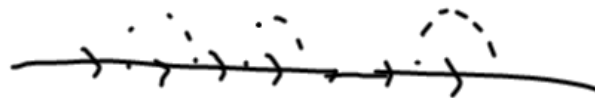


$$\mathcal{Q} = \rightarrow + \rightarrow_{1'}^{1''} + \text{loop} + \text{loop} + \dots$$

$$\hat{U}(\beta, 0) = T_{\tau} e^{-\int d\tau_1 d\tau_2 \hat{V}(\tau_1, \tau_2)}$$

$$\mathcal{Q} = - \langle T_{\tau} \hat{U}(\beta, 0) \hat{\Psi}(\tau) \hat{\Psi}^{\dagger}(0) \rangle_c$$



A diagram showing a horizontal line with an arrow pointing right. Two vertical dashed lines extend upwards from the horizontal line, each ending in a small circle. Below the horizontal line, there is an arrow pointing up towards the line, and the equation  $\mathcal{D}_0 = \frac{1}{ik_n - \mathcal{E}_n}$ .

$$\mathcal{D}_0 = \frac{1}{ik_n - \mathcal{E}_n}$$

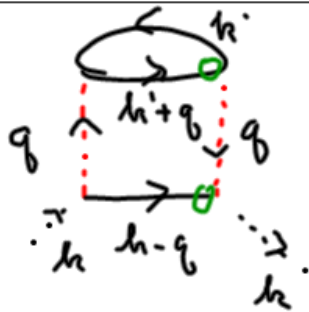
A diagrammatic equation: a double-lined arrow equals a single-lined arrow plus a single-lined arrow with a circle containing the Greek letter sigma (Σ) on top. Below this, the sigma symbol is defined as the sum of four diagrams: a dashed line with a red dashed line below it and a circle on top; a dashed line with a loop on top; a dashed line with a loop on the bottom; and a dashed line with a loop on the side. An arrow points from the sigma symbol in the expansion to the sigma symbol in the equation  $\mathcal{D} = \mathcal{D}_0 + \mathcal{D}_0 \Sigma \mathcal{D} + \dots$ .

$$\mathcal{D} = \mathcal{D}_0 + \mathcal{D}_0 \Sigma \mathcal{D} + \dots$$

$$\mathcal{D}_0^{-1} \mathcal{D} = 1 + \Sigma \mathcal{D}$$

$$(\mathcal{D}_0^{-1} - \Sigma) \mathcal{D} = 1$$

$$\mathcal{D} = \frac{1}{ik_n - \mathcal{E}_n - \Sigma(k, ik_n)}$$



$$\langle a_1^+ a_1 a_2^+ a_2 \rangle$$

$$\langle a_1^+ a_2 a_2^+ a_1 \rangle$$

## 5.2 Modes collectifs

1. Dif. + prolongement
2. Fonction de Lindhard  
(cas sans interaction)
3. Paramètre de développement  $r_s$
4. Écrantage + plasma (élémentaire)
5. Oscillations de densité en présence d'interaction

5.2 Modes collectifs.

$$\frac{1}{\epsilon^L(q, \omega)} = 1 - \frac{4\pi}{q^2} \chi_{nn}^R(q, \omega)$$

$$S_{nn}(q, \omega) = \frac{2}{1 - e^{-\beta\hbar\omega}} \text{Im} \chi_{nn}^R(q, \omega)$$

$$\chi_{nn}(q, iq_n)$$

$$= \int d^3r e^{-iq(r-r')} \int_0^\beta d\tau e^{iq_n \tau}$$

$$\langle T_\tau [\delta n(\vec{r}, \tau) \delta n(r', 0)] \rangle$$

$$= \frac{1}{V} \int_0^\beta d\tau \langle T_\tau [\delta n_{\vec{q}}(\tau) \delta n_{-\vec{q}}(0)] \rangle$$



$$\delta n(q, \tau) \equiv n(q, \tau) - \langle n(q, \tau) \rangle$$

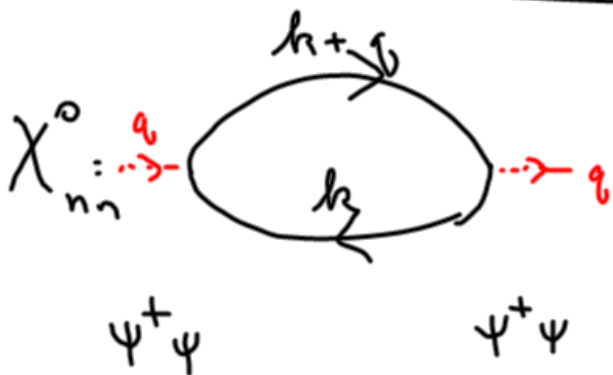
$$= n(q, \tau) - n_0 (2\pi)^3 \delta(q)$$

$\chi_{nn}$   $\equiv$  susceptibilité de charge  
fonction de réponse pour la charge

$$\chi_{nn}^R(q, \omega) = \frac{1}{V} \int dt e^{i\omega t} i \langle [\delta n_{\vec{q}}(t), \delta n_{-\vec{q}}(0)] \rangle$$

$$\chi_{nn}^R(q, \omega) = \lim_{\eta \rightarrow 0^+} \chi_{nn}(q, \omega + i\eta) \theta(t)$$

## 5.2.2 Cas sans interaction (Lindhard)



$$X_{nn}^0(q, iq_n) = -2T \sum_{ih_n} \int \frac{d^3h}{(2\pi)^3} \mathcal{G}^0(\vec{h}+\vec{q}, ih_n+iq_n) \mathcal{G}^0(\vec{h}, ih_n)$$

$$\begin{aligned}
 n_g &= \int d^3r e^{-ig \cdot r} n(r) \\
 &= \int d^3r e^{-ig \cdot r} \sum_{\sigma} \psi_{\sigma}^{\dagger}(r) \psi_{\sigma}(r) \\
 &= \int d^3r e^{-ig \cdot r} \sum_{\sigma} \frac{1}{\sqrt{V}} \sum_h c_{h\sigma}^{\dagger} e^{-ih \cdot r}
 \end{aligned}$$

$$\begin{aligned}
 \alpha^{\dagger} &= \sum_n c_n^{\dagger} \langle n | \alpha \rangle \\
 | \alpha \rangle &= \sum_n | n \rangle \langle n | \alpha \rangle
 \end{aligned}$$

$$\frac{1}{\sqrt{V}} \sum_{h'} e^{+ih' \cdot r} c_{h'\sigma}$$

$$n_g = \sum_{\sigma} \sum_h c_{h\sigma}^{\dagger} c_{h+g\sigma}$$



$$\begin{aligned}
& \chi_{nn}^{\circ}(q, iq_n) \\
&= \frac{1}{V} \int dz e^{iq_n z} \sum_{\sigma} \sum_{\sigma'} \sum_{h} \sum_{h'} \\
& \left[ \langle T_{\tau} \left[ \overset{+}{\underbrace{C_{h\sigma}(\tau) C_{h+q\sigma}(\tau)}_{\text{red}}} \overset{+}{\underbrace{C_{h'\sigma'}(0) C_{h'-q\sigma'}(0)}_{\text{blue}}} \right] \right\rangle \\
& \quad - \left. \left[ \overset{+}{\underbrace{\langle C_{h\sigma} C_{h\sigma} \rangle}_{\text{green}}} \overset{+}{\underbrace{\langle C_{h'\sigma'} C_{h'\sigma'} \rangle}_{\text{green}}} \delta_{q,0} \right] \right]
\end{aligned}$$

Évaluation:

$$\begin{aligned}
 & T \sum_n \mathcal{Q}^\circ(k+\gamma, ik_n + iq_n) \mathcal{Q}^\circ(k, ik_n) \\
 &= T \sum_n \frac{1}{ik_n + iq_n - S_{k+\gamma}} \frac{1}{ik_n - S_k} \\
 &= T \sum_n \left[ \frac{1}{ik_n + iq_n - S_{k+\gamma}} - \frac{1}{ik_n - S_k} \right] \frac{1}{-iq_n + S_{k+\gamma} - S_k} \\
 &= \frac{f(S_{k+\gamma}) - f(S_k)}{-iq_n + S_{k+\gamma} - S_k}
 \end{aligned}$$

$$\chi_{nn}^{\text{OR}}(q, \omega) = -2 \int \frac{d^3k}{(2\pi)^3} \frac{f(S_k) - f(S_{k+q})}{\omega + iq + S_k - S_{k+q}}$$

$\psi^\dagger(\tau) \psi(z)$        $\psi^\dagger(\tau) \psi(\tau)$   
 fonction de Lindhard  $e^{iq_n \tau}$  ↑

$$\langle (x - \langle x \rangle)^2 \rangle$$

$$= \langle x^2 \rangle - 2\langle x \rangle \langle x \rangle + \langle x \rangle^2$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

$$\langle (n - \langle n \rangle)^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2$$

$$\langle n \rangle = \sum_{\sigma} \sum_{\mu} \langle c_{\mu}^{\dagger} c_{\mu} \rangle$$

$$\text{Im} \chi_{nn}^R(q, \omega)$$

$$= 2\pi \int \frac{d^3k}{(2\pi)^3} (f(S_k) - f(S_{k+q}))$$

canal particulier-trou

$$= 2\pi \int \frac{d^3k}{(2\pi)^3} f(S_k) \left[ \delta(\omega + S_k - S_{k+q}) - \delta(\omega + S_{k-q} - S_k) \right]$$

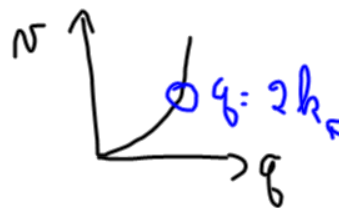
$$= \frac{2\pi}{(2\pi)^3} \int_0^\infty k^2 dk \int_{-1}^1 d(\cos\theta) f(S_k) 2\pi$$

$$\left[ \delta\left(\frac{\omega - E_q}{\hbar q/m} - \cos\theta\right) \frac{m}{\hbar q} - \delta\left(\frac{\omega + E_q}{\hbar q/m} - \cos\theta\right) \frac{m}{\hbar q} \right]$$

$$\frac{\hbar q}{m} \cos\theta = \omega - E_q$$



Self-énergie pour les phonons!



Aperçu

$r_s$

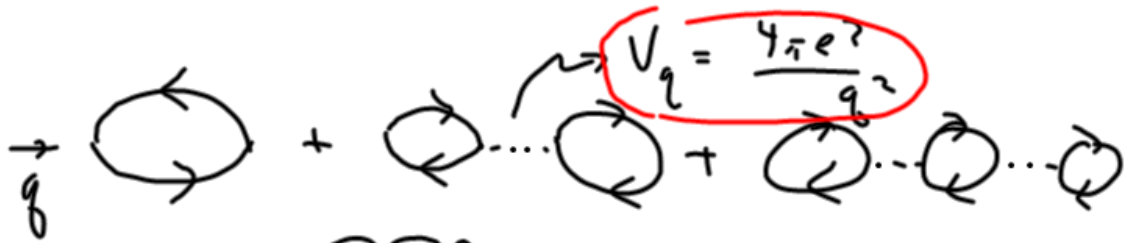
$$n_0 = \frac{1}{\frac{4\pi}{3} (r_s a_0)^3}$$

$a_0 =$  rayon de Bohr

$$\frac{e^2}{a_0} = \frac{\hbar^2}{a_0^2 m}$$

$$r_s \text{ pas de dim.} \\ = \frac{\text{P.t.}}{\text{Cin.}}$$

$$a_0 = \frac{1}{mc^2}$$



RPA



$$\frac{4\pi e^2}{(h_2 - h'_2)^2}$$

$$\int d^3h \int d^3h'$$

