48. TPSC : aspects physiques

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48. TPSC : aspects physiques

$$\chi_{ch}(q=0,\omega\neq 0)=0 \qquad \frac{\chi_0}{1-\frac{U}{2}\chi_0}$$

$$\chi_{sp}=\frac{\chi_0}{1-\frac{U_{sp}}{2}\chi_0} \qquad \chi_{ch}=\frac{\chi_0}{1+\frac{U_{ch}}{2}\chi_0} \qquad \begin{array}{l} U_{sp}\to F_0^a \\ U_{ch}\to F_0^s \end{array}$$

$$\frac{T}{N}\sum_q \chi_{sp} = n - 2\langle n_\uparrow n_\downarrow\rangle$$

$$\frac{T}{N}\sum_q \chi_{ch} = n + 2\langle n_\uparrow n_\downarrow\rangle - n^2$$

$$\boxed{U_{sp}=\frac{U\langle n_\uparrow n_\downarrow\rangle}{\langle n_\uparrow\rangle\langle n_\downarrow\rangle}}$$

$$U_{sp}\langle n_\uparrow\rangle\langle n_\downarrow\rangle = U\langle n_\uparrow n_\downarrow\rangle$$

$$\uparrow$$

$$\frac{T}{N}\sum_q \frac{\chi_0}{1-\frac{U_{sp}}{2}\chi_0} \simeq \frac{T}{N}\sum_q \chi_0\left(1+\frac{U_{sp}}{2}\chi_0\right) = n - 2\,\frac{U_{sp}}{U}\,\frac{n^2}{4}$$

$$= \cancel{n} - \cancel{2}\frac{n^{2}}{\cancel{4}} + \frac{U_{sp}}{\cancel{2}}\Big[\overbrace{\underbrace{\frac{T}{N}\sum_q(\chi_0)^2}_{n^2}}^{\Omega}\Big] = \cancel{n} - \frac{U_{sp}}{U}\frac{n^2}{\cancel{2}}$$

$$U_{sp}\Omega = 1 - \frac{U_{sp}}{U} \qquad U_{sp}\left[\Omega+\frac{1}{U}\right]=1$$

$$\boxed{U_{sp}=\frac{U}{\Omega U+1}} \quad \swarrow U_{sp}^{max} \sim \frac{1}{\Omega}$$

Mermin-Wegner:

$$\frac{T}{N}\sum_{q}\frac{\chi_0}{1-\boxed{\frac{U\langle n_\uparrow n_\downarrow\rangle}{2\langle n_\uparrow\rangle\langle n_\downarrow\rangle}}\chi_0} = n - 2\langle n_\uparrow n_\downarrow\rangle$$

$U_{sp}/2$

$$\chi(q, iq_n) = \int\frac{d\omega'}{\pi}\frac{\omega'\chi''(\omega')}{q_n^2+\omega'^2}$$

près d'une transition de phase: $\to 0$

$$1-\frac{U_{sp}}{2}\chi_0(q, iq_n) \to \boxed{1-\frac{U_{sp}}{2}\chi_0(Q,0)} - \frac{1}{4}U_{sp}\frac{\partial^2\chi^0}{\partial q^2}(q-Q)^2$$

$$-\frac{1}{2}U_{sp}\boxed{\frac{\partial\chi^0}{\partial iq_n}}iq_n$$

$$\chi_{sp} = \frac{\chi^{(0)}}{1-\frac{U_{sp}}{2}\chi^0}\;\frac{1}{1+q^2\xi^2+iq_n\frac{\xi^2}{\omega_{sp}\xi_0^2}}$$

$$\boxed{\xi^2 = \frac{\frac{1}{4}U_{sp}\frac{\partial^2\chi^0}{\partial q^2}}{1-\frac{U_{sp}}{2}\chi_0}}$$

$$\sim\frac{a\,\xi^2}{1+q^2\xi^2+iq_n\frac{\xi^2}{\omega_{sp}\xi_0^2}}$$ Ornstein-Zernicke

$$T\int\frac{d^dq}{(2\pi)^d}\frac{a\,\xi^2}{1+q^2\xi^2} = C(T)$$

$$\boxed{\omega_{sp}^0\,\xi^{-2} \ll T}$$

$$T\int\frac{d^dq}{(2\pi)^d}\frac{a}{\xi^{-2}+q^2} = C(T)$$

iq_n $\boxed{\omega_{sp} \ll T}$

$d=2$

$$T\int dq\frac{q}{\xi^{-2}+q^2}\sim T\ln(q^2+\xi^{-2})\Big|_0^\Lambda\sim C(T)$$

$$\sim T\ln\frac{\Lambda^2}{\xi^{-2}}\sim C(T)$$

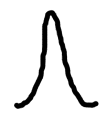
$$A\,e^{C/T} = \Lambda^2\xi^2$$

Heisenberg

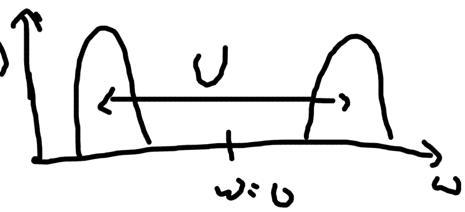
$\to$ σ-non-linéaire

avr. 1-09:51

49 Transition de Mott

Brinkmann-Rice:  $m^* \to \infty$

Mott: $N(\omega)$



49.1 Impureté quantique (Anderson)

$$K = H_f + H_c + H_{cf} - \mu N$$

$$K_f = (\epsilon-\mu)(f_\uparrow^\dagger f_\uparrow + f_\downarrow^\dagger f_\downarrow) + U f_\uparrow^\dagger f_\uparrow f_\downarrow^\dagger f_\downarrow \leftarrow$$

$$K_c = \sum_{k\sigma}(\epsilon_k-\mu)\, c_{k\sigma}^\dagger c_{k\sigma} \leftarrow$$

$$H_{cf} = \sum_{k\sigma} V_{ik}(c_{k\sigma}^\dagger f_{i\sigma}) + (V_{ik}^* f_{i\sigma}^\dagger c_{k\sigma}) \leftarrow$$

$$\frac{\partial c_{k\sigma}}{\partial\tau} = [K, c_{k\sigma}] = -(\epsilon_k-\mu)c_{k\sigma} - V_{ik} f_{i\sigma}$$

$$\frac{\partial f_{i\sigma}}{\partial\tau} = [K, f_{i\sigma}] = -(\epsilon-\mu)f_{i\sigma} - \sum_k V_{ik}^* c_{k\sigma} - U f_{i-\sigma}^\dagger f_{i-\sigma} f_{i\sigma}$$

$$\mathcal{G}_{ff_\sigma}(\tau) = -\langle T_\tau f_{i\sigma}(\tau) f_{i\sigma}^\dagger\rangle \qquad \mathcal{G}_{cf\sigma}(\vec{k}, i;\tau) = -\langle T_\tau c_{k\sigma}(\tau) f_{i\sigma}^\dagger\rangle$$

$$\frac{\partial \mathcal{G}_{ff\sigma}}{\partial\tau} = -\delta(\tau) - (\epsilon-\mu)\mathcal{G}_{ff}(\tau) - \sum_k V_{ik}^* \mathcal{G}_{cf\sigma}(\vec{k}, i;\tau)$$
$$+ U\langle T_\tau f_{i-\sigma}^\dagger(\tau) f_{i-\sigma}(\tau) f_{i\sigma}(\tau) f_{i\sigma}^\dagger\rangle$$

$$\frac{\partial \mathcal{G}_{cf\sigma}}{\partial\tau} = -(\epsilon_k-\mu)\mathcal{G}_{cf\sigma} - V_{ik}\mathcal{G}_{ff\sigma}$$

$$[i\omega_n - (\epsilon-\mu)]\mathcal{G}_{ff} = 1 + \sum_k V_{ik}^* \mathcal{G}_{cf\sigma} \; \boxed{- U \int \langle f_{i-\sigma}^\dagger(\tau) f_{i-\sigma}(\tau) f_{i\sigma}(\tau) f_{i\sigma}^\dagger\rangle e^{i\omega_n\tau} d\tau}$$

$$[i\omega_n - (\epsilon_k-\mu)]\mathcal{G}_{cf} = V_{ik}\mathcal{G}_{ff}$$

$$\Big[\underbrace{i\omega_n - (\epsilon-\mu) - \sum_k V_{ik}^* \frac{1}{i\omega_n - (\epsilon_k-\mu)} V_{ik}}_{\mathcal{G}_{0\,ff}^{-1}}\Big]\mathcal{G}_{ff\sigma}(i\omega_n) = 1 + \Sigma_{ff}(i\omega_n)\mathcal{G}_{ff}(i\omega_n)$$

Fonction d'hybridation $\Delta(i\omega_n)$

$$\langle f_\uparrow^\dagger(\tau) f_\uparrow(\tau) f_\uparrow(\tau') f_\uparrow^\dagger(\tau')\rangle$$

49.2 Modèle pour lesquels champ moyen est exact

$$\mathcal{H} = -\frac{1}{2N}\sum_{ij} S_i S_j - h\sum_i S_i \qquad S_i = \pm 1$$

$$= -\frac{1}{2N}\left(\sum_i S_i\right)^2 - h\sum_i S_i \qquad \int e^{-N f(x)}$$

$$\int d\lambda\, e^{-\frac{N\beta}{2}\lambda^2 + \beta\lambda\left(\sum_i S_i\right) + \beta h \sum_i S_i} \sqrt{\frac{N\beta}{2\pi}}^{\,N}$$

$$= e^{-\beta H}$$

49.3 $\boxed{d=\infty}$

$$-2\,\boxed{t\sqrt{d}}\sum_{\substack{\langle ij\rangle\\ \sigma}} \underbrace{\langle c^{\dagger}_{i\sigma} c_{j\sigma}\rangle}_{} \rightarrow \quad \boxed{\left|\langle \psi_j | \psi_i\rangle\right|^2 \propto \frac{1}{d}}$$

fini

$\epsilon_1 = -2t\cos k_1 \qquad \dfrac{d\epsilon}{\sqrt{1-\epsilon^2}}$

$$N(\omega) = \sum_k \delta(\omega - \epsilon_k) = \int_{-\pi}^{\pi} \frac{dk_1}{2\pi}\frac{dk_2}{2\pi}\cdots\frac{dk_d}{2\pi}\,\delta(\omega - \epsilon_1 - \epsilon_2 \ldots \epsilon_d)$$

$$= \int P(\epsilon_1)\,P(\epsilon_2)\ldots P(\epsilon_d)\,\delta(\omega - \epsilon_1 - \epsilon_2 - \ldots - \epsilon_d)\; d\epsilon_1 \ldots d\epsilon_d$$

$$\int \epsilon^2 P(\epsilon)\,d\epsilon = \int \frac{dk}{2\pi}\cos k^2 = \frac{1}{2}$$

$$N(\omega) \propto \frac{1}{\sqrt{2\pi\,\frac{1}{2}\sqrt{d}^{\,2}}}\; e^{-\dfrac{\omega^2}{2\,\frac{1}{2}\sqrt{d}^{\,2}(2t)^2}}$$

$\boxed{2t\sqrt{d} \text{ soit fini}}$