51. Instabilité de la phase normale

1. Cas sans interaction + rotation
2. Effet des interactions
3. χ et paramagnons.
4. Modes collectifs
   1. χ transversale
   2. Thermodynamique et Mermin-Wagner
   3. Kanamori Brueckner

52. Antiferromagnétisme près du demi-remplissage
   1. Pseudogap et régime classique-renormalisé
   2. " dopés aux électrons

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30.2 Théorème des graphes connexes

1. Pour moyennes normalisées
2. Pour fct caractéristique ou énergie libre

53. Interactions électron-phonon dans les métaux (jellium)

1. H et éléments de matrice
2. Interaction e-e effective
3. RPA
4. m^*, Z, Migdal

51.4 Modes collectifs

1. χ transversale

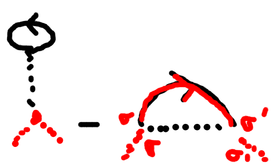
$$\chi^{R}_{-+} = \langle \psi^{+}_{\downarrow} \psi_{\uparrow} \; \psi^{+}_{\uparrow} \psi_{\downarrow} \rangle$$

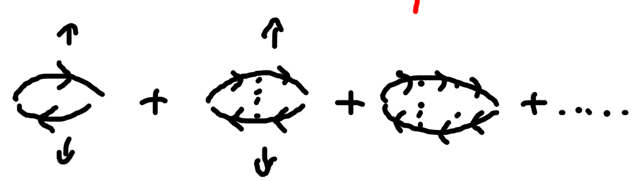
$$-\psi^{+}(\bar{1})\, \varphi(\bar{1},\bar{2})\, \psi(\bar{2})$$

$$\mathcal{G}_{\sigma\sigma'} = -\langle T_c \psi_{\sigma} \psi^{+}_{\sigma'} [S] \rangle / \langle T_c [S] \rangle \qquad [S] = e$$

$$\frac{\delta \mathcal{G}_{\sigma\sigma'}}{\delta \varphi_{\sigma\sigma'}} = -\langle T_c \psi_{\sigma} \psi^{+}_{\sigma'} \psi^{+}_{\sigma} \psi_{\sigma'} \rangle - \cancel{\mathcal{G}}\,\cancel{\mathcal{G}}$$

$$\frac{\delta \mathcal{G}_{\sigma\sigma'}}{\delta \varphi_{\sigma\sigma'}} = -\mathcal{G} \frac{\delta \mathcal{G}^{-1}}{\delta \varphi} \mathcal{G} = \mathcal{G}_{\sigma\sigma} \mathcal{G}_{\sigma'\sigma'} + \mathcal{G}_{\sigma\sigma} \frac{\delta \Sigma_{\sigma\sigma'}}{\delta \mathcal{G}_{\bar{\sigma}\bar{\sigma}}} \frac{\delta \mathcal{G}_{\bar{\sigma}\bar{\sigma}}}{\delta \varphi_{\sigma\sigma'}} \mathcal{G}_{\sigma'\sigma'}$$





$$\chi^{R}_{-+} = \frac{\chi^{0}_{-+}}{1 - U \chi^{0}_{-+}}$$

$$\chi^0_{-+} = \quad = -\frac{1}{N} T \sum_k \frac{f(\xi_{k+q\uparrow}) - f(\xi_{k\downarrow})}{\omega + i\eta + \xi_{k+q\uparrow} - \xi_{k\downarrow}}$$

$$\xi_{k+q\uparrow} = \epsilon_{k+q} - \mu + U\langle n_\downarrow \rangle$$

$$\xi_{k\downarrow} = \epsilon_k - \mu + U\langle n_\uparrow \rangle$$

$$\xi_{k+q} - \xi_k = \epsilon_{k+q} - \epsilon_k + \Delta$$

$$\Delta = U(\langle n_\uparrow \rangle - \langle n_\downarrow \rangle)$$

$$= v_F \cdot q + \cancel{\frac{q^2}{2m}}^{\sim 0} + \Delta$$

$$\chi^0_{-+} = -\frac{T}{N}\sum_k \frac{f(\xi_{k+q}) - f(\xi_k)}{\omega + i\eta + v_F \cdot q - \Delta}$$

$$v_F \cdot q \ll \Delta \Rightarrow$$

$$= -\frac{T}{N}\sum_k \frac{f(\xi_{k+q\uparrow}) - f(\xi_{k\downarrow})}{\omega + i\eta - \Delta}\left[1 - \frac{\overbrace{v_F \cdot q}^{\cos\theta}}{\omega + i\eta - \Delta} + \left(\frac{v_F \cdot q}{\omega + i\eta - \Delta}\right)^2 + \cdots\right]$$

$$v_F = \frac{P}{M}$$

$$= -\frac{(\langle n_\uparrow \rangle - \langle n_\downarrow \rangle)}{\omega + i\eta - \Delta}\left[1 + \Theta q^2\right]$$

$$= -\frac{\Delta}{U}\frac{1}{\omega + i\eta - \Delta}\left(1 + \Theta(q^2)\right)$$

$$f(\xi_{k+q\uparrow}) - f(\xi_{k\downarrow})$$

$$\frac{1}{N}\sum_k \rightarrow \int \frac{d^dk}{(2\pi)^d}$$

$$\chi^R_{-+} = \frac{-\frac{\Delta}{U}\frac{1}{\omega + i\eta - \Delta}\left(1 + \Theta(q^2)\right)}{1 - U\left[-\frac{\Delta}{U}\frac{1}{\omega + i\eta - \Delta}\left(1 + \Theta(q^2)\right)\right]} = \frac{\chi^0_{-+}}{1 - U\chi^0_{-+}}$$

$$= \frac{-\Delta/U\left(1 + \Theta(q^2)\right)}{\omega + i\eta - \cancel{\Delta} + \cancel{\Delta}\left(\cancel{1} + \Theta(q^2)\right)} \approx \frac{-\Delta/U}{\omega + i\eta - Dq^2}$$

Mode de Goldstone $\boxed{\omega \sim Dq^2}$

$$\chi^R_{-+} + \chi^R_{+-} = \frac{-\Delta/U}{\omega + i\eta - Dq^2} + \frac{\Delta/U}{\omega + i\eta + Dq^2}$$

$$\mathrm{Im}\left[\chi^R_{-+} + \chi^R_{+-}\right] = +\frac{\Delta}{U}\pi\delta(\omega - Dq^2) - \frac{\Delta}{U}\pi\delta(\omega + Dq^2)$$

$$\chi^R_{-+}(q,0) + \chi^R_{+-}(q,0) = \int \frac{d\omega'}{\pi}\frac{\chi''(\omega')}{\omega'} = \frac{\Delta}{U}\left[\frac{2}{Dq^2}\right]$$

$$\langle S_x^2 \rangle + \langle S_y^2 \rangle = \frac{T}{N}\sum_q \left[\chi_{-+}(q) + \chi_{+-}(q)\right]$$

$$= \frac{T}{N}\sum_q \sum_{iq_n} \frac{-\Delta/U}{iq_n - Dq^2} + \frac{\Delta/U}{iq_n + Dq^2}$$

$$= 2\sum_q \frac{\Delta/U}{e^{\beta D q^2} - 1}$$

Pour q petit

$$\frac{1}{e^{\beta D q^2} - 1} \sim \frac{1}{\beta D q^2} \sim \frac{T}{Dq^2}$$

$$Z = \int \mathcal{D}S_q \; e^{-\beta(\vec{q}\,S_q)^2} = \int \mathcal{D}S_q \; e^{-\beta q^2 S_q^2}$$

$$\langle q^2 S_q^2 \rangle = \frac{1}{2}T \qquad \langle S_q^2 \rangle \propto \frac{T}{q^2}$$

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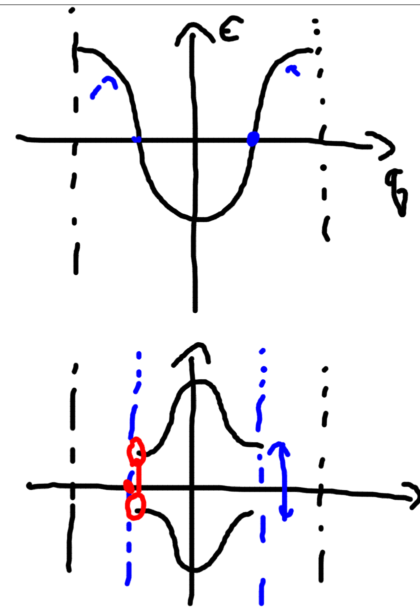
Antiferro aimant



$n=1$

Ferro:

Kanamori - Bruckner

$$U_{eff} = \frac{U}{1+U\Lambda}$$

$$\propto \frac{1}{\Lambda}$$

$$1 - \frac{U}{2}\chi_0 = 0$$

30.2 Graphes connexes

$$\boxed{\langle x^2\rangle - \langle x\rangle^2 = \langle x^2\rangle_c}$$

1. $\langle e^{-f(x_1,\dots)} A(x_1,\dots x_n)\rangle = \langle e^{-f} A\rangle$

$$= \sum_{n=0}^{\infty} \frac{1}{n!}\langle (-f)^n A\rangle = \sum_{n=0}^{\infty} \frac{1}{\cancel{n!}} \sum_{m,l} \boxed{\frac{\cancel{n!}}{m!\,l!}}\, \delta_{n,l+m}$$

$$\langle (-f)^l A\rangle_c \langle (-f)^m\rangle$$

$\langle (-f)^n A\rangle =$

$$= \langle e^{-f} A\rangle_c \langle e^{-f}\rangle$$

$$\boxed{\frac{\langle e^{-f} A\rangle}{\langle e^{-f}\rangle} = \langle e^{-f} A\rangle_c}$$

2. $\ln \langle e^{-f} \rangle = \ln \mathrm{Tr}\left[e^{-\beta H} T_\tau e^{-\int V d\tau} \right]$

$$\frac{\partial}{\partial \lambda} \langle e^{-\lambda f} \rangle = \langle e^{-\lambda f} (-f) \rangle = \langle e^{-\lambda f} (-f) \rangle_c \langle e^{-\lambda f} \rangle$$

$$= \langle \frac{\partial}{\partial \lambda} e^{-\lambda f} \rangle_c \langle e^{-\lambda f} \rangle$$

$$\int_0^1 \frac{\partial}{\partial \lambda} \ln \langle e^{-\lambda f} \rangle = \frac{\partial}{\partial \lambda} \langle e^{-\lambda f} \rangle_c$$

$$\ln \langle e^{-f} \rangle - \ln 1 = \langle e^{-f} \rangle_c - 1$$

$$\ln \langle e^{-f} \rangle = \langle e^{-f} \rangle_c - 1$$

$$\boxed{\langle e^{-f} \rangle = e^{\sum_{n=1}^{\infty} \frac{1}{n!} \langle (-f)^n \rangle_c}}$$

53. Int. el-ph dans métaux

1. H et éléments de matrice (jellium)

$$H = K_e + V_{ee} + K_i + V_{ii} + V_{ei}$$

$$\omega_{ip} = \frac{Z^2 e^2 n}{M \epsilon_i}$$

$$V_{ei} = \sum_{i\alpha\sigma} \int d^3r\, \psi^\dagger_\sigma(r)\psi_\sigma(r) V(\vec{r} - \vec{R}_{i\alpha})$$

$$\vec{R}_{i\alpha} = \vec{u}_{i\alpha} + \vec{R}_{0i\alpha}$$

$$= V^0_{ei} + \sum_{i\sigma\alpha} \int d^3r\, \psi^\dagger_\sigma(r)\psi_\sigma(r)\, \vec{u}_{i\alpha} \cdot \vec{\nabla}_{R_{i\alpha}} V(\vec{r} - \vec{R}^0_{i\alpha})$$

$$\sum_{k'} e^{-ik'\cdot r} \qquad \frac{1}{\sqrt{\Omega}} \sum_k e^{ik\cdot r} c_{k\sigma} \qquad \int_Q V_Q\, e^{i\vec{Q}\cdot(\vec{r} - \vec{R}_{i\alpha})}$$

$$= V^0_{ei} + \frac{1}{\sqrt{\Omega}} \sum_{k'k} c^\dagger_{k'\sigma} c_{k\sigma} \boxed{V(Q)\, i\vec{Q}} \cdot \boxed{\sum_{i\alpha} \vec{u}_{i\alpha} e^{-i\vec{Q}\cdot\vec{R}^0_{i\alpha}}}$$

$$\vec{Q} = \vec{k} - \vec{k}'$$

$$= V^0_{ei} + \frac{1}{\sqrt{\Omega}} \sum_{kk'\sigma} c^\dagger_{k'\sigma} c_{k\sigma} \boxed{\vec{W}_{k'-k}} \cdot \frac{\vec{e}_{k-k'}}{\sqrt{2\omega_{k-k'}\rho_M}} \left(a_{k'-k} + a^\dagger_{k-k'}\right)$$

$$W_{k'-k} = i(\vec{k} - \vec{k}')\left[-\frac{Ze^2 n_0}{\epsilon_0 |k - k'|^2}\right] \qquad \rho_M = \boxed{\frac{NM}{\Omega}} = \frac{M}{v}$$

$$= V^0_{ei} + \frac{1}{\sqrt{\Omega}} \sum_{kk'\sigma} M_{k'-k}\, c^\dagger_{k'\sigma} c_{k\sigma} \left[a_{k'-k} + a^\dagger_{k-k'}\right]$$

$$Z = Z_e^0 Z_i^0 T_\tau \left[\frac{e^{-\beta(K_e + V_{ee})}}{Z_e^0} \frac{e^{-\beta(K_i + V_{ii})}}{Z_i^0} T_\tau e^{-\int_0^\beta d\tau\, V_{ei}(\tau)} \right]$$

$$= Z_e^0 Z_i^0 \left\langle \left\langle T_\tau e^{-\int_0^\beta d\tau\, V_{ei}(\tau)} \right\rangle_i \right\rangle_e$$

$$\boxed{e^{-\beta(K+V)} = e^{-\beta K} T_\tau e^{-\int_0^\beta V(\tau)\, d\tau}}$$

$$= Z_e^0 Z_i^0 \left\langle T_\tau e^{\left\langle \sum_{n=1}^{\infty} \left[\int_0^\beta d\tau (-V_{ei}) \right]^n \frac{1}{n!} \right\rangle_{i\,c}} \right\rangle_e$$

$$= Z_e^0 Z_i^0 \left\langle T_\tau e^{\left\langle \frac{1}{2} \int_0^\beta d\tau \int_0^\beta d\tau'\, V_{ei}(\tau) V_{ei}(\tau') \right\rangle_{c\, i}} \right\rangle_e$$