle B(r,t)\rangle = \frac{i}{\hbar}\int_{t_0}^{t} dt'\ \langle \delta H_I(t')\, B_I(r,t) - B_I(r,t)\, \delta H_I(t')\rangle$$

$$\delta\langle B(r,t)\rangle = -\frac{i}{\hbar}\int_{t_0}^{t} dt' \int d^3r'\ \langle A_I(r',t')\, B_I(r,t) - B_I(r,t)\, A_I(r',t')\rangle\ a(r',t')$$

$$\delta\langle B(r,t)\rangle = \iint_{t_0 = -\infty}^{t} dt'\, 2\chi''_{BA}(r,t;r',t')\, a(r',t')\, d^3r'$$

$$\boxed{\chi''_{BA}(r,t;r',t') = \frac{1}{2\hbar}\langle [B(r,t), A(r',t')]\rangle}$$

$$\delta\langle B(r,t)\rangle = i\int_{-\infty}^{\infty} dt'\, d^3r'\, 2\chi''_{BA}(r,t\,;r',t')\,\theta(t-t')\,\underbrace{a(r',t')}$$

Retardée

$$\boxed{\chi^{R}_{BA}(r,t\,;r',t') = 2i\,\chi''_{BA}(r,t\,;r',t')\,\theta(t-t')}$$

$$\delta\langle B(r,t)\rangle = \int d^3r' \int_{-\infty}^{\infty} dt'\, \chi^{R}_{BA}(r,t\,;r',t')\,a(r',t')$$

Causalité

II. Propriétés générales.

1.

2. Symétries de $\mathcal{H}$ et de X"

$$[H, S] = 0$$

$$HS - SH = 0 \qquad \boxed{S^{-1} H S = H}$$

$$\boxed{\langle O \rangle} = \mathrm{Tr}[\rho O] \qquad S^{-1} \rho S = \rho$$

$\langle p \rangle$

$$= \mathrm{Tr}[S^{-1} \rho S O]$$

$$= \mathrm{Tr}[\rho S O S^{-1}] = \boxed{\langle S O S^{-1} \rangle}$$

$$= -\langle p \rangle$$

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Invariance sous translation R $\quad T_R^{-1} B A T_R$ $\;\; T_R T_R^{-1}$

$$T_R^{-1} A(r,t) T_R \rightarrow A(r+R,t)$$

$$\chi''_{B A}(r,t;r',t') = \chi''_{BA}(r+R,t;r'+R,t')$$

$$= \chi''_{BA}(r-r';t,t')$$

translation t

$$\mathrm{Tr}\left[\rho\, B(r,t)\, A(r',t')\right]$$

$$\mathrm{Tr}\left[e^{iHt_0}\rho\, e^{-iHt_0}\, e^{iHt} B e^{-iHt}\, e^{iHt'} A e^{-iHt'}\right]$$

Parité $\quad S_p^{-1}\, r\, S_p \rightarrow -r$

$$S_p^{-1}\, r\, S_p = \epsilon_p\, r \qquad \epsilon_p = -1$$

Signature

$$\chi''_{BA}(r,t;r',t') = \epsilon_B^P \epsilon_A^P\, \chi''_{BA}(r,t;-r',t')$$

Inversion du temps.

$$S_T^{-1}\,\vec{r}\,S_T = \vec{r}$$

$$S_t^{-1}\,\vec{p}\,S_t = -\vec{p}$$

$$\vec{p} = \sum_{r_\alpha} \frac{\hbar}{i}\nabla_{r_\alpha}$$

Cas simple, sous S_t $\qquad S_t\Psi = \Psi^*$

$$i\hbar\frac{\partial}{\partial t}\Psi = H\Psi$$

$$-i\hbar\frac{\partial}{\partial t}\Psi^* = H\Psi^*$$

$$\Psi(t) = e^{-iHt/\hbar}\Psi(0)$$

$$\Psi^*(t) = e^{iHt/\hbar}\Psi^*(0)$$

$\langle\psi|\mathcal{O}|\psi'\rangle$

$= \left(\langle\psi|\mathcal{O}\right)|\psi'\rangle = \langle\psi|\left(\mathcal{O}|\psi'\rangle\right)$

$\langle\psi|\underset{\leftarrow}{K}\mathcal{O}\underset{\rightarrow}{K}|\psi'\rangle \qquad \langle\psi|A\underset{\rightarrow}{K}|\psi'\rangle$

$\langle\underset{\leftarrow}{K}\mathcal{O}\underset{\rightarrow}{K}\rangle \longrightarrow \langle i|\underset{\leftarrow}{K}\mathcal{O}\underset{\rightarrow}{K}|j\rangle$

$= \left(\langle i|\underset{\leftarrow}{K}\right)\left(\underset{\rightarrow}{K}\left(\mathcal{O}^*|j\rangle\right)\right)$

$= \langle\mathcal{O}^* j|i\rangle$

$= \langle i|\mathcal{O}^* j\rangle^*$

$= \langle j|\mathcal{O}^{*+}|i\rangle$

$\langle a|A|b\rangle^*$

$= \langle A^+ a|b\rangle^*$

$$\boxed{\begin{aligned}\langle\underset{\leftarrow}{K}\mathcal{O}\underset{\rightarrow}{K}\rangle &= \langle\mathcal{O}^{*+}\rangle\\ &= \epsilon_T\langle\mathcal{O}^+\rangle\end{aligned}}$$

$\chi''_{BA}(r,t;r',t') = \epsilon^T_A\epsilon^T_B\,\chi_{AB}(r',t',r,t)$.

Spin

$\underset{\rightarrow}{K}U \qquad S_t|\uparrow\rangle = -e^{-i\delta}i|\downarrow\rangle$

$U = e^{-i\delta}\sigma_y \qquad S_t|\downarrow\rangle = e^{-i\delta}i|\uparrow\rangle$

Choisit $e^{-i\delta} = i$

$S_tS_t|\uparrow\rangle = -|\uparrow\rangle \quad S_t|\uparrow\rangle = |\downarrow\rangle$

$S_t|\downarrow\rangle = -|\uparrow\rangle$

$\vec{S} \to \vec{r}\times\vec{p}$

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