55. Supraconductivité

1. Brisure de symétrie, analogie avec ferroaimant
2. Hamiltonien BCS réduit ✓
3. $\mathcal{G}$ de Nambu ✓
4. Équation du gap ✓
5. DOS ✓
6. Eliashberg

56. Energie libre et fonctionnelles

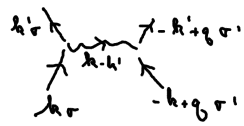
1. Luttinger-Ward
2. Kadanoff-Baym
3. Potthoff : relation DMFT

55.2 BCS

$AB = \langle A\rangle B + A\langle B\rangle - \langle A\rangle\langle B\rangle$

$\langle AB\rangle = \langle A\rangle\langle B\rangle$

$$\mathcal{H} = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \frac{1}{2N}\sum_{\substack{kk'q\\ \sigma\sigma'}} c^\dagger_{k\sigma} c^\dagger_{-k+q\sigma'} V_{k-k'} c_{-k'+q\sigma'} c_{k'\sigma}$$



$$\boxed{\begin{aligned}&\langle c^\dagger_{k\uparrow} c^\dagger_{-k+q\downarrow}\rangle = b^*_k \delta_{q,0}\\ &= -\langle c^\dagger_{k\downarrow} c^\dagger_{-k\uparrow}\rangle\end{aligned}}$$

$$\mathcal{H} = \sum_{k\sigma} \epsilon^{HF}_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma} + \frac{2}{2N}\sum_{kk'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} V_{k-k'} \langle c_{-k'\downarrow} c_{k'\uparrow}\rangle$$

$$+ \frac{2}{2N}\sum_{kk'} \langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\rangle V_{k-k'} c_{-k'\downarrow} c_{k'\uparrow}$$

$$\boxed{\Delta_k = -\frac{1}{N}\sum_{k'} V_{k-k'} \langle c_{-k'\downarrow} c_{k'\uparrow}\rangle}$$

$$K = \sum_{k\sigma} \xi_k c^\dagger_{k\sigma} c_{k\sigma} - \sum_k \left(c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \Delta_k + c_{-k\downarrow} c_{k\uparrow} \Delta^*_k \right)$$

$$\boxed{\Psi_k = \begin{pmatrix} c_{k\uparrow} \\ c^\dagger_{-k\downarrow}\end{pmatrix} \quad \Psi^\dagger_k = \left(c^\dagger_{k\uparrow} \;\; c_{-k\downarrow}\right)}$$

$$\{\Psi^\alpha_k, \Psi^{\dagger\beta}_{k'}\} = \delta_{\alpha\beta}\delta_{kk'}$$

$$K = \sum_k \left(c^\dagger_{k\uparrow} \;\; c_{-k\downarrow}\right) \begin{pmatrix} \xi_k & -\Delta_k \\ -\Delta^*_k & -\xi_k \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^\dagger_{-k\downarrow}\end{pmatrix}$$

$$K = \sum_k \Psi^\dagger_k \left(\xi_k \tau_3 - \Delta^k_1 \tau_1 + \Delta^k_2 \tau_2 \right) \Psi_k$$

$\Delta_1 = \mathrm{Re}\,\Delta$

$\Delta_2 = \mathrm{Im}\,\Delta$

$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}$ $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix}$

$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}$

$$\boxed{\mathcal{G}^{\alpha\beta}(1,2) = -\langle T_\tau \Psi^\alpha(1) \Psi^{\dagger\beta}(2)\rangle}$$

$$\frac{\partial \mathcal{G}_k(\tau)}{\partial\tau} = -\delta(\tau) - \langle T_\tau \frac{\partial \Psi_k}{\partial\tau}(\tau) \Psi^\dagger_k\rangle$$

$$\frac{\partial \Psi^\alpha_k}{\partial\tau} = [K, \Psi^\alpha_k]$$

$$\left[\sum_{k'} \Psi^{\dagger\gamma}_{k'} \tau_i^{\gamma\beta} \Psi^\beta_{k'}, \Psi^\alpha_k\right]$$

$$= -\sum_{k'} \{\Psi^{\dagger\gamma}_{k'}, \Psi^\alpha_k\} \Psi^\beta_{k'} \tau_i^{\gamma\beta}$$

$$= -\tau_i^{\alpha\beta} \Psi^\beta_k$$

$$K = \sum_k \Psi^\dagger_k |n|\, \hat{n}\cdot\vec{\tau}\, \Psi_k \qquad \vec{n} = \left(-\Delta^k_1 \;\; \Delta^k_2 \;\; \xi_k\right)$$

$$\frac{\partial\mathcal{G}}{\partial\tau} = -\delta(\tau) I - |n|\,\hat{n}\cdot\vec{\tau}\,\mathcal{G}$$

$\vec{n} = |n|\,\hat{n}$

$$\boxed{\begin{aligned}&(aI + \vec{b}\cdot\vec{\tau})(aI - \vec{b}\cdot\vec{\tau})\\ &\quad = a^2 - (\vec{b}\cdot\vec{\tau})(\vec{b}\cdot\vec{\tau}) = a^2 - b^2\\ &\text{car } (\vec{a}\cdot\vec{\tau})(\vec{c}\cdot\vec{\tau}) = \vec{a}\cdot\vec{c}\, I + i(\vec{a}\times\vec{c})\cdot\vec{\tau}\end{aligned}}$$

$$(ik_n I - \vec{n}\cdot\vec{\tau})\mathcal{G} = I$$

$$\mathcal{G} = (ik_n I - \vec{n}\cdot\vec{\tau})^{-1} =$$

$$\boxed{\mathcal{G} = \frac{ik_n I + \vec{n}\cdot\vec{\tau}}{(ik_n)^2 - E_k^2}}$$ où $E_k^2 = \xi_k^2 + |\Delta_k|^2$

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55.4 Équation du gap

$$\Delta_k = -\sum_{k'} V_{k-k'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$$

$$= -\frac{1}{N}\sum_{k'} V_{k-k'} \langle \Psi^{\dagger 2}_{k'} \Psi^{1}_{k'} \rangle$$

$$= +\frac{1}{N}\sum_{k'} V_{k-k'} \langle T_\tau \Psi^{1}_{k'}(\tau\to 0^-) \Psi^{\dagger 2}_{k'} \rangle$$

$$= -\frac{1}{N}\sum_{k'} V_{k-k'}\, T \sum_{ik_n} e^{-ik_n 0^-} \mathcal{G}^{12}_{k'}(ik_n)$$

$$\Delta_k = \frac{1}{N}\sum_{k'} V_{k-k'}\, T \sum_{ik_n} \frac{\Delta_{k'} e^{-ik_n 0^-}}{(ik_n)^2 - E_{k'}^2}$$

$$T\sum_{ik_n} \frac{e^{-ik_n 0^-}}{2E_{k'}} \left[\frac{1}{ik_n - E_{k'}} - \frac{1}{ik_n + E_{k'}} \right]$$

$$= \frac{1}{2E_{k'}} \left[f(E_{k'}) - f(-E_{k'}) \right] \qquad f(-E_k) = 1 - f(E_k)$$

$$\boxed{\Delta_k = -\int \frac{d^3k'}{(2\pi)^3} \frac{\boxed{V_{k-k'}}}{2E_{k'}} \left[1 - 2f(E_{k'}) \right] \Delta_{k'}}$$

Équation du gap BCS

$$\zeta_k = \sqrt{\frac{1-2f(E_k)}{2E_k}} \qquad E_k = \sqrt{\xi_k^2 + |\Delta|^2}$$

$$e^{i\theta}\left[\zeta_k \Delta_k\right] = -\int \frac{d^3k'}{(2\pi)^3} \left(C_k V_{k-k'} C_{k'} \right) \left[C_{k'} \Delta_{k'} \right] e^{i\theta} \quad \frac{1}{e^{\beta E_k}+1}$$

À $T=0$

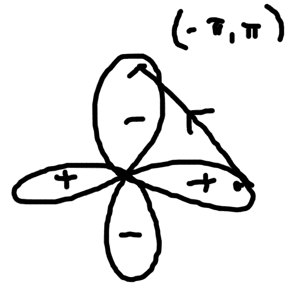
$$\cancel{\Delta_k} = -\int_{-\omega_D}^{\omega_D} N(0)\, d\xi_{k'} \frac{V_{k-k'} \cancel{\Delta_{k'}}}{2\sqrt{\xi_k^2 + |\Delta_k|^2}}$$

Approx $\Delta \ll \omega_D$

$$\boxed{\frac{2\Delta}{k_B T_c} = 3.53}$$

V_Q

$$\Delta_k = - V_Q \, \Delta_{k+Q} \frac{\left(1 - 2f(E_{k+Q})\right)}{2E_{k+Q}}$$



55.5 Densité d'états

$$A(k,\omega) = -2\,\mathrm{Im}\, G^R_\uparrow(k,\omega)$$

$$= -2\,\mathrm{Im}\left[\frac{\omega+\xi_k}{(\omega+i\eta)^2-E_k^2}\right]$$

$$= -2\,\mathrm{Im}\,\frac{\omega+\xi_k}{2E_k}\left(\frac{1}{\omega+i\eta-E_k}-\frac{1}{\omega+i\eta+E_k}\right)$$

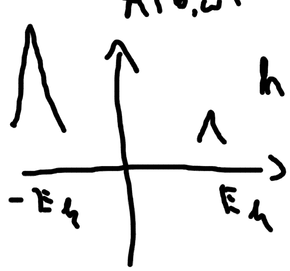
$$= +2\pi\left[\frac{\omega+\xi_k}{2E_k}\delta(\omega-E_k)-\frac{\omega+\xi_k}{2E_k}\delta(\omega+E_k)\right]$$

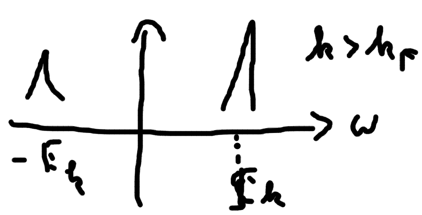
$$= 2\pi\left[\frac{1}{2}\left(1+\frac{\xi_k}{E_k}\right)\delta(\omega-E_k)+\frac{1}{2}\left(1-\frac{\xi_k}{E_k}\right)\delta(\omega+E_k)\right]$$

facteurs de cohérence

$$= 2\pi\left[u_k^2\,\delta(\omega-E_k)+v_k^2\,\delta(\omega+E_k)\right]$$

$A(k,\omega)$





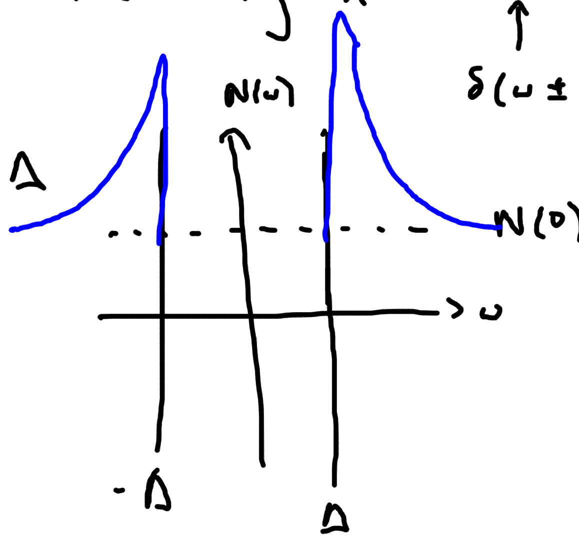
$$E_k = \sqrt{\xi_k^2+|\Delta|^2}$$

$$N(\omega) = \int \frac{d^3k}{(2\pi)^3} A(k,\omega) = N(0) \int d\xi_k \, A(k,\omega)$$

$$\delta\left(\omega \pm \sqrt{\xi_k^2 + \Delta^2}\right)$$

$$N(\omega) = \frac{\omega}{\sqrt{\omega^2 - \Delta^2}} \quad \omega > \Delta$$

$$= 0 \quad \omega < \Delta$$



55.6 Eliashberg.

$$\tilde{K}^0 = H_0 + (H_\chi + H_\Delta) - \mu N$$

$$H'_{int} = H_{int} - (H_\chi + H_\Delta)$$

$$= \frac{1}{2} \sum_{k k' q} V_q \left(\Psi^\dagger_{k+q} \tau_3 \Psi_k\right)\left(\Psi^\dagger_{k'-q} \tau_3 \Psi_{k'}\right)$$

$$- \left(H_\chi + \sum_k \Psi^\dagger_k \left(\chi_k \tau_3 - \Delta_1 \tau_1 + \Delta_2 \tau_2\right) \Psi_k\right)$$

$\Sigma(\cancel{k}, \omega) =$  $= 0$

$\langle \psi \psi \rangle$

$$\Psi_{BCS} = \prod_k (u_k + v_k e^{i\theta} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) |0\rangle$$

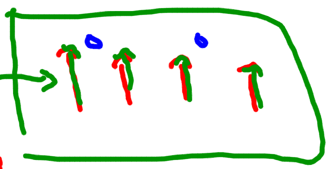
$$\langle c_{k\uparrow} c_{-k\downarrow} \rangle \neq 0 \qquad |BCS\rangle = \dots e^{iN\theta} |N\rangle + e^{i(N+2)\theta} |N+2\rangle + \dots$$

$$\boxed{|N\rangle = \int d\theta \, e^{-iN\theta} |BCS\rangle} \qquad |\theta\rangle = \sum_N e^{iN\theta} |N\rangle$$

$$[N, \theta] = i \qquad \theta$$

$$\langle N | \theta | N \rangle$$

$$\left[\cdot\cdot \qquad \cdot\cdot \right] \theta \left[\cdot\cdot e^{i\theta N} |N\rangle + e^{i(1+N)\theta} |N+1\rangle \cdots\cdots \right]$$



$$\langle S_z \rangle = 0$$

$$\frac{d\theta}{dt} = \frac{2eV}{h}$$

$$\boxed{\left[\hat{N} . e^{\pm i\hat{\theta}} \right] = \pm e^{\pm i\hat{\theta}}}$$

$$|N\rangle + |N+1\rangle$$

$$\int d\theta \rightarrow |N\rangle$$

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