12. Kubo pour conductivité

1. Couplage minimal
2. Réponse de j^A à $\vec{A}$ et φ
3. Kubo transverse
4. Kubo longitudinal
    - Règle de somme f + invariance de jauge
    - Autre expression

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13. Poids de Drude, métal, isolant, S.C.

1. Poids de Drude
✓ 2. Métal
✓ 3. Isolant
✓ 4. Supraconducteur
✓ 5. Comparaison
✓ 6. λ_L à partir d'optique Ferrel-Glover-Tinkham

$$\frac{\hbar}{i}\nabla \longrightarrow \frac{\hbar}{i}\vec{\nabla} - e\vec{A} \qquad\qquad i\hbar\frac{\partial}{\partial t} \to i\hbar\frac{\partial}{\partial t} - e\varphi$$

$$S_{e\text{-}m} = e\int \boxed{A_\mu \frac{dr^\mu}{dt}}\, dt \qquad A_\mu = \left(-\frac{\varphi}{c}, \vec{A}\right) \quad r^\mu = \begin{pmatrix} ct \\ \vec{r} \end{pmatrix}$$

$$= e\int \left(A_\mu + \partial_\mu \Lambda\right)\frac{dr^\mu}{dt}\, dt$$

$$= e\int \frac{d\Lambda}{dt}\, dt$$

$$\boxed{L_{e\text{-}m} = -e\varphi + e\vec{A}\cdot\vec{v}}$$

$$\boxed{L = \frac{1}{2}mv^2 - e\varphi + e\vec{A}\cdot\vec{v}} + L_{em}$$

$$\boxed{p_a = \left(\frac{\partial L}{\partial v_a}\right)_{\vec{r},\vec{A}} = mv_a + eA_a}$$

$$\frac{\hbar}{i}\vec{\nabla} \quad \text{M.Q.}$$

Courant:

Euler-Lagrange pour $\vec{A}, \varphi$

$$L = \vec{p}\cdot\vec{v} - \vec{H}$$

$$\boxed{\left(\frac{\partial L_{e\text{-}m}}{\partial A_a}\right)_{\vec{r},\vec{v}} = e v_a = -\left(\frac{\partial H}{\partial A_a}\right)_{p,r}}$$

12.2 réponse de j à A et φ

$$\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla}-e\vec{A}\right)^2=-\frac{\hbar^2}{2m}\nabla^2-\frac{\hbar e}{2mi}\left(\vec{\nabla}\cdot\vec{A}+\vec{A}\cdot\vec{\nabla}\right)+\frac{e^2A^2}{2m}$$

$$\delta\mathcal{H}(t)=\delta\mathcal{H}_A+\delta\mathcal{H}_\varphi$$

$$\boxed{\delta\mathcal{H}_A(t)=-\int d^3r\, j(r)\cdot A(r)}$$

$$\longrightarrow j(r)=\sum_\alpha\frac{\hbar e}{2mi}\left(\nabla_{r_\alpha}\delta(r-r_\alpha)+\delta(r-r_\alpha)\cdot\nabla_{r_\alpha}\right)$$

Courant paramagnétique $\quad e\dfrac{\partial\psi^*\psi}{\partial t}=-\vec{\nabla}\cdot\vec{j}$

$$\psi^*\left(i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi\right)$$

$$\psi\left(-i\hbar\frac{\partial\psi^*}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi^*\right)$$

$$\vec{j}^{A}(r,t) = \frac{e}{2m}\sum_{\alpha}\left(\frac{\hbar}{i}\vec{\nabla}_{\alpha} - e\vec{A}\right)\cdot\delta(r-r_\alpha)$$

$$+\,\delta(r-r_\alpha)\cdot\left(\frac{\hbar}{i}\vec{\nabla} - e\vec{A}\right)$$

$$= j(r,t) - \frac{e^2}{m}\sum_{\alpha}\delta(r-r_\alpha)\vec{A}$$

courant diamagnétique

$$\boxed{\vec{j}^{A}(r,t) = \vec{j}(r,t) - \frac{e}{m}\vec{A}\,\rho(r,t)}$$

paramagnétique

$$\boxed{\begin{aligned}\rho(r,t) &= \sum_{\alpha} e\,\delta(r-r_\alpha)\\ &= e\,n(r)\end{aligned}}$$

$$\delta\langle \vec{j}^{A}_{a}(q,\omega)\rangle = \chi^{R}_{j_a j_b}(q,\omega)\,A_b(q,\omega) - \frac{ne^2}{m}A_a(q,\omega)$$

$$-\,\chi_{j_a\rho}(q,\omega)\,\varphi(q,\omega)$$

$$H + e\varphi(r_\alpha,t) \longrightarrow H + e\int d^3r\sum_{\alpha}\delta(r-r_\alpha)\,\varphi(r,t)$$

$$\boxed{\begin{aligned}\delta\langle \vec{j}^{A}(q,\omega)\rangle &= \left[\chi^{R}_{j_a j_b}(q,\omega) - \frac{ne^2}{m}\delta_{ab}\right]\frac{A_b(q,\omega)\,i(\omega+i\eta)}{i(\omega+i\eta)}\\ &\quad -\chi^{R}_{j_a\rho}(q,\omega)\,\varphi(q,\omega)\end{aligned}}$$

$$j = \sigma E \qquad E = -\frac{\partial A}{\partial t} - \nabla\varphi$$

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12.3 σ transversal

$$\vec{A} = \vec{A}_L + \vec{A}_T \quad \text{où} \quad \vec{\nabla}\cdot\vec{A}_T = 0 \quad \longrightarrow \hat{q}\cdot(\vec{A}_T) = 0$$

$$\vec{\nabla}\times\vec{A}_L = 0 \longrightarrow \vec{q}\times(\hat{q}A_L) = 0$$

en T.F.

$$\vec{A}(q,\omega) = \hat{q}\,\hat{q}\cdot\vec{A} + (I - \hat{q}\hat{q})\cdot\vec{A}$$

$$\vec{A} \to \vec{A} + \vec{\nabla}\Lambda$$

$$\varphi \to E_L$$

$$f(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} f(\omega) \qquad \omega \to \omega + i\eta$$

$$\frac{\partial f}{\partial t} = \int \frac{d\omega}{2\pi}(-i\omega)e^{-i\omega t} f(\omega)$$

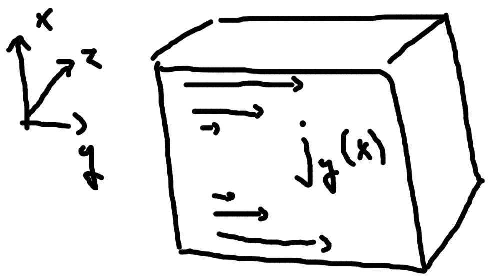
$$E_a(q,\omega) = i(\omega + i\eta)A_a(q,\omega)$$

$$\boxed{\text{T.F.}\frac{\partial f}{\partial t}} = -i\omega\,\boxed{\text{TF } f}$$

$$\text{T.F.}\frac{\partial f}{\partial r_a} = i q_a f(q)$$

$$E^T = -\frac{\partial A^T}{\partial t} - \cancel{\nabla\varphi}$$

$$\delta\langle \dot{j}_a^A(q,\omega)\rangle = \underbrace{\boxed{\frac{1}{i(\omega+i\eta)}\left[\chi^R_{\dot{j}_a\dot{j}_b}(q,\omega) - \frac{ne^2}{m}\delta_{ab}\right]}}_{\sigma^T_{ab}} E_b(q,\omega)$$



$$\langle \delta \dot{j}_y^A(q_x,\omega)\rangle$$

$$= \sigma_{\dot{j}_y\dot{j}_y}(q_x,\omega)\, E_y(q_x,\omega)$$

19.4 $\boxed{\text{conduct. longitudinale}}$

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}\cdot\vec{j} = \rightarrow \boxed{\frac{\partial \rho}{\partial t} = -i q_x j_x}$$

$$\frac{\partial}{\partial t}\chi^R_{j_x\rho}(q_x,t) = \frac{\partial}{\partial t}\left(\frac{i}{\hbar V}\langle[j_x(q_x,0),\rho(-q_x,-t)]\rangle\theta(t)\right)$$

$$= \frac{i}{\hbar V}\langle[j_x(q_x,0),\rho(-q_x,0)]\rangle\delta(t)$$

$$-iq_x\frac{i}{\hbar V}\langle[j_x(q_x,0),j_x(-q_x,-t)]\rangle\theta(t)$$

$$-i(\omega+i\eta)\chi^R_{j_x\rho}(q_x,\omega) = -iq_x\chi^R_{j_xj_x}(q_x,\omega)$$

$$+\frac{i}{\hbar V}\langle[j_x(q_x,0),\rho(-q_x,0)]\rangle$$

$$\chi''_{j_x\rho}(q_x,t) = \frac{1}{2\hbar}\langle[j_x,\rho(-t)]\rangle \qquad \int\frac{d\omega}{2\pi}e^{-i\omega t}f(t)\delta(t) = \frac{f(0)}{2\pi}$$

$$\chi''_{j_x\rho}(q_x,t=0) = \int\frac{d\omega'}{2\pi}\chi''_{j_x\rho}(q_x,\omega')\frac{iq_x}{iq_x}$$

$$\boxed{i\omega'\rho = iq_xj_x}$$

$$= \int\frac{d\omega'}{2\pi}\frac{\cancel{i}\omega'}{\cancel{i}q_x}\chi''_{\rho\rho}(q_x,\omega') = \frac{1}{q_x}\frac{q_x^2ne^2}{2m}$$

$$+q_x\frac{ne^2}{2m}$$

$$\boxed{\chi^R_{j_x\rho}(q_x,\omega) = \frac{1}{-i(\omega+i\eta)}(-iq_x)\left[\chi^R_{j_xj_x}(q_x,\omega) - \frac{ne^2}{m}\right]}$$

$$\boxed{\sigma_{xx}(q_x,\omega) = \frac{1}{i(\omega+i\eta)}\left[\chi^R_{j_x j_x}(q_x,\omega) - \frac{ne^2}{m}\right]}$$

$E_x(q_x,\omega)$

$$\delta\langle j^A(q_x,\omega)\rangle = \sigma_{xy}(q_x,\omega)\left(i(\omega+i\eta)A - iq_x\varphi\right)$$

Soit $\varphi = 0$

$$\boxed{\Lambda(x,\omega=0)}$$

$$-\frac{\partial A}{\partial t} - \nabla\varphi$$

$$A \to A + \nabla\Lambda$$

$$\varphi \to \varphi - \frac{\partial\Lambda}{\partial t}$$

inv. de jauge $\Rightarrow$

$$\boxed{\chi^R_{j_x j_x}(q_x,\omega=0) = \frac{ne^2}{m}}$$

Règle de somme f

$$\int\frac{d\omega'}{\pi}\frac{\chi''_{j_x j_x}(q_x,\omega')}{\omega'}$$

$$\omega'\rho = q_x j_x$$

$$j_x = \frac{\omega'}{q_x}\rho$$

$$\mathrm{Re}\,\chi^R_{j_x j_x} = \mathcal{P}\int\frac{d\omega'}{\pi}\frac{\chi''_{j_x j_x}(q_x,\omega')}{\omega'}$$

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13. Drude, métal, isolant, S.C.

1. Drude :

Longitudinale

$$\boxed{\sigma_{xy}(q_T,\omega) = \frac{1}{i(\omega+i\eta)}\left[\chi^R_{j_x j_y}(q_T,\omega) - \frac{ne^2}{m}\right]}$$

$$\mathrm{Re}\,\sigma_{xx}(q_T,\omega) = \mathcal{P}\frac{\chi''_{j_x j_x}}{\omega} - \pi\,\delta(\omega)\left[\mathrm{Re}\,\chi^R(q_T,\omega) - \frac{ne^2}{m}\right]$$

$$\boxed{D = \pi\left(\frac{ne^2}{m} - \mathrm{Re}\,\chi^R(q_T,\omega=0)\right)}$$

$$\chi^R = \mathrm{Re}\,\chi + i\chi''$$

$$\frac{1}{i(\omega+i\eta)} \to \frac{1}{i}\mathcal{P}\frac{1}{\omega} - \pi\,\delta(\omega)$$

2. Métal

Libres $j = n e v$

$$\frac{\partial j}{\partial t} = ne \frac{\partial v}{\partial t} = \frac{ne^2 E}{m}$$

$$-i(\omega + i\eta)\, j = \frac{ne^2}{m} E$$

$$j = \boxed{\frac{ne^2}{-i(\omega + i\eta)\, m}} E$$

$$\mathrm{Re}\,\sigma = \mathrm{Re}\left[-\frac{1}{i} \mathcal{P}\left(\frac{1}{\omega}\right) + \pi \delta(\omega) \right] \frac{ne^2}{m}$$

$$\boxed{\mathrm{Re}\,\sigma = \pi \frac{ne^2}{m} \delta(\omega)}$$

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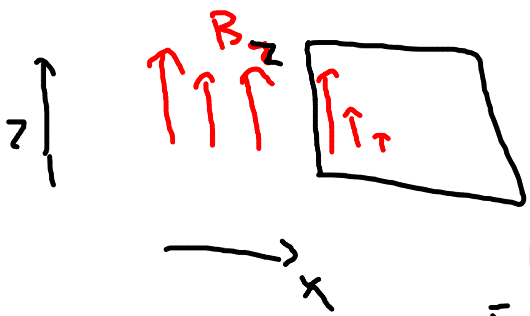
3. Isolant

$$\frac{ne^2}{m} = \mathrm{Re}\, \chi^R_{j_x j_x}(q_x, 0)$$

$$= \mathcal{P} \int \frac{d\omega'}{\pi} \frac{\chi''_{j_x j_x}(q_x, \omega')}{\omega'}$$

4. Supraconducteur



$$\langle \delta j_y^A (q_x, 0) \rangle$$

$$= \left[\chi^R_{j_y j_y} (q_x, 0) - \frac{ne^2}{m} \right] A_y(q_x, 0)$$

$$\boxed{\langle \delta j_y^A (q_x, 0) \rangle = - \frac{n_s e^2}{m} A_y(q_x, \omega)}$$

$$\frac{n_s e^2}{m} = \left[\frac{ne^2}{m} - \chi^R_{j_y j_y} (q_x, 0) \right]$$

$$B_z = B_z(0) e^{-x/\lambda_L}$$

$$\frac{1}{\lambda_L^2} = \frac{n_s e^2}{m} \mu_0$$

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$$\lim_{q_x \to 0} \left[\int d^3r \; e^{-iq_x(x-x')} \chi^R_{j_x j_x}(r-r') \right.$$

$$\left. \neq \int d^3r \; e^{-iq_x(x-x')} \chi^R_{j_y j_y}(r-r') \right]$$

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