53. Interaction e-ph dans les métaux

1. H et éléments de matrice
2. Interaction effective
✓ 3. Approximation RPA
4. m^*, Z, Migdal

54. Instabilité de la phase normale: Problème de Cooper

55. BCS

1. Brisure de symétrie, analogie avec ferroaimant
2. Hamiltonien BCS réduit
3. $\mathcal{G}$ de Nambu
4. Équation du gap

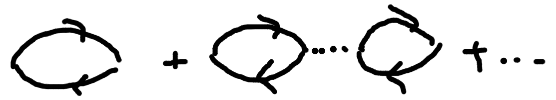
$$\boxed{V_q\, \delta(\tau-\tau') + |M_q|^2\, \mathcal{D}(q,\tau-\tau')}$$

$$\underset{V_c}{\uparrow} \qquad \underset{V_p}{\uparrow}$$

$$M_q = \frac{i Z e^2 n_0}{\sqrt{2\rho_M \omega_{iq}}\, |q|\, \epsilon_0}$$

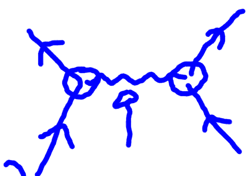
$$V_{ei}(R_i + u_i) \to V_{ei}(R) + \vec{\nabla} V_{ei}(R) \cdot \vec{u}$$

53.3 RPA



$$V_{eff} = \frac{V_c + V_p}{1 + (V_c + V_p)\chi}$$

$$= \boxed{\frac{V_c}{1+V_c\chi}} + \left[\frac{M_q}{1+V_c\chi}\right]\left[\frac{\mathcal{D}}{1 + \frac{V_p\chi}{1+V_c\chi}}\right]\left[\frac{M_{-q}}{1+V_c\chi}\right]$$



$$V_p = M^2\mathcal{D}$$

$$\frac{\mathcal{D}}{1+\frac{M_q^2\mathcal{D}}{1+V_c\chi}\chi} = \frac{-2\omega_{ip}}{q_n^2+\omega_{ip}^2 + \boxed{|M_q|^2(-2\omega_{ip})}\frac{\chi}{1+V_c\chi}}$$

$$-\omega_{ip}^2 V_c$$

$$= \frac{-2\omega_{ip}}{q_n^2+\omega_{ip}^2\left[1-\frac{V_c\chi}{1+V_c\chi}\right]}$$

$$\omega_q^2 = \frac{\omega_{ip}^2}{1+V_c\chi} = \frac{\omega_{ip}^2}{\epsilon^{RPA}(q)}$$

$$\epsilon^{RPA}(q) = \frac{q^2+q_{TF}^2}{q^2} \rightarrow \omega_q^2 \underset{\sim}{=} \left[\frac{\omega_{ip}^2}{q_{TF}^2}\right] q^2 = c_s^2 q^2$$

Bohm-Staver $c_s = \left[\frac{Z\,m}{3M}\right]^{1/2} v_F$

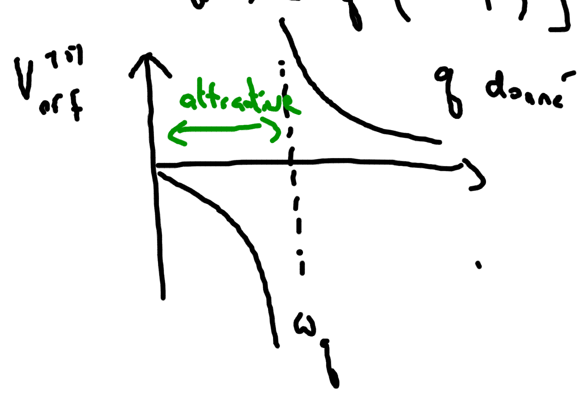
$$\frac{M_q}{\epsilon(q,0)} \; \frac{-g\,\omega_{ip}}{q_n^2+\omega_q^2} \; \frac{M_{-q}}{\epsilon(-q,0)}$$

$$= \frac{-g\,\omega_{ip}\,|M_q|^2}{\epsilon^2(q,0)} \; \frac{1}{q^2+\omega_q^2} = \frac{V(q)}{\epsilon(q,0)}\left[\frac{-\omega_q^2}{q_n^2+\omega_q^2}\right]$$

$$V^{TOT}_{eff} = \frac{V(q)}{\epsilon(q)}\left[1-\frac{\omega_q^2}{q_n^2+\omega_q^2}\right] = \frac{V(q)}{\epsilon(q)}\left[\frac{q_n^2}{q_n^2+\omega_q^2}\right]$$

$$V^{Ret}_{eff} = \frac{V(q)}{\epsilon(q,0)}\left[\frac{-(\omega+i\eta)^2}{\omega_q^2-(\omega+i\eta)^2}\right]$$

$iq_n \to \omega + i\eta$

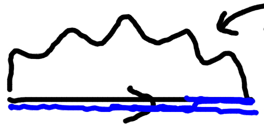


$$\frac{V(q)}{\epsilon(q) - \frac{\omega_q^2}{\omega^2}\,\epsilon(q)}$$

$$1+\frac{q_{TF}^2}{q^2} - \frac{\omega_{ip}^2}{\omega^2}$$

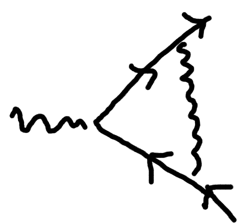
$$\frac{V(q)}{1+(\epsilon-1)_{el}+(\epsilon-1)_{i}}$$

53.4 m^*, Z, Migdal

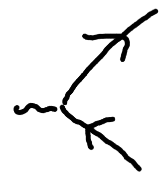


$\Sigma \longrightarrow m^* \;, Z,$

Migdal:



$= \sqrt{\frac{m}{M}}$



§4. Instabilité de la phase normale : Problème de Cooper

$$\chi = \frac{\delta \langle M \rangle}{\delta h} = \langle M\, M \rangle \longrightarrow \infty \text{ à } T_c$$

Susceptibilité de paire.

$$\langle T_\tau \psi_\uparrow(1)\, \psi_\downarrow(1)\;\; \psi_\downarrow^+(2)\, \psi_\uparrow^+(2) \rangle$$

$$\Psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow^+ \end{pmatrix} \qquad \Psi^+ = \begin{pmatrix} \psi_\uparrow^+ & \psi_\downarrow \end{pmatrix}$$ Espace de Nambu

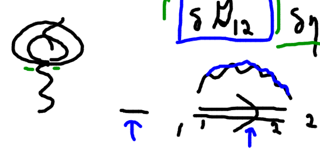
$$\left\{ \Psi^\alpha(1),\, \Psi^{+\beta}(2) \right\} = \delta_{\alpha\beta}\, \delta(1-2)$$

$$\mathcal{G}^{\alpha\beta}(1,2) = -\langle T_\tau\, \Psi(1)\, \Psi^+(2) \rangle \qquad -\Psi^+(\bar 1)\, \eta(\bar 1, \bar 2)\, \Psi(\bar 2)$$

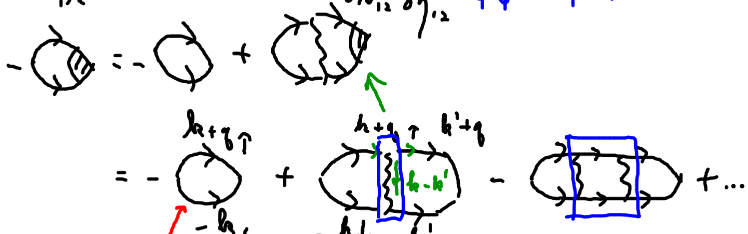
$$\mathcal{G}(1,2) = -\langle T_\tau\, \Psi(1)\, \Psi^+(2)\; e \qquad \rangle$$

$$\left.\frac{\delta \mathcal{G}^{12}(1,1)}{\delta \eta_{12}(2,2)}\right|_{\eta=0} = + \langle T_\tau\, \psi_\uparrow(1)\, \psi_\downarrow(1)\, \psi_\uparrow^+(2)\, \psi_\downarrow^+(2) \rangle$$

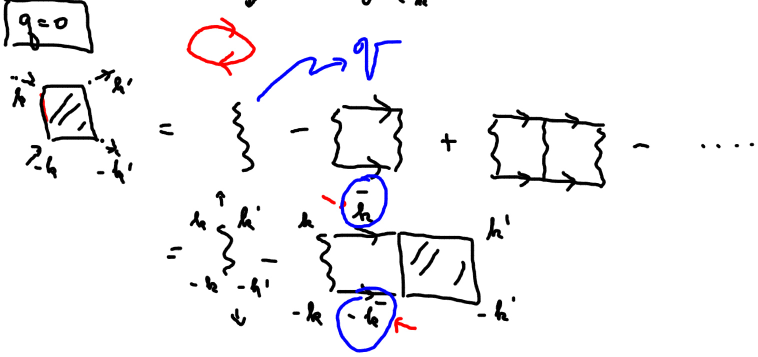
$$\frac{\delta \mathcal{G}^{12}(1,1)}{\delta \eta^{12}(2,2)} = \mathcal{G}_{11}\, \mathcal{G}_{22} + \mathcal{G}_{11} \boxed{\frac{\delta \Sigma_{12}}{\delta \mathcal{G}_{12}}} \boxed{\frac{\delta \mathcal{G}_{12}}{\delta \eta^{12}}}\, \mathcal{G}_{22}$$



$$-\frac{\delta \mathcal{G}_{12}}{\delta \eta_{12}} = -\mathcal{G}_{1}\, \mathcal{G}_{22} - \mathcal{G}_{11}\, \frac{\delta \Sigma_{12}}{\delta \mathcal{G}_{12}}\, \frac{\delta \mathcal{G}_{12}}{\delta \eta_{12}}\, \mathcal{G}_{22}$$



$\boxed{q=0}$



$$V(\vec{k}-\vec{k}', ik_n - ik_n') = \begin{cases} V(\vec{k}-\vec{k}') & \left.\begin{matrix}|k_n| < \omega_D \\ |k_n'| < \omega_D\end{matrix}\right\} \\ 0 & \text{autrement} \end{cases}$$

$$V(\vec{k}-\vec{k}') = V(\hat{k}\cdot\hat{k}') = \sum_\ell V_\ell P_\ell(\cos\theta)$$

$$\boxed{V_0 < 0}$$

$$\Omega^0_{\uparrow\downarrow}(ik_n, ik_n') = V_0\, \theta(\omega_D - |k_n|)\, \theta(\omega_D - |k_n'|)$$

$$-T\sum_{i\bar{k}_n} \int d\bar{\xi}\, N(0)\, \frac{V_0\, \theta(\omega_D - |\bar{k}_n|)\, \theta(\omega_D - |k_n'|)}{(i\bar{k}_n - \bar{\xi})(-i\bar{k}_n - \bar{\xi})}\, \Omega^0_{\uparrow\downarrow}(i\bar{k}_n, ik_n')$$

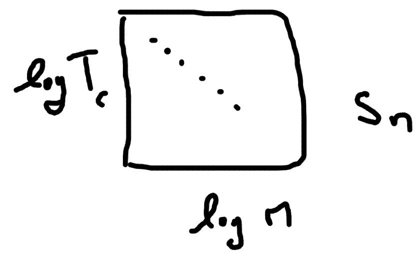
$$\boxed{\Omega^{00}_{\uparrow\downarrow}} = V_0 - N(0)\,T \sum_{i\bar{k}_n}^{\omega_0/2\pi T} \frac{\pi}{|\bar{k}_n|}\, V_0\, \Omega^{00}_{\uparrow\downarrow}$$

$$\int d\bar{\xi}\, \frac{1}{i\bar{k}_n - \bar{\xi}}\, \frac{1}{-i\bar{k}_n - \bar{\xi}} = \int d\bar{\xi}\, \frac{1}{\bar{k}_n^2 + \bar{\xi}^2} = \frac{1}{|\bar{k}_n|} \arctan \frac{\bar{\xi}}{|\bar{k}_n|}\Bigg|_{-\infty}^{\infty} = \frac{\pi}{|\bar{k}_n|}$$

$$-N(0)\,2\cancel{T} \sum_{n=0}^{\omega_0/2\pi T} \frac{\cancel{\pi}}{(2n+1)\cancel{\pi}\cancel{T}} = -N(0)\left[\gamma + \ln\left(\frac{2\omega_0}{\pi T}\right)\right]$$

γ : Cte Euler $0.577\,215\,665\ldots$

$$\Lambda^{00}_{qL} = \frac{V^0}{1 + aV^0} \quad \text{où}$$



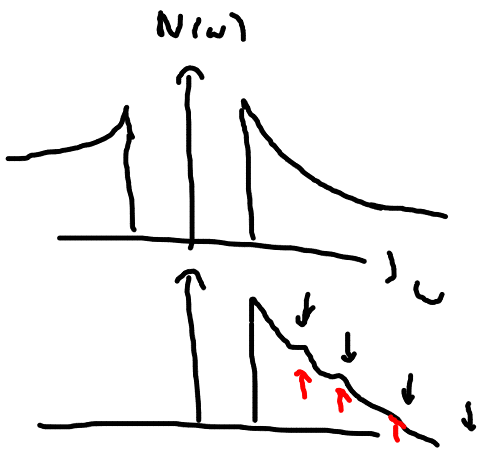
$V^0 < 0$ diverge..

$$1 = -N(0)V_0\left[\gamma + \ln\left(\frac{2\omega_D}{\pi T_c}\right)\right]$$

$$e^{-1/N(0)V_0} = e^{\gamma}\,\frac{2\omega_D}{\pi T_c}$$

$$\boxed{T_c = \left(\frac{2e^{\gamma}}{\pi}\right)\omega_D\, e^{1/N(0)V_0}}$$

$$\boxed{V_0 < 0}$$


N(ω)
ω


ω

54 BCS

Analogie avec Ferro.

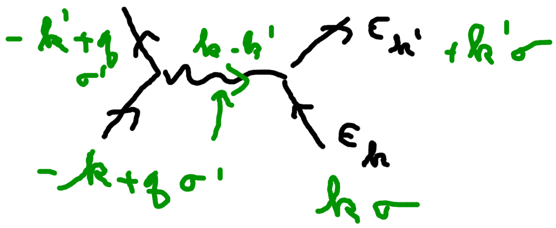
$\langle M_{Q=0}(\omega=0) M_{-Q=0}(\omega=0)\rangle \to \infty$ à T_c	$\langle \Delta(\omega=0) \; \Delta^{+}(\omega=0) \rangle \to \infty$ à T_c
$T<T_c$ $\quad \langle M^2_{Q=0}\rangle \neq 0$	$\langle \Delta_{Q=0} \rangle \neq 0$
$\langle c^{+}_{k\uparrow} c_{k\uparrow} - c^{+}_{k\downarrow} c_{k\downarrow} \rangle \neq 0$	$0 \neq \langle c^{+}_{k\uparrow} c^{+}_{-k\downarrow} - c^{+}_{k\downarrow} c^{+}_{-k\uparrow} \rangle$
	$\langle c^{+}_{k\uparrow} c^{+}_{-k\downarrow} \rangle = \langle c^{+}_{-k\uparrow} c^{+}_{k\downarrow} \rangle$

symétrie brisée : inv. sous rotation	sym. brisée : inv. de jauge.
Brisure d'ergodicité. $\mathrm{Tr}\left[e^{-\beta H}\right] \to \mathrm{Tr}\left[e^{-\beta(H - h^2 M^2)}\right]$ $V \to \infty$ premier $h \to 0$ dernier	$\mathrm{Tr}\left[e^{-\beta(H - \mu N)}\right]$ $= \mathrm{Tr}\left[e^{-\beta\left(H - \mu N - \eta \sum_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \text{c.h.}\right)}\right]$
$M^2 \sim \cos\theta \quad \theta \to 0$	$\langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \rangle \propto e^{i\theta} \ldots$ Goldstone $\to$ plasma $\leftrightarrow$ Meissner

55.2 H BCS "résumé"

$$\langle U n_\uparrow n_\downarrow \rangle \to \boxed{U\langle n_\uparrow \rangle n_\downarrow + U\langle n_\downarrow \rangle n_\uparrow} - U\langle n_\uparrow \rangle \langle n_\downarrow \rangle$$

$U\langle n_\uparrow \rangle\langle n_\downarrow \rangle$



$\omega < \omega_D$

$(\epsilon_k - \epsilon_{k'}) < \omega_D$

$$H = \sum_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma} \epsilon_k + \frac{1}{2N} \sum_{\substack{k k' q \to 0 \\ \sigma\sigma'}} c^\dagger_{k\sigma} c^\dagger_{-k+\underset{0}{\cancel{q}}\,\sigma'} V_{k-k'} c_{-k'+\underset{0}{\cancel{q}}\,\sigma'} c_{k'\sigma}$$

$$\left\{ \begin{array}{l} \langle c^\dagger c^\dagger \rangle V c c \\ + c^\dagger c^\dagger V \langle c c \rangle = P' \end{array} \right.$$

→ s'annule sauf pour $q = 0$