✓ 17 Self-énergie

✓ 18. Théorie des perturbations pour G^R

✓ 1. $\hat{G}^R = \hat{G}_0^R + \hat{G}_0^R V \hat{G}^R$

\- 2. Diag. de Feynmann

- Position
- Quantité de mouvement

→ 3. Dyson et self-irréductible

~~19. Prop. formelles~~

~~20. Moyenne sur impuretés~~

~~21. Resommation~~

22. Intégrale de chemin

Partie IV Fonctions de Green à T finie

23. Résultats principaux 2ème quant.

1. Espace de Fock : création-annihilation
   1. Fermions
   2. Bosons
2. Changement de base
3. Opérateurs à 1 corps
4. " à 2 corps

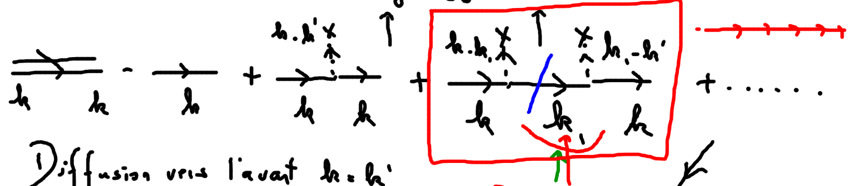
18.3 Dyson + Self-irréductible

Rappel: $\langle r|\hat{G}^R(\omega)|r'\rangle = \langle r|\dfrac{1}{\omega+i\epsilon-H}|r'\rangle$

$H = H_0 + V$

$$\frac{1}{\omega+i\epsilon-H_0-V} = \hat{G}^R = \hat{G}_0^R + \hat{G}_0^R V \hat{G}_0^R + \hat{G}_0^R V \hat{G}_0^R V \hat{G}_0^R + \dots$$

$$= G_0^R + G_0^R V \hat{G}^R$$



Diffusion vers l'avant $k = k'$

$$\Rightarrow \boxed{\Sigma^{(1)R} = \overset{\times}{\vdots}\,\circ}$$

$$\frac{1}{G_0^{-1R}(\omega) - \Sigma^R} = \frac{1}{1-G_0^R\Sigma^R}G_0^R = G_0^R + G_0^R\Sigma^R G_0^R + G_0^R\Sigma^R G_0^R\Sigma^R G_0^R + \dots$$

$$\frac{1}{\omega+i\eta-\epsilon_k-\langle k|V|k\rangle}$$

$$\Sigma^{(2)R} = k.k_1 \quad k_1.k \qquad k_1 \neq k$$

Tous les termes au 2ème ordre du développement

$$G_0^R\Sigma^{(1)R}G_0^R\Sigma^{(1)R}G_0^R + G_0^R\Sigma^{(2)R}G_0^R$$

$$\int_{k_1 \neq k} \frac{d^3k_1}{(2\pi)^3}\frac{|\langle k|V|k_1\rangle|^2}{\omega+i\eta-\epsilon_{k_1}}$$

$$\boxed{G^R = \frac{1}{G_0^{-1R} - \Sigma^{(1)R} - \Sigma^{(2)R}}} \quad \text{à l'ordre 2}$$

$$\boxed{G^R = G_0^R + G_0^R\Sigma^R G^R}$$

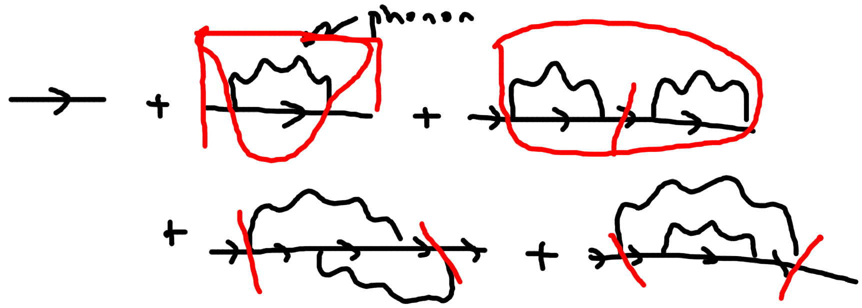
Équation de Dyson $\forall$ ordre

$$G_0^{-1R}G^R = 1 + \Sigma^R G^R$$

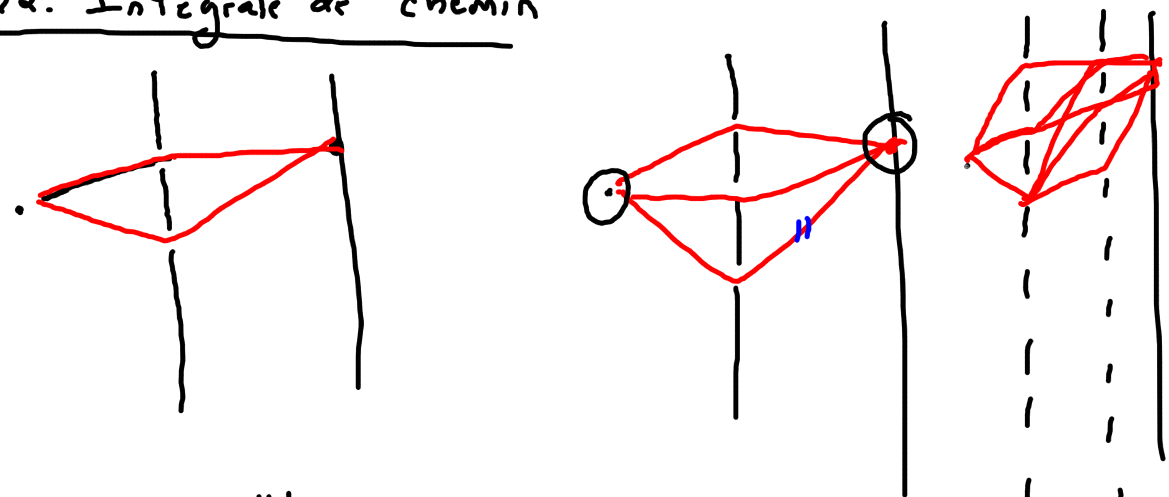
Σ^R = self-énergie irréductible à 1 particule

$$\boxed{G^R = G_0^R + G_0^R\tilde{\Sigma}^R G_0^R}$$

$\tilde{\Sigma}^R \to$ Self-énergie réductible


phonon

22. Intégrale de chemin



$$\langle x_f | e^{-iHt} | x_i \rangle = G^{>}(x_f t ; x_i 0)$$

$$G^R = -i \, G^{>} \, \theta(t)$$

Sur petit intervalle

$$\langle x_f | e^{-i\epsilon(K+V)} | x_i \rangle \qquad \text{temps } \epsilon \text{ infinitésimal}$$

$$e^A e^B = e^M \qquad \text{où } M = \underbrace{A+B}_{\epsilon} + \underbrace{\tfrac{1}{2}[A,B]}_{\epsilon^2} + \underbrace{a_2[A,[A,B]]}_{\epsilon^3}$$

$$\langle x_f | e^{-i\epsilon K} \, e^{-i\epsilon V(x_i)} | x_i \rangle$$

$$\int \frac{dp}{2\pi} \langle x_f | e^{-i\epsilon K} | p \rangle \langle p | e^{-i\epsilon V(x_i)} | x_i \rangle$$

$$\int \frac{dp}{2\pi} \langle x_f | e^{-i\epsilon \frac{p^2}{2m}} | p \rangle \langle p | e^{-i\epsilon V(x_i)} | x_i \rangle$$

$$\langle x_f | p \rangle = e^{ipx_f}$$

$$\int \frac{dp}{2\pi} \, e^{ip(x_f - x_i) - i\epsilon \frac{p^2}{2m} - i\epsilon V(x_i)}$$

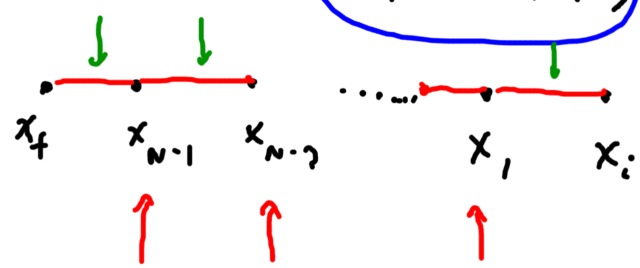
$$\boxed{\int dx \, e^{-\frac{x^2}{2\sigma^2} - xa}} \qquad \sqrt{\frac{m}{2\pi i \epsilon}} \, e^{i\epsilon \left(\frac{x_i - x_f}{\epsilon}\right)^2 \frac{1}{2m}}$$

$$\langle x_f | e^{-iHt} | x_i \rangle = \int \left(\prod_{j=1}^{N-1} dx_j \right) \langle x_f | e^{-i\epsilon H} | x_{N-1} \rangle$$

$$\langle x_{N-1} | e^{-i\epsilon H} | x_{N-2} \rangle \; \ldots\ldots\ldots \; \langle x_1 | e^{-i\epsilon H} | x_i \rangle$$

$$e^{-i\epsilon H} \; e^{-i\epsilon H} \; e^{-i\epsilon H} \; \ldots\ldots$$

où $\epsilon = \dfrac{t}{N}$



$$\langle x_f | e^{-iHt} | x_i \rangle = \int \left(\prod_{j=1}^{N-1} dx_j \right) \left(\sqrt{\frac{m N}{2\pi i t}} \right)^N$$

$$e^{i\epsilon \left(\left(\frac{x_f - x_{N-1}}{\underbrace{\epsilon}_{N^2}} \right)^2 \frac{m}{2} - V(x_{N-1}) \right)} \; e^{i\epsilon \left(\frac{m}{2} \left(\frac{x_{N-1} - x_{N-2}}{\epsilon} \right)^2 - V(x_{N-2}) \right)}$$

$$\cdots\cdots \quad e^{i\epsilon \left(\left(\frac{x_1 - x_i}{\epsilon} \right)^2 \frac{m}{2} - V(x_i) \right)}$$

$$\langle x_f | e^{-iHt/\hbar} | x_i \rangle = \int \mathcal{D}x \; e^{i \int dt \, L(x, \dot{x})}$$

$$= \int \mathcal{D}x \; e^{i S(x, \dot{x}) / \hbar}$$

$$e^{-\beta H}$$

23.1 Espace de Fock, création annihilation.

2 ptcles distinctes, e.g. électron et proton

$$|e\rangle \otimes |p\rangle$$

2 électrons, identiques: $\Psi(x_1, x_2) = \pm \Psi(x_2, x_1)$

$$\frac{1}{2}\left(|\alpha_1\rangle|\alpha_2\rangle - |\alpha_2\rangle|\alpha_1\rangle \right)$$

$$|0\rangle$$

$$V_0 \oplus V_1 \oplus V_1 \otimes V_1 \oplus V_1 \otimes V_1 \otimes V_1$$

a

$a^\dagger$

23.1

2. Fermions

$|\alpha_i\rangle$ orthogonaux $\langle\alpha_i|\alpha_j\rangle = \delta_{ij}$

$|0\rangle$ vide $|\alpha_i\rangle = a^\dagger_{\alpha_i}|0\rangle$

$|\alpha_1\alpha_2\rangle = -|\alpha_2\alpha_1\rangle$ $|\alpha_1\alpha_2\alpha_3\rangle = -|\alpha_2\alpha_1\alpha_3\rangle$

$$a^\dagger_{\alpha_1}a^\dagger_{\alpha_2}|0\rangle = -a^\dagger_{\alpha_2}a^\dagger_{\alpha_1}|0\rangle$$

$$\{a^\dagger_{\alpha_1}, a^\dagger_{\alpha_2}\} = 0 \qquad a^\dagger_{\alpha_1}a^\dagger_{\alpha_2} + a^\dagger_{\alpha_2}a^\dagger_{\alpha_1} = 0$$

si $\alpha_1 = \alpha_2$

$a^\dagger_{\alpha_1}a^\dagger_{\alpha_1} = -a^\dagger_{\alpha_1}a^\dagger_{\alpha_1}$

$$\{a_{\alpha_1}, a_{\alpha_2}\} = 0$$

$a^\dagger_{\alpha_1}|0\rangle = |\alpha_1\rangle$ $\langle\alpha_1|0\rangle = 0$

$\langle 0|a_{\alpha_1} = \langle\alpha_1|$ $\langle 0|\boxed{a_{\alpha_1}|0\rangle} = \boxed{0}$

Cas a et a^+

$$\langle \alpha_i | \alpha_j \rangle = \delta_{ij}$$

$$\langle 0 | a_{\alpha_i} a^+_{\alpha_j} | 0 \rangle = \delta_{ij}$$

$$a_{\alpha_i} a^+_{\alpha_j} \pm a^+_{\alpha_i} a_{\alpha_j} = \delta_{ij}$$

Opérateur nombre :

$$\boxed{\hat{n}_{\alpha_i} = a^+_{\alpha_i} a_{\alpha_i}}$$

$$a^+_{\alpha_j} a^+_{\alpha_k} \cdots a^+_{\alpha_\ell} |0\rangle$$

α_i jamais dans la liste.

$$\hat{n}_{\alpha_i} a^+_{\alpha_j} a^+_{\alpha_k} \cdots a^+_{\alpha_i} \cdots a^+_{\alpha_\ell} |0\rangle$$

$$\left(a^+_{\alpha_i} a_{\alpha_i}\right)\left(a^+_{\alpha_j} a^+_{\alpha_k} \cdots a^+_{\alpha_i} \cdots a^+_{\alpha_\ell}\right)|0\rangle$$

$$a^+_{\alpha_i} a_{\alpha_i} a^+_{\alpha_i} = a^+_{\alpha_i}\left(1 - a^+_{\alpha_i} a_{\alpha_i}\right)$$

févr. 4-10:19