III - Intro. propagateur à une particule G

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15. Définition

$$\Psi(\vec{r},t)\,\theta(t-t') = i\int d^3r'\, G^R(r,t;r',t')\,\Psi_0(r',t')$$

$$G^R(r,t;r',t') = -i\,\langle r|\,e^{-iH(t-t')}\,|r'\rangle\,\theta(t-t')$$

16.1 Info dans G :

$$\boxed{G^R(r,r';\omega) = \sum_n \frac{\Psi_n(r)\,\Psi_n^*(r')}{\omega+i\eta-E_n}} = \langle r|\,\hat{G}^R(\omega)\,|r'\rangle$$

$$\hat{G}^R(\omega) = \frac{1}{\omega+i\eta-H} \qquad \hat{G}^R(t) = -i\,e^{-iHt}\,\theta(t)$$

$$\hat{G}^A(\omega) = \frac{1}{\omega-i\eta-H} \qquad \hat{G}^A(t) = i\,e^{-iHt}\,\theta(-t)$$

Pas d'interaction, invariant sous transl.

$$\langle k|\,\hat{G}^R(\omega)\,|k'\rangle = \frac{\langle k|k'\rangle}{\omega+i\eta-\epsilon_k} \qquad H|k\rangle = \epsilon_k|k\rangle$$

16.2. Relation à DOS

$$\rho(E) = -\frac{1}{\pi}\,\mathrm{Tr}\left\{\mathrm{Im}\,\hat{G}^R(\omega)\right\}$$

16.3 / Repr. spectrale et K.K.

$$G^R(r,r';\omega) = \sum_n \frac{\psi_n(r)\psi_n^*(r')}{\omega + i\eta - E_n} = \int \frac{d\omega'}{2\pi} \frac{\boxed{\sum_n \psi_n(r)\psi_n^*(r') 2\pi\delta(\omega'-E_n)}}{\underbrace{\omega+i\eta-\omega'}}$$

$A(r,r';\omega')$ poids spectrale

$$\sigma = \frac{1}{i(\omega+i\eta)}\left[\chi^R_{jj}(\omega) - \frac{ne^2}{m}\right]$$

$$\chi^R_{jj}(\omega) = \int \frac{d\omega'}{\pi} \frac{\chi''_{jj}(\omega')}{\underbrace{\omega'-\omega-i\eta}}$$

Repr spectrale

$$\boxed{G^R(r,r';\omega) = \int \frac{d\omega'}{2\pi} \frac{A(r,r';\omega')}{\omega+i\eta-\omega'}} \Rightarrow \boxed{A(r,r';\omega) = -2\,\mathrm{Im}\,G^R(r,r';\omega)}$$

Poids spectral

$$\frac{1}{\omega+i\eta-\omega'} = \mathcal{P}\left(\frac{1}{\omega-\omega'}\right) - i\pi\delta(\omega-\omega')$$

Relations K.K. pour G

$$\mathrm{Re}\left[G^R(r,r';\omega)\right] = \mathcal{P}\int \frac{d\omega'}{\pi} \frac{\mathrm{Im}\,G^R(r,r';\omega')}{\omega'-\omega}$$

$$\mathrm{Im}\left[G^R(r,r';\omega)\right] = -\mathcal{P}\int \frac{d\omega'}{\pi} \frac{\mathrm{Re}\,G^R(r,r';\omega')}{\omega'-\omega}$$

16.3.2 Règles de somme

$$A(r,r';\omega) = \sum_n \psi_n(r)\,\psi_n^*(r')\,\delta(\omega - E_n)\,2\pi$$

$$E_n = \langle n|H|n\rangle$$

$$\int \frac{d\omega}{2\pi} A(r,r';\omega) = \sum_n \psi_n(r)\psi_n^*(r') = \sum_n \langle r|n\rangle\langle n|r'\rangle = \langle r|r'\rangle$$

$$= \delta^3(r-r')$$

$$\int d^3r \int \frac{d\omega}{2\pi}\,\omega^n A(r,r;\omega) = \int d^3r \sum_m E_m^n\,\psi_m(r)\,\psi_m^*(r)$$

$$= \int d^3r\,\langle r|H^n|r\rangle = \boxed{\mathrm{Tr}\left[H^n\right]}$$

16.3.3 Dév. haute fréquence

$\langle \omega^n \rangle$

$$G^R(r,r';\omega) = \int \frac{d\omega'}{2\pi} \frac{A(r,r';\omega')}{\omega + i\eta - \omega'} = \sum_{n=0}^{\infty} \boxed{\int \frac{d\omega'}{2\pi}} \frac{\boxed{\omega'^n \, A(r,r';\omega')}}{\omega^{n+1}}$$

$$\mathrm{Tr}\left[\hat{G}^R(\omega)\right] = \int \frac{d\omega'}{2\pi} \frac{\mathrm{Tr}\left[-2\,\mathrm{Im}\,\hat{G}^R\right]}{\omega + i\eta - \omega'} = \sum_{n=0}^{\infty} \int \frac{d\omega'}{2\pi} \frac{\omega'^n}{\omega^{n+1}} \mathrm{Tr}\left[2\pi\,\delta(\omega' - \bar{H})\right]$$

$$\hat{G}^R(\omega) = \frac{1}{\omega + i\eta - H} \qquad \mathrm{Im}\,\hat{G}^R = -\pi\,\delta(\omega - H) \qquad \sum_{n=0}^{\infty} \frac{\mathrm{Tr}\left[H^n\right]}{\omega^{n+1}}$$

$$\frac{1}{\omega - \omega'} = \frac{1}{\omega\left(1 - \frac{\omega'}{\omega}\right)} = \frac{1}{\omega}\left(1 + \frac{\omega'}{\omega} + \left(\frac{\omega'}{\omega}\right)^2 + \dots\right)$$

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16.4 Relation aux fluctuations

$$G^R = -i\, e^{-iHt}\,\theta(t)$$
$$G^A = i\, e^{-iHt}\,\theta(-t)$$

$$S_{\rho_k \rho_{-k}}(t) = \frac{1}{V}\langle \rho_k(t)\rho_{-k}(0)\rangle$$

$$\boxed{\int dt\, e^{i\omega t} S_{\rho_k \rho_k}(t)}$$

$$= \frac{1}{V}\langle e^{+iHt}\rho_k e^{-iHt}\rho_{-k}\rangle$$

$$= \frac{1}{V}\langle i\left(G^R(-t) - G^A(-t)\right)\rho_k\left(G^R(t) - G^A(t)\right)i\, \rho_{-k}\rangle$$

$$G^R(-t)G^R(t) = 0$$

$$= \frac{+1}{V}\langle G^R(-t)\rho_k G^A(t)\rho_{-k}\rangle$$
$$+ \frac{1}{V}\langle G^A(-t)\rho_k G^R(t)\rho_{-k}\rangle$$

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16.5 Relation aux E.D.

$$\Psi(r,t)\theta(t-t') = i\int d^3r'\; G^R(r,t;r',t')\,\Psi_0(r',t')$$

$$i\frac{\partial}{\partial t}\left[\Psi(r,t)\theta(t-t')\right] = H(r)\,\Psi(r,t)\,\theta(t-t') + i\,\Psi(r,t)\,\delta(t-t')$$

$$\left(i\frac{\partial}{\partial t}-H\right)\left[\Psi(r,t)\,\theta(t-t')\right] = i\,\Psi(r,t)\,\delta(t-t')$$

$$\left(i\frac{\partial}{\partial t}-H\right)\cancel{i}\int d^3r'\, G^R(r,t;r',t')\,\Psi_0(r',t') = \cancel{i}\,\Psi_0(r,t')\,\delta(t-t')$$

$$\Psi_0(r',t') = \delta^3(r'-r'')$$

$$\boxed{\left(i\frac{\partial}{\partial t}-H\right)G^R(r,t;r'',t') = \delta^3(r-r'')\,\delta(t-t')}$$

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Opérateurs.

$$\hat{G}^R(t) = -i\, e^{iHt}\, \theta(t)$$

$$i\frac{\partial}{\partial t}\hat{G}^R(t) = H\,\hat{G}^R(t) + \delta(t)$$

$$\left(i\frac{\partial}{\partial t} - H\right)\hat{G}^R(t) = \delta(t)$$

$$\boxed{\hat{G}^R(t) = \frac{1}{i\frac{\partial}{\partial t} - H}\,\delta(t)}$$

$\langle r|$ $\quad\quad |r'\rangle$

$$i\frac{\partial}{\partial t}\psi = H\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi$$

$$i\frac{\partial}{\partial t}\langle r|\psi\rangle = \int d^3r'\, \langle r|H|r'\rangle\langle r'|\psi\rangle$$

$$\int \frac{d\omega'}{\pi} \frac{1}{\omega - i\eta - \omega'} G^R(r,r';\omega') = G^R(r,r';\omega)$$

$$\frac{1}{\omega - i\eta - \omega'} = \mathcal{P}\frac{1}{\omega - \omega'} + i\pi\delta(\omega - \omega')$$

17. Phénoménologie pour Σ^R self-énergie

Cas sans int.

$$\langle k | \hat{G}_0^R(\omega) | k' \rangle = \frac{\langle k | k' \rangle}{\omega + i\eta - \epsilon_k} = G_0^R(k,\omega) \langle k | k' \rangle$$

$$G_0^R(k,\omega) = \frac{1}{\omega + i\eta - \epsilon_k}$$

$$A_0(k,\omega) = -2 \,\mathrm{Im}\, G^R(k,\omega) = 2\pi \delta(\omega - \epsilon_k) \checkmark$$

$$G_0^R(k,\omega) = \int \frac{d\omega'}{2\pi} \frac{A_0(k,\omega')}{\omega + i\eta - \omega'}$$

Cas non inv. sous translation

$$G^R(k,\omega) = \int \frac{d\omega'}{2\pi} \frac{A(k,\omega')}{\omega + i\eta - \omega'}$$

Soit

$$A(k,\omega') = \frac{2\Gamma}{(\omega' - \tilde{\epsilon}_k)^2 + \Gamma^2}$$

$$A(k,\omega') = -2 \,\mathrm{Im}\, G^R(k,\omega')$$

$$G^R(k,\omega) = \frac{1}{\omega + \cancel{i\eta} - \tilde{\epsilon}_k + i\Gamma}$$

$$\mathrm{Im}\, G^R(k,\omega) = \mathrm{Im}\left[\frac{\omega - \tilde{\epsilon} - i\Gamma}{(\omega - \tilde{\epsilon}_k + i\Gamma)(\omega - \tilde{\epsilon} - i\Gamma)} \right]$$

$$G^R(k,\omega) = \frac{1}{\omega - \epsilon_k - \Sigma^R(k,\omega)}$$

Définition de Σ^R

Cas particulier

$$\boxed{\Sigma^R(k,\omega) = \tilde{\epsilon}_k - \epsilon_k - i\Gamma}$$

Général

$$\boxed{G^R(k,\omega) = \frac{1}{G_0^{R^{-1}}(k,\omega) - \Sigma^R(k,\omega)}}$$

$$G^{R^{-1}}(k,\omega) = G_0^{R^{-1}}(k,\omega) - \Sigma^R(k,\omega)$$

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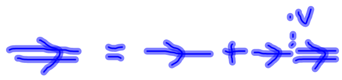
18. Théorie des peturbations

$$H = H_0 + V(r) \qquad H_0|k\rangle = \frac{k^2}{2m}|k\rangle$$

$$\hat{G}^R = \frac{1}{\omega + i\eta - H} = \frac{1}{\omega + i\eta - H_0 - V} \rightarrow (\omega + i\eta - H_0 - V)\hat{G}^R = 1$$

$$(\omega + i\eta - H_0)\hat{G}^R = 1 + V\hat{G}^R$$



$$\hat{G}_0^{R-1}\hat{G}^R = 1 + V\hat{G}^R$$

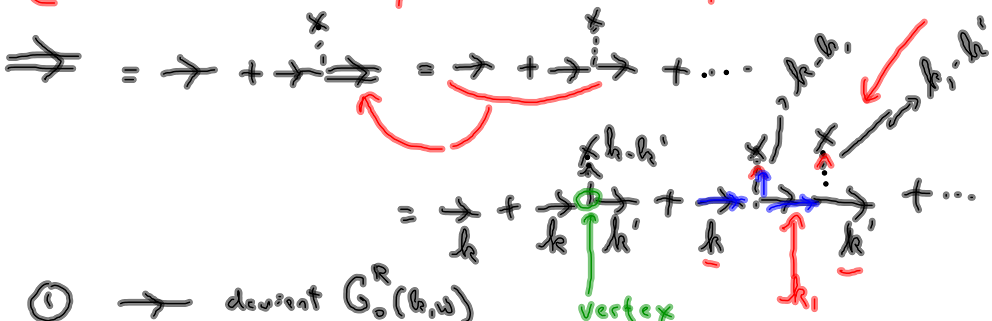
$$\boxed{\hat{G}^R = \hat{G}_0^R + \hat{G}_0^R V \hat{G}^R} = \hat{G}_0^R + \hat{G}_0^R V \hat{G}_0^R + \cdots$$

$$= \hat{G}_0^R + \hat{G}_0^R V \hat{G}_0^R + \hat{G}_0^R V \hat{G}_0^R V \hat{G}_0^R + \hat{G}_0^R V \hat{G}_0^R V \hat{G}_0^R V \hat{G}_0^R + \cdots$$

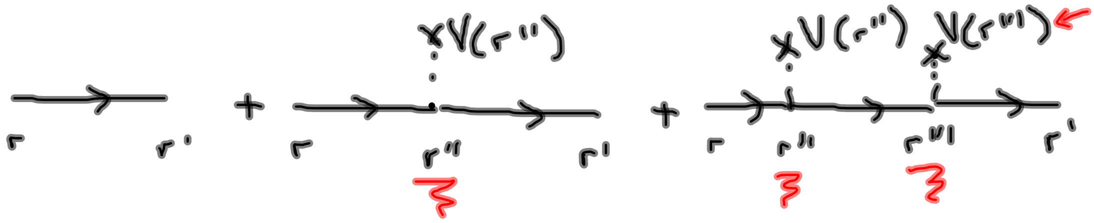
$$\langle k|\hat{G}^R|k'\rangle = \langle k|\hat{G}_0^R|k'\rangle + \int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}\langle k|\hat{G}_0^R|k_1\rangle\langle k_1|V|k_2\rangle\langle k_2|\hat{G}_0^R|k'\rangle + \cdots$$

$$= G_0^R(k,\omega)\langle k|k'\rangle + G_0^R(k,\omega)\langle k|V|k'\rangle G_0^R(k',\omega)$$

$$\int\frac{d^3k_1}{(2\pi)^3} G_0^R(k,\omega)\langle k|V|k_1\rangle G_0^R(k_1,\omega)\langle k_1|V|k'\rangle G_0^R(k',\omega)$$



1. $\rightarrow$ devient $G_0^R(k,\omega)$
2. $\overset{\times}{\vdots}\, k-k'$ devient $\langle k|V|k'\rangle$
3. $\int\frac{d^3k_i}{(2\pi)^3}$ de tous les k_i non déterminés par conservation de k
4. k est conservé à chaque vertex



Série de Born

Si on se tanne vite.

$$\langle k|G^R|k'\rangle = \langle k|k'\rangle G_0^R(k,\omega) + G_0^R(k,\omega)\, V(k-k')\, G_0^R(k',\omega)$$

$$-2\,\mathrm{Im}\, G = \sum_n \psi_n(k)\,\psi_n^*(k')\; 2\pi\, \delta(\omega - E_n)$$

$$\mathrm{Im}\left[\left(\frac{1}{\omega+i\eta-E_k}\right)^2\right] = \frac{\partial}{\partial\omega}\mathrm{Im}\left[\frac{1}{\omega+i\eta-E_k}\right]$$