$$\delta \langle j_a^A (q_x, 0) \rangle = \left[\chi^R_{j_a j_b}(q_x, 0) - \frac{ne^2}{m}\delta_{ab} \right] A_b(q_x, 0)$$

$$\boxed{\sigma_{ab}(q_x, \omega) = \frac{1}{i(\omega + i\eta)} \left[\chi^R_{j_a j_b}(q_x, \omega) - \frac{ne^2}{m}\delta_{ab} \right]}$$

$$\chi^R_{j_x j_x}(q_x, 0) = \frac{ne^2}{m}$$

$$\langle \delta \vec{j}^{\,T}(q,0)\rangle = -\frac{n_s e^2}{m}\vec{A}^T(q,0)$$

$$\nabla \times \langle \delta j^T(q,0)\rangle = -\frac{n_s e^2}{m}\vec{B}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 j - \cancel{\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}} \to 0$$

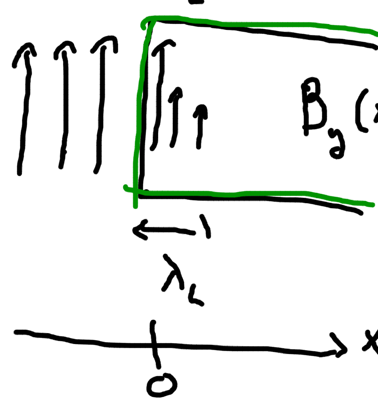
$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = -\frac{n_s e^2}{m}\mu_0\vec{B}$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \vec{\nabla}^2\vec{B} = -\frac{n_s e^2}{m}\mu_0\vec{B} = -\frac{1}{\lambda_L^2}\vec{B}$$

$$\boxed{\frac{1}{\lambda_L^2} = \frac{n_s e^2}{m}\mu_0}$$

$$\nabla^2\vec{B} = \frac{1}{\lambda_L^2}\vec{B}$$

$$\frac{\partial^2 B_y}{\partial x^2} = \frac{1}{\lambda_L^2}B_y$$



$$B_y(x) = e^{-x/\lambda_L}B(0)$$

$$\sigma_{j_y j_y}(q_x,\omega)$$

$$= \frac{1}{i(\omega+i\eta)}\left[\chi^R_{j_y j_y}(q_x,\omega) - \frac{ne^2}{m}\right]$$

$$\mathrm{Re}\,\sigma_{j_y j_y}(q_x,\omega)$$

$$= \mathcal{P}\,\frac{\chi''_{j_y j_y}(q_x,\omega)}{\omega}$$

$$-\pi\delta(\omega)\left[\mathrm{Re}\,\chi^R_{j_y j_y}(q_x,\omega) - \frac{ne^2}{m}\right]$$

$$\underline{\int \frac{d\omega}{2\pi}\,\mathrm{Re}\,\sigma_{j_y j_y}(q_x,\omega)} = \boxed{\mathcal{P}\int \frac{d\omega}{2\pi}\cancel{\frac{\chi''_{j_y j_y}(q_x,\omega)}{\omega}}} - \frac{\pi}{2\pi}\left[\cancel{\mathrm{Re}\,\chi^R_{j_y j_y}(q_x,0)} - \frac{ne^2}{m}\right] = +\frac{ne^2}{2m}$$

$$\boxed{\chi^R_{jj}(q,\omega) = \int \frac{d\omega'}{\pi}\,\frac{\chi''_{jj}(q,\omega')}{\omega'-\omega-i\eta}}$$

Cas supra:

$$\int \frac{d\omega}{2\pi}\,\mathrm{Re}\,\sigma(q_x,\omega) = \frac{(n-n_s)e^2}{2m}$$

$$\nabla\times B = \mu_0 j - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \quad \sigma E$$

$$-\frac{\partial B}{\partial t} = \nabla\times E$$

$$\frac{1}{c^2} = \mu_0\epsilon_0$$

	D	D_s
Metal	D	0
Isolant	0	0
S.C.	D	D_s

Fréquences de Matsubara et temps imaginaire.

$$\boxed{\chi_{A_iA_j}(\tau) = \frac{1}{\hbar}\left[\langle A_i(\tau) A_j\rangle \theta(\tau) + \langle A_j A_i(\tau)\rangle \theta(-\tau)\right]}$$

$e^{\beta H}$ $\quad e^{-\beta H}$

$$A_i(\tau) = e^{\tau H/\hbar} A_i e^{-\tau H/\hbar}$$

$$\boxed{A_i(t) = e^{itH/\hbar} A_i e^{-itH/\hbar}}$$

$e^{-\beta H}$

$$\frac{\partial U_I}{\partial \tau} = \delta H_I(\tau)\, U_I(\tau)$$

$$\chi_{A_iA_j}(\tau)$$

$$= \frac{1}{\hbar}\langle T_\tau A_i(\tau) A_j\rangle$$

$$\chi_{A_iA_j}(\tau) = \frac{1}{\hbar}\Big(\langle A_i(\tau)A_j\rangle\,\theta(\tau) + \langle A_j A_i(\tau)\rangle\,\theta(-\tau)\Big)$$

$$\boxed{-\beta\hbar \leq \tau \leq \beta\hbar}$$

cas $\tau < 0$ $\qquad \sum_{n,m} \langle n| \qquad |m\rangle\langle m|\,|n\rangle$

Soit $\tau < 0$

$$\boxed{\chi_{A_iA_j}(\tau) = \frac{1}{\hbar}\langle A_j A_i(\tau)\rangle} = \frac{1}{\hbar}\,\mathrm{Tr}\left[\frac{e^{-\beta H}}{Z}A_j A_i(\tau)\right]$$

$$= \frac{1}{\hbar}\,\mathrm{Tr}\left[e^{-\beta H}e^{\beta H}A_i(\tau)\frac{e^{-\beta H}}{Z}A_j\right]$$

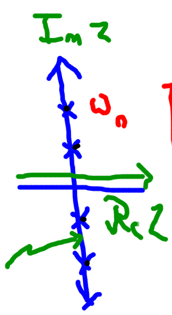
$$= \frac{1}{\hbar}\langle A_i(\tau+\beta\hbar)A_j\rangle = \boxed{\chi_{A_iA_j}(\tau+\beta\hbar)}$$

$$\Longrightarrow \chi_{A_iA_j}(\tau) = \frac{1}{\beta\hbar}\sum_n e^{-i\omega_n\tau}\chi_{A_iA_j}(i\omega_n)$$

Fréquences de Matsubara $\quad \boxed{\omega_n = \frac{2\pi n}{\beta\hbar}} \qquad e^{i\omega_n(\tau+\beta\hbar)} = e^{i\omega_n\tau}$

$$\Longrightarrow \chi_{A_iA_j}(i\omega_n) = \int_0^{\beta\hbar} d\tau\, e^{i\omega_n\tau}\chi_{A_iA_j}(\tau)$$

Temps imaginaire



$$\boxed{\chi_{A_iA_j}(\underbrace{i\omega_n}_{z}) = \int \frac{d\omega'}{\pi}\,\frac{\chi''_{A_iA_j}(\omega')}{\omega' - \underbrace{i\omega_n}_{z}}}$$

$i\omega_n \longrightarrow \omega + i\eta$

χ^R

Soit la fct. d'une variable complexe z

$$f(z) = z^2 + 2z$$

Quel est $f(z)$ si $z = 2$ $\qquad$ si $z = 2i + 4$

si $z = 3i$

Partie $\overline{\underline{III}}$

15. Définition du propagateur (fonction de Green)

16. Information dans G

1. Repr. en opérateur
2. Relation à DOS
3. Repr. spectrale

15. Définition (fct de Green ou propagateur)

$$\psi(\vec{r},t) = \langle r | e^{-iHt/\hbar} | \psi_H \rangle \qquad \hbar = 1$$

$$|\psi_S(t')\rangle = e^{-iHt'/\hbar} |\psi_H\rangle$$

$$\psi(\vec{r},t) = \langle r | e^{-iH(t-t')} |\psi_0(t')\rangle$$

$$\int d^3r' \, |r'\rangle\langle r'|$$

$$\psi(\vec{r},t)\,\theta(t-t') = \int d^3r' \, \langle r | e^{-iH(t-t')} | r'\rangle \, \theta(t-t') \langle r' | \psi_0(t')\rangle$$

$$\psi(\vec{r},t)\,\theta(t-t') = i \int d^3r' \, G^R(r,t;r',t') \, \psi_0(r',t')$$

$$\boxed{G^R(r,t;r',t') = -i \langle r | e^{-iH(t-t')} | r'\rangle \, \theta(t-t')}$$

- Indép. cond. in.
- Toute l'information, i.e. E_n, $\langle \vec{r} | \psi_n \rangle$ états propres
- Analogue F. Green en e.m
- Perturbations peut être formulée naturellement avec G
- Généralisable à N particules

$$G^R(r,t;r',t') = -i\langle r|e^{-iH(t-t')}|r'\rangle\theta(t-t')$$

$$G^R(r,r';\omega) = \int_0^\infty d(t-t')\,e^{i\omega(t-t')}(-i)\langle r|e^{-iH(t-t')}|r'\rangle e^{-\eta(t-t')}$$

$\sum_n |n\rangle\langle n|$

$$= \sum_n \int_0^\infty d(t-t')\,e^{i\omega(t-t')}(-i)\langle r|n\rangle\langle n|r'\rangle e^{-iE_n(t-t')-\eta(t-t')}$$

$$= \sum_n \frac{(-i)\langle r|n\rangle\langle n|r'\rangle}{i(\omega+i\eta)-iE_n} e^{i(\omega+i\eta)(t-t')-iE_n(t-t')}\Big|_0^\infty$$

$$= \boxed{\sum_n \frac{\langle r|n\rangle\langle n|r'\rangle}{\omega+i\eta-E_n}} = \sum_n \frac{\psi_n(\vec r)\,\psi_n^*(r')}{\omega+i\eta-E_n}$$

E

Pôles sont situés aux énergies propres

Résidus nous renseignent sur états propres

$$G^R(r,r',\omega) = \langle r|\frac{1}{\omega+i\eta-H}|r'\rangle$$

$$\sum_n \langle r|n\rangle\langle n|\frac{1}{\omega+i\eta-H}|n\rangle\langle n|r'\rangle$$

16. 1. Repr. en opérateur

$$\hat{G}^R(\omega) = \frac{1}{\omega + i\eta - H} \longrightarrow \hat{G}^R(t) = -i e^{-iHt} \theta(t)$$

$$\hat{G}^A(\omega) = \frac{1}{\omega - i\eta - H} \longrightarrow G^A(t) = i e^{-iHt} \theta(-t)$$

$$\langle k | \frac{1}{\omega + i\eta - H} | k' \rangle = G^R(k, k', \omega) = \frac{\langle k | k' \rangle}{\omega + i\eta - E_k}$$

janv. 28-11:10

16.2 Relation à densité d'états (DOS)

$$\rho(E) = \sum_n \delta(E - E_n) = \sum_n \langle n | n \rangle \delta(E - E_n)$$

$$= \int d^3r \sum_n \langle n | r \rangle \langle r | n \rangle \delta(E - E_n)$$

$$= \int d^3r \left(-\frac{\mathrm{Im}}{\pi} G^R(r, r, E) \right)$$

$$= -\frac{1}{\pi} \int d^3r \; \mathrm{Im}\, G^R(r, r; E) = \int d^3r \; \rho(r, E)$$

DOS locale

$$= -\frac{1}{\pi} \int d^3r \langle r | \mathrm{Im} \frac{1}{E + i\eta - H} | r \rangle$$

$$= -\frac{1}{\pi} \mathrm{Tr} \left[\mathrm{Im}\, \hat{G}^R \right]$$

$$\boxed{\rho(r, E) = -\frac{1}{\pi} \mathrm{Im}\, G^R(r, r, E)}$$