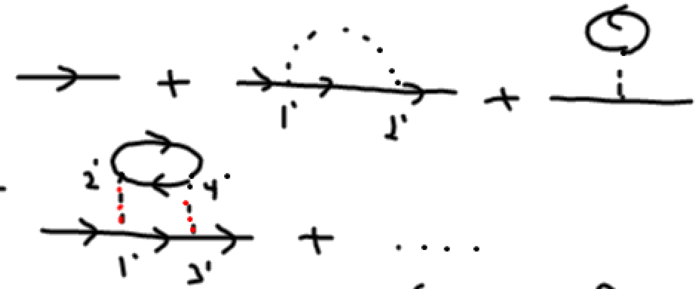
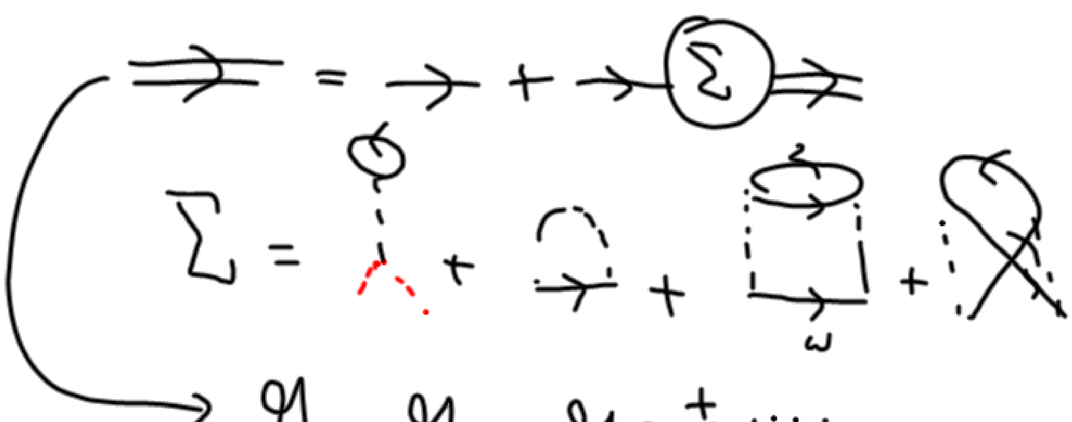
$\mathcal{G} =$



$$\hat{U}(\beta,0) = T_\tau \, e^{-\int d\tau_1 d\tau_2 \, \hat{V}(\tau_1,\tau_2)}$$

$$\mathcal{G} = -\langle T_\tau \, \hat{U}(\beta,0) \, \hat{\Psi}(\tau) \, \hat{\Psi}^{+}(0) \rangle_c$$

$$\mathcal{G}_0 = \frac{1}{ik_n - \xi_k}$$

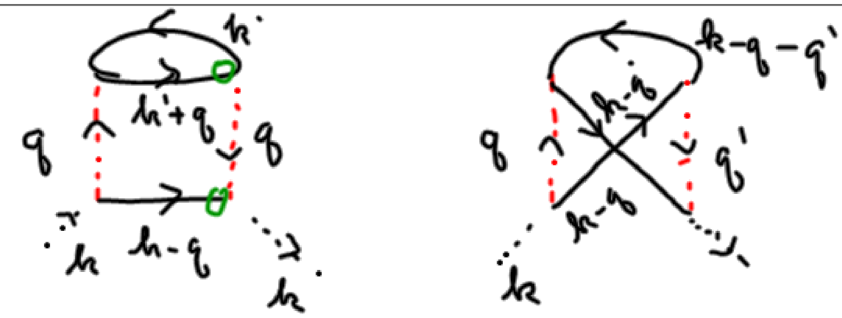


$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0 \Sigma \mathcal{G} + \cdots$$

$$\mathcal{G}_0^{-1} \mathcal{G} = 1 + \Sigma \mathcal{G}$$

$$\left(\mathcal{G}_0^{-1} - \Sigma\right) \mathcal{G} = 1$$

$$\mathcal{G} = \frac{1}{ik_n - \xi_k - \Sigma(k, ik_n)}$$



$$\langle a_1^\dagger a_1 \, a_2^\dagger a_2 \rangle$$

$$\langle a_1^\dagger a_2 \, a_2^\dagger a_1 \rangle$$

5.2 Modes collectifs

1. Déf. + prolongement
2. Fonction de Lindhard
   (cas sans interaction)
3. Paramètre de développement r_s
4. Écrantage + plasma (élém.)
5. Oscillations de densité en présence d'interaction

5.2 Modes collectifs.

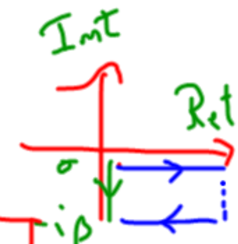
$$\frac{1}{\epsilon^L(q,\omega)} = 1 - \frac{4\pi}{q^2}\chi^R_{nn}(q,\omega)$$

$$S_{nn}(q,\omega) = \frac{2}{1-e^{-\beta\omega}}\,\mathrm{Im}\,\chi^R_{nn}(q,\omega)$$

$$\boxed{\chi_{nn}(q, iq_n)}$$

$$= \int d^3r \, e^{-iq(r-r')} \int_0^\beta d\tau \, e^{iq_n\tau}$$

$$\langle T_\tau [\delta n(\vec{r},\tau)\, \delta n(r',0)] \rangle$$



$$\boxed{= \frac{1}{V} \int_0^\beta d\tau \, \langle T_\tau [\delta n_q(\tau)\, \delta n_{-q}(0)] \rangle}$$

$$\delta n(q,\tau) \equiv n(q,\tau) - \langle n(q,\tau) \rangle$$

$$= n(q,\tau) - n_0 (2\pi)^3 \delta(q)$$

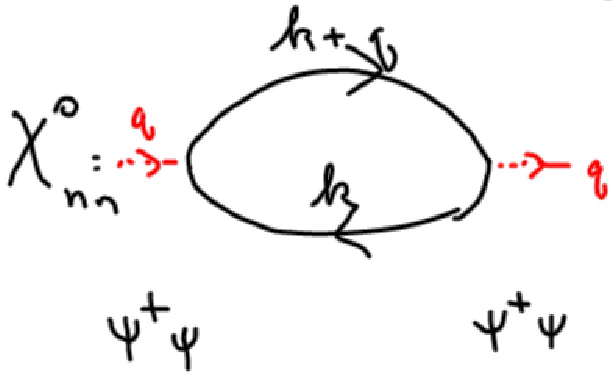
$\chi_{nn} \equiv$ susceptibilité de charge

fonction de réponse pour la charge

$$\chi^R_{nn}(q,\omega) = \frac{1}{V} \int dt \, e^{i\omega t} \, i \, \langle [\delta n_q(t), \delta n_{-q}(0)] \rangle \, \theta(t)$$

$$\chi^R_{nn}(q,\omega) = \lim_{iq_n \to \omega + i\eta} \chi_{nn}(q, iq_n)$$

5.2.2 Cas sans interaction (Lindhard)



$$\chi^0_{nn}(q, iq_n) = -2T\sum_{ik_n}\int\frac{d^3k}{(2\pi)^3}\,\mathcal{G}^0(\vec{k}+\vec{q}, ik_n+iq_n)\,\mathcal{G}^0(\vec{k}, ik_n)$$

$$n_q = \int d^3r\, e^{-iqr}\, n(r)$$

$$= \int d^3r\, e^{-iqr} \sum_\sigma \psi^+_\sigma(r)\, \psi_\sigma(r)$$

$$= \int d^3r\, e^{-iqr} \sum_\sigma \frac{1}{\sqrt{V}} \sum_k c^+_{k\sigma} e^{-ik.r}$$

$$\frac{1}{\sqrt{V}} \sum_{k'} e^{+ik'.r} c_{k'\sigma}$$

$$\boxed{\begin{aligned} \alpha^+ &= \sum_n c^+_n \langle n|\alpha\rangle \\ |\alpha\rangle &= \sum_n |n\rangle\langle n|\alpha\rangle \end{aligned}}$$

$$n_q = \sum_\sigma \sum_k c^+_{k\sigma} c_{k+q\,\sigma}$$

$$\chi^{0}_{nn}(q, iq_n)$$

$$= \frac{1}{V} \int d\tau \, e^{iq_n \tau} \sum_{\sigma} \sum_{\sigma'} \sum_{k} \sum_{k'}$$

$$\Big[\big\langle T_\tau \big[c^{+}_{k\sigma}(\tau) c_{k+q\,\sigma}(\tau) \, c^{+}_{k'\sigma'}(0) \, c_{k'-q\,\sigma'}(0) \big] \big\rangle$$

$$- \langle c^{+}_{k\sigma} c_{k\sigma} \rangle \langle c^{+}_{k'\sigma'} c_{k'\sigma'} \rangle \, \delta_{q,0} \Big]$$

Évaluation:

$$T\sum_n \mathcal{G}^0(k+q, ik_n+iq_n)\,\mathcal{G}^0(k, ik_n)$$

$$= T\sum_n \frac{1}{ik_n+iq_n-\zeta_{k+q}}\,\frac{1}{ik_n-\zeta_k}$$

$$= T\sum_n \left[\frac{1}{ik_n+iq_n-\zeta_{k+q}} - \frac{1}{ik_n-\zeta_k}\right]\frac{1}{-iq_n+\zeta_{k+q}-\zeta_k}$$

$$= \frac{f(\zeta_{k+q})-f(\zeta_k)}{-iq_n+\zeta_{k+q}-\zeta_k}$$

$$\boxed{\chi^{0R}_{nn}(q,\omega) = -2\int \frac{d^3k}{(2\pi)^3}\,\frac{f(\zeta_k)-f(\zeta_{k+q})}{\omega+i\eta+\zeta_k-\zeta_{k+q}}}$$

$$\underbrace{\psi^\dagger(\tau)\,\psi(\tau)}\qquad \underbrace{\psi^\dagger(0)\,\psi(0)}$$

fonction de Lindhard $\quad e^{iq_n\tau}$

$$\langle (x - \langle x \rangle)^2 \rangle$$

$$= \langle x^2 \rangle - 2 \langle x \rangle \langle x \rangle + \langle x \rangle^2$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

$$\boxed{\langle (n - \langle n \rangle)^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2}$$

$$\boxed{\langle n \rangle = \sum_{\sigma} \sum_{k} \langle c^{+}_{k\sigma} c_{k\sigma} \rangle}$$

canal particule-trou

$$\mathrm{Im}\,\chi^R_{nn}(q,\omega)$$

$$= 2\pi \int \frac{d^3k}{(2\pi)^3} \left(f(\xi_k) - f(\xi_{k+q}) \right) \delta(\omega + \xi_k - \xi_{k+q})$$

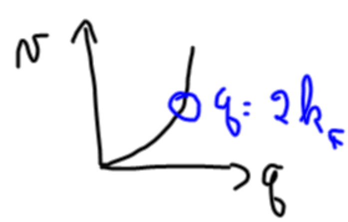
$$= 2\pi \int \frac{d^3k}{(2\pi)^3} f(\xi_k) \Big[\delta(\omega + \xi_k - \xi_{k+q}) - \delta(\omega + \xi_{k-q} - \xi_k) \Big]$$

$$= \frac{2\pi}{(2\pi)^3} \int_0^\infty k^2 dk \int_{-1}^{1} d(\cos\theta)\; f(\xi_k)\; 2\pi$$

$$\left[\delta\left(\frac{\omega - \epsilon_q}{kq/m} - \cos\theta \right) \frac{m}{kq} - \delta\left(\frac{\omega + \epsilon_q}{kq/m} - \cos\theta \right) \frac{m}{kq} \right]$$

$$\frac{kq}{m} \cos\theta = \omega - \epsilon_q$$

Self-énergie pour les phonons!



Aperçu

r_s $\qquad n_0 = \dfrac{1}{\frac{4\pi}{3}(r_s a_0)^3}$

a_0 = rayon de Bohr

$$\frac{e^2}{a_0} = \frac{\hbar^2}{a_0^2 m} \qquad = \frac{\text{P.t.}}{\text{Cin.}}$$

r_s pas de dim.

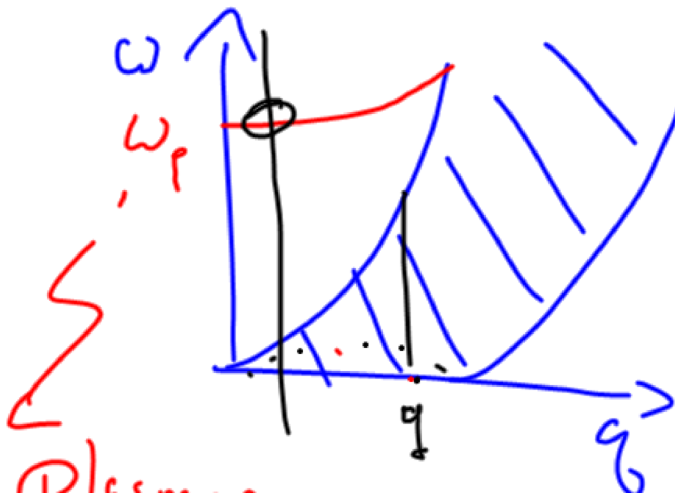
$$\boxed{a_0 = \frac{1}{mc^2}}$$

$$V_q = \frac{4\pi e^2}{q^2}$$

RPA

$$\frac{4\pi e^2}{|k - k'|^2}$$

$$\int d^3k \int d^3k'$$



Plasmon

(Mode collectif)

$\epsilon_1(q, \omega)$