5.4.2: "Guérison" de HF

Liquide de Fermi :

m^*, ℓ_k,

Calcul de l'énergie totale $\int \frac{d\lambda}{\lambda} \langle V \lambda \rangle$

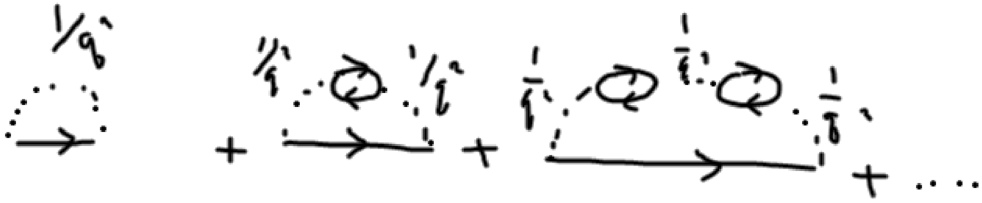
5.5 Théorie des pert.

et séries asymptotiques.

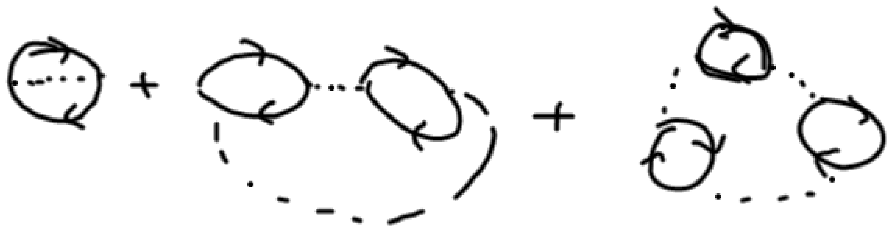
5.6 Diagrammes squelettes

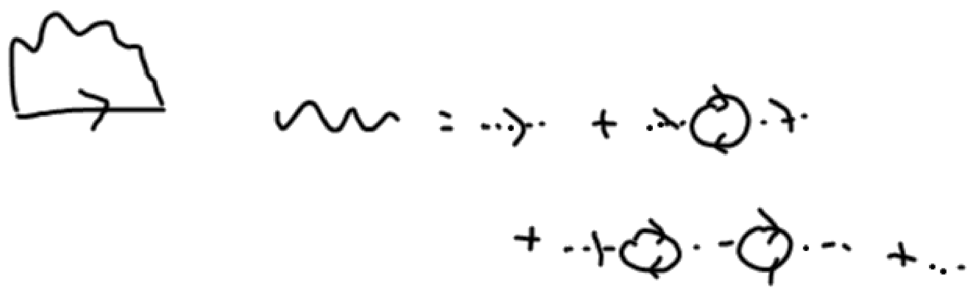
fonctions de vertex etc...

au-delà de RPA.



$$\Sigma G = V \chi_{nn}$$



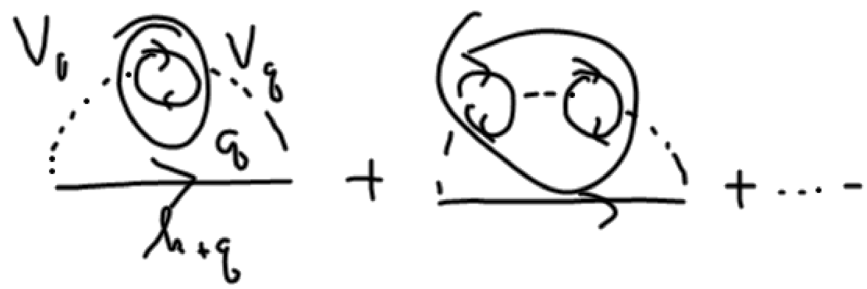


χ

$$\chi(q, iq_n) = \int \frac{d\omega'}{\pi} \frac{\chi''(q,\omega')}{\omega' - iq_n}$$

$\mathcal{G}$

$$\int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega'}{\pi}\, T \sum_{iq_n} \left[\frac{V_q\, \chi''(q,\omega')\, V_q}{\omega' - iq_n} \right] \frac{1}{ik_n + iq_n - \xi_{k+q}}$$



$$\Sigma''(k,\omega) = -\pi \int \frac{d^3q}{(2\pi)^3} \int \left[n_B(\omega') + f(\omega+\omega') \right] V_q^2$$

$$\underbrace{\chi''(q,\omega')}_{\to\ \underbrace{f(q)\,\omega'}} \,\delta(\omega+\omega' - \xi_{k+q}) \frac{d\omega'}{\pi}$$

$\omega' = -\omega$

$\omega' = 0$

à $T = 0$

$$\propto \left[\omega^2 + (\pi T)^2 \right]$$

Liquide de Fermi

$$\omega - \xi_k - \mathrm{Re}\,\Sigma(\xi_k, \omega) = 0 \longrightarrow \text{sol: } \omega = E_k - \mu$$

$$E_k = \epsilon_k - 0.17\, r_s (\ln r_s + 0.2) \frac{k_F k}{2m} + \text{cte.}$$

$$\rightarrow m^* = \frac{m}{1 - 0.08\, r_s (\ln r_s + 0.2)}$$

$$\Gamma_k(\omega = E_k - \mu) = 0.25\, r_s^{1/2} \frac{(k - k_F)^2}{2m}$$

$$E_k = v_k^* (k - k_F)$$

$$\Gamma_k / Z_k = \frac{\sqrt{3}\,\pi^2}{128}\, \omega_p \left(\xi_k / E_F\right)^2$$

Energie libre

$$\xrightarrow{T\to 0} \sum_{k\sigma} \xi_k + \frac{V}{2}\int \frac{d^3q}{(2\pi)^3} \int_0^1 d\lambda \; V_q \left[-\mathrm{Im} \int \frac{d\omega'}{\pi} \frac{\chi^{0R}(q,\omega')}{1+\lambda V_q \chi^0(q,\omega')} - n_0 \right]$$

$$= \sum_{k<k_F} \xi_k$$

$$+ \frac{V}{2}\int_0^1 \frac{d\lambda}{\lambda} \; \lambda \int \frac{d^3q}{(2\pi)^3} V_q \left[T \sum_{iq_n} \chi_{nn}(q, iq_n) - n_0 \right]_\lambda$$

$$T \sum_{iq} \chi_{nn}(q, iq_n) = T \sum_{iq_n} \int \frac{d\omega'}{\pi} \frac{\chi''(q,\omega')}{\omega' - iq_n}$$

$$= \int \frac{d\omega'}{\pi} n_B(\omega') \chi''(q,\omega')$$

$\lim T \to 0 \quad n_B(\omega') = \dfrac{1}{e^{\beta\omega'} - 1}$

$$-\int_{-\infty}^0 \frac{d\omega'}{\pi} \chi''(q,\omega') = -\int_0^\infty \frac{d\omega'}{\pi} \chi''(q, \omega')$$

$$2V\int\frac{d^3k}{(2\pi)^3}\left(\epsilon_k-\mu\right)$$

$$+V\int\frac{d^3q}{(2\pi)^3}\left[-V_q n_0-\int_0^\infty\frac{d\omega'}{\pi}\,\mathrm{Im}\,\ln\left[1+V_q\chi^{0R}_{nn}(q,\omega')\right]\right]$$

Correlation.

$$\frac{E^{RPA}_{TOT}(T=0)}{N}=\frac{2.21}{r_s^2}-\frac{0.91}{r_s}+0.0622\ln r_s$$

$$-0.142+\mathcal{O}\left(r_s\ln r_s\right)$$

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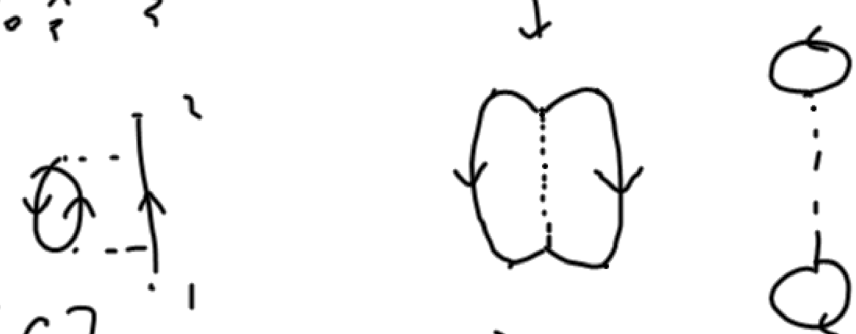
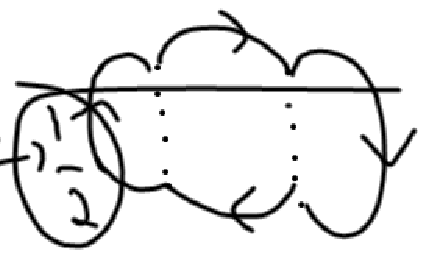
$\int_0^1 \frac{d\lambda}{\lambda} \langle \lambda V \rangle_\lambda$

$\psi^\dagger \psi^\dagger \psi \psi$



$\Phi[G] =$



$\frac{1}{2}$ +

+ $\frac{1}{3}$ + ...

m^*

$$\frac{1}{\ell_k} = \frac{\Gamma_k}{v_k} = -\frac{2\,\mathrm{Im}\,\Sigma}{v_k}$$

$$\frac{d^2\sigma}{d\omega\, d\Omega} \propto \int \frac{d^3k}{(2\pi)^3}\, n_k\, \delta\left(\omega + \epsilon_k - \epsilon_{k+q}\right)$$

$$\propto \int \frac{d^3k}{(2\pi)^3}\, n_k\, \delta\left(\omega - \epsilon_q - \frac{kq}{m}\cos\theta\right)$$

$$\propto \int k^{\cancel{2}}\, dk\, n_k \frac{m}{\cancel{k}q}\, \delta\left(\frac{\omega - \epsilon_q}{\frac{kq}{m}} - \cos\theta\right)$$

$$\propto \int_{|Q|}^{\infty} \frac{dk\, k\, n_k}{q} \qquad \left|\frac{m(\omega - \epsilon_q)}{q}\right| < k$$

$$|Q| = \frac{m(\omega - \epsilon_q)}{q}$$

5.5. Théorie des pert. + séries asymptotiques

$$Z(g) = \int \frac{dx}{\sqrt{2\pi}} \, e^{-\frac{x^2}{2} - \frac{g}{4}x^4}$$

$$Z(g=0) = 1 \qquad\qquad \alpha = \frac{e^2}{\hbar c}$$

$$Z(g<0) = \infty$$

$$Z(g) = \sum_{n=0}^{\infty} g^n Z_n$$

$$Z_n = \frac{(-1)^n}{4^n n!} \int \frac{dx}{\sqrt{2\pi}} e^{-x^2/2} x^4$$

$$= \frac{(-1)^n}{4^n n!} \frac{(4n-1)!!}{2^n} = \frac{(-1)^n}{16^n n!} \frac{(4n)!}{(2n)!}$$

$$n! \simeq \sqrt{2\pi} \, n^{n+1/2} e^{-n}$$

$$Z_n \propto \frac{(4n)^{4n+1/2} e^{-4n}}{n^{n+1/2} e^{-n} (2n)^{2n+1/2} e^{-2n}}$$

$$\propto \frac{e^{-n}}{n^{1/2}} n^n \propto \left(\frac{n}{e}\right)^n \frac{1}{n^{1/2}}$$

$$R_n = \left| Z(g) - \sum_{m=0}^{n} g^m Z_m \right|$$

erreur

$$= \int \frac{dx}{\sqrt{2\pi}} e^{-x^2/2} \left| e^{-\frac{g}{4}x^4} - \sum_{m=0}^{n} \frac{(-1)^m}{4^m m!} g^m x^{4m} \right|$$

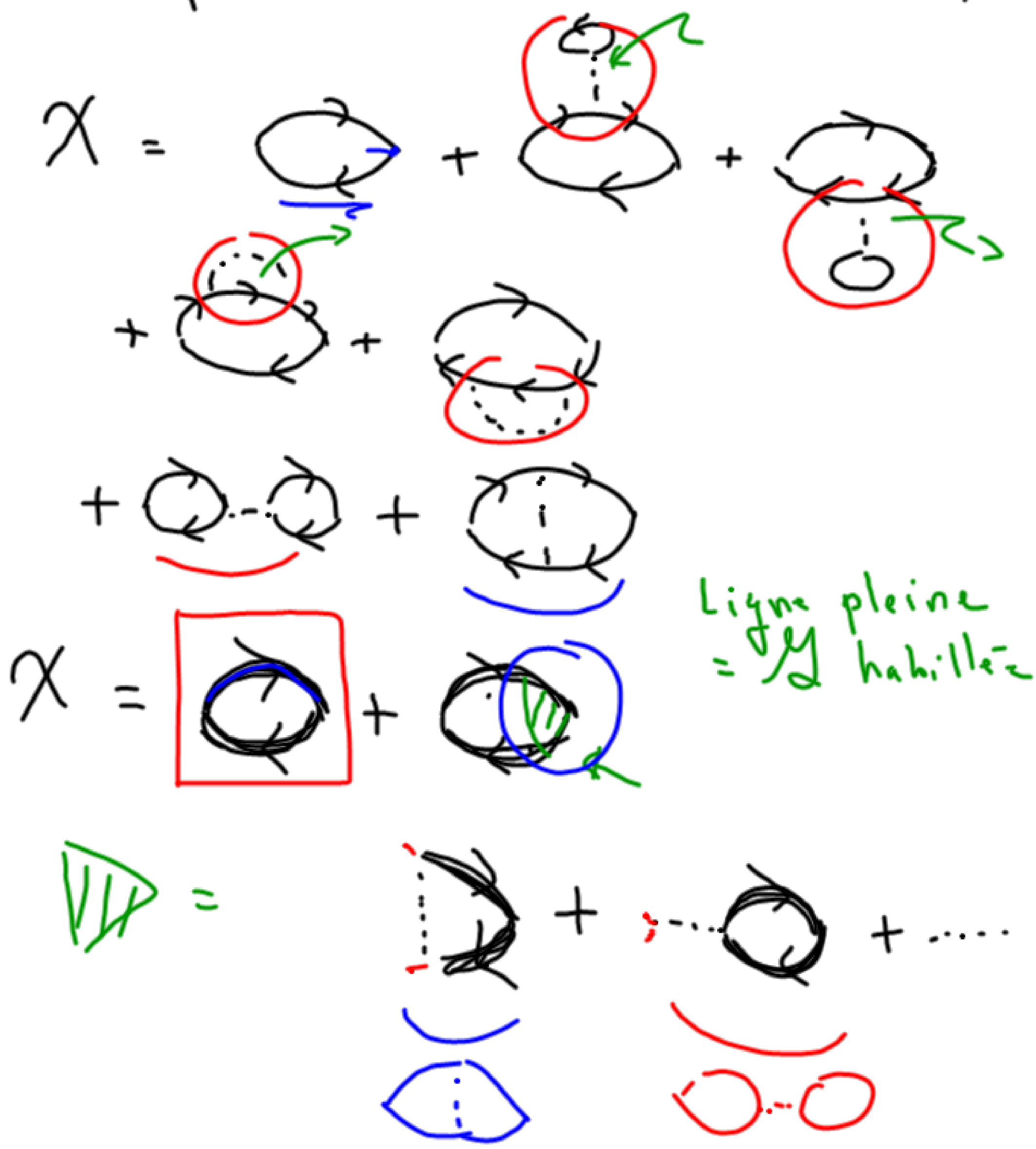
$$= \int \frac{dx}{\sqrt{2\pi}} e^{-x^2/2} \left| \sum_{m=n+1}^{\infty} \frac{(-1)^m}{4^m m!} g^m x^{4m} \right|$$

$$a_{n+1} - (a_{n+2} - a_{n+3}) - (a_{n+4} - a_{n+5}) - \ldots$$

$$a_{n+2} > a_{n+3} \qquad a_{n+4} > a_{n+5} \quad \ldots$$

$$\leq g^{n+1} \left| Z_{n+1} \right| \propto g^{n+1} \frac{n^{n+1}}{e^{n+1}} \frac{1}{\sqrt{n}}$$

5.6 Squelettes, au-delà de la RPA.



$$= 2\int \frac{d^3k}{(2\pi)^3}\, T\sum_{ik_n} \int \frac{d\omega'}{2\pi}\, \frac{A(k,\omega')}{ik_n-\omega'} \int \frac{d\omega''}{2\pi}\, \frac{A(k+q,\omega'')}{ik_n+iq_n-\omega''}$$

$$2\int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega'}{2\pi}\,\frac{d\omega''}{2\pi}\, A(k,\omega')\,A(k\quad,\omega'')\; (\omega''-\omega')\,\frac{\left(f(\omega'')-f(\omega')\right)}{q_n^2+(\omega''-\omega')^2}$$

$q=0$

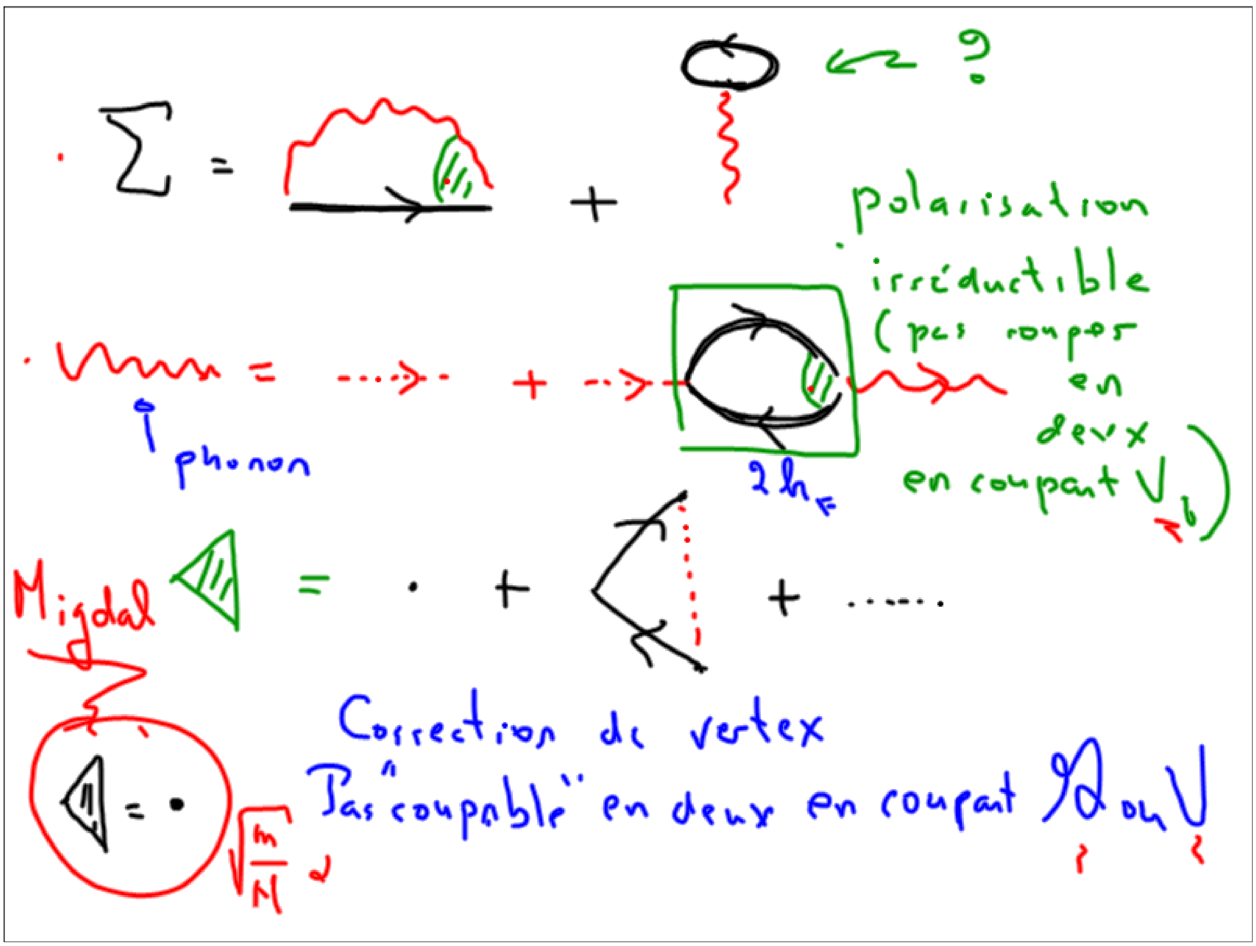
Loi cons. de N

$$\chi_{nn}(q=0, iq_n \neq 0) = 0$$

$$\frac{1}{1+V_q\chi_0} \to \frac{1}{1-\frac{V_q}{4}\,GG}$$

$$G \propto \frac{Z_k}{i\omega_n - E_k + \mu}$$

$$Z_k \sim \frac{1}{2} \text{ à } 0.7$$



polarisation
irréductible
(pas coupée
en
deux
en coupant V)
phonon
2k_F
Migdal
Correction de vertex
Pâs "coupable" en deux en coupant g ou V
√(m/M)