5.3 Formalisme

1. Relation entre Σ et énergie pot. $\rightarrow$ fluctuations de densité

2. Théorème général pour calcul de F

5.4.2 Σ_{RPA} "réparation" de H.F.

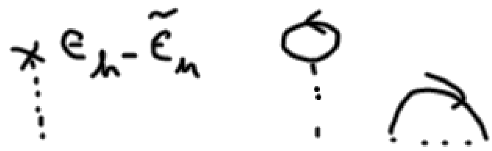
Variationelle

$$\tilde{H}_0 = \sum_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma} \tilde{\epsilon}_k$$

$$-T \ln Z \leq -T \ln Z_0 + \langle H - \tilde{H}_0 \rangle_0$$

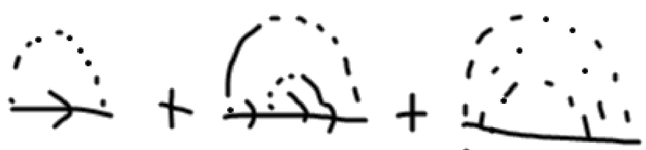
"Milieu effectif"

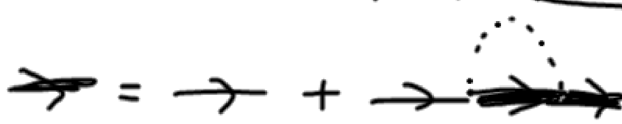
$\tilde{H}_0$ $\quad \tilde{\Sigma} =$



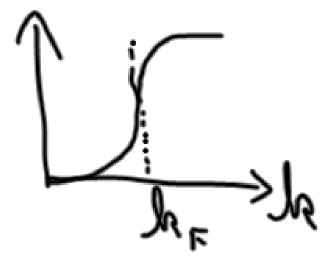
$= 0$

"Sommer diagrammes en arc-en-ciel"





$\Sigma^{HF}(k)$



$m^* = 0$

5.3.1 Σ vs V

à prouver :

$$\boxed{\begin{aligned}\int_1\int_{1'} \Sigma(1,1')\,\mathcal{G}(1',1^+) &= 2\langle V\rangle\beta \\ &= \int_1\int_{1'} \left\langle T_\tau\left[\psi^\dagger(1^+)\,\psi^\dagger(1')\,V(1-1')\,\psi(1')\,\psi(1)\right]\right\rangle\end{aligned}}$$

$$\psi(1) = e^{K\tau_1}\psi_s(\vec{r}_1)e^{-K\tau_1}$$

$$\frac{\partial\psi(1)}{\partial\tau_1} = [K, \psi(1)]$$

$$= \frac{\nabla_1^2}{2m}\psi(1) + \mu\psi(1)$$

$$- \int_{1'} \psi^+(1')V(1-1')\psi(1')\psi(1)$$

$$\mathcal{G}(1,2) = -\langle T_\tau\, \psi(1)\psi^+(2)\rangle$$

$$-\left(\frac{\partial}{\partial\tau_1} - \frac{\nabla^2}{2m} - \mu\right)\mathcal{G}(1,2) = +\delta(1-2)$$

$$- \langle T_\tau \left[\int_{1'} \psi^+(1')V(1-1')\psi(1')\psi(1)\,\psi^+(2)\right]\rangle$$

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0\Sigma\mathcal{G}$$

$$\rightarrow \mathcal{G}_0^{-1}\mathcal{G} = 1 + \Sigma\mathcal{G}$$

$$2\langle V\rangle\beta = \sum_{\sigma_1}\int d^3x_1 \int_{1'} \int d\tau_1$$

$$\langle T_\tau \left[\psi^+(1^+)\psi^+(1')V(1-1')\psi(1')\psi(1)\right]\rangle$$

$$\int_{1''} \Sigma(1,1'')\mathcal{G}(1'' 2)$$

$$= -\langle T_\tau \left[\int_{1''} \psi^+(1'') V(1-1'') \psi(1'') \psi(1) \psi^+(2)\right]\rangle$$

$$\int_1 \int_{1''} \Sigma(1,1'')\mathcal{G}(1'',1^+)$$

$$= + \langle T_\tau \left[\int_{1''}\int_1 \psi^+(1^+) \psi^+(1'') V(1-1'') \psi(1'') \psi(1)\right]\rangle$$

$$= 2\langle V\rangle\beta$$

$$= \beta \int d^3x_1 \int d^3x_{1''} \langle n(x_{1''})\, v(x_1 - x_{1''})\, n(x_1)\rangle$$

$$-\beta v(0) n_0 V$$

$$= -\beta v(0) n_0 V + \beta \int d^3x_1\, d^3x_{1''}\, v(x_1 - x_{1''})\, \chi_{nn}(x_1 0, x_{1''} 0)$$

$$= -\beta v(0) n_0 V + \beta V \int \frac{d^3q}{(2\pi)^3} V_q\, T\sum_{iq_n} \chi_{nn}(q, iq_n)$$

$$\int \frac{d^3q}{(2\pi)^3} V_q$$

$$= \beta V \left[\int \frac{d^3q}{(2\pi)^3} V_q \left[T\sum_{iq_n} \chi_{nn}(q, iq_n) - n_0\right]\right]$$

5.3.2: Calcul de F

$$-\frac{1}{\beta}\frac{\partial \ln Z}{\partial \lambda} = -\frac{1}{\beta}\frac{1}{Z}\frac{\partial}{\partial \lambda}\mathrm{Tr}\, e^{-\beta(H_0+\lambda V-\mu N)}$$

$$= \cancel{-}\frac{1}{\cancel{\beta}}\frac{1}{Z}\,\mathrm{Tr}\left(e^{-\beta(H_0+\lambda V-\mu N)}(\cancel{-\beta}V)\right)$$

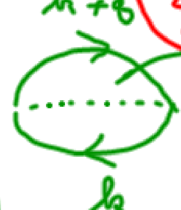
$$= \frac{1}{\lambda}\langle \lambda V\rangle_\lambda$$

couplage.
constante de
intégration sur la

$$\boxed{-\frac{1}{\beta}\ln Z = -\frac{1}{\beta}\ln Z_{\lambda=0} + \int_0^1 \frac{d\lambda}{\lambda}\langle \lambda V\rangle_\lambda}$$

5.4.2. "Réparation" de H.-F

$$\Sigma G = \quad \Sum_{ik'_n} \frac{1}{ik'_n - \xi_{k'}} \quad \Sum_{ik_n} \frac{1}{ik_n - \xi_k}$$

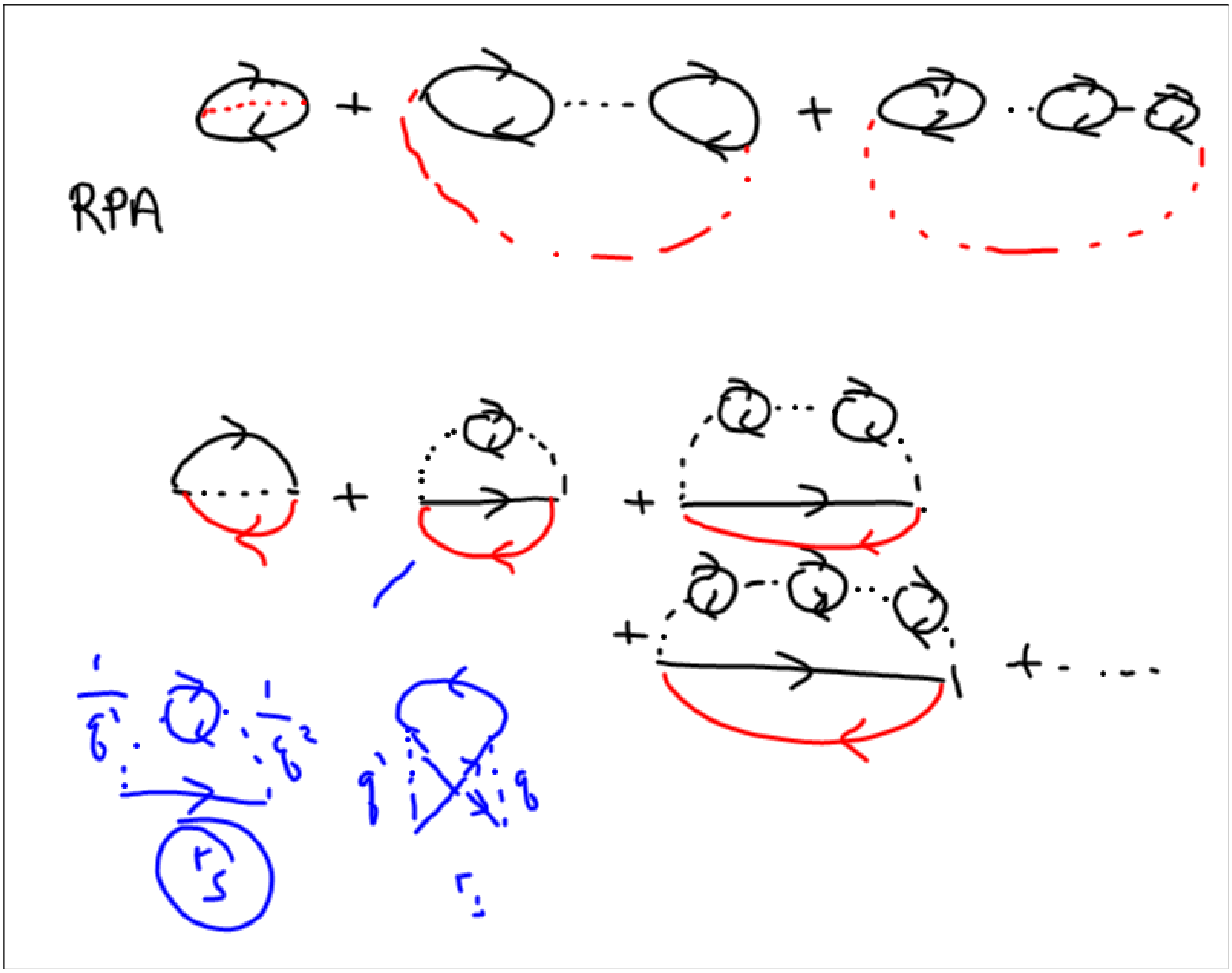


$$\Sum_{iq_n} \Sum_{ik_n} \frac{1}{ik_n - \xi_k} \frac{1}{ik_n + iq_n - \xi_{k+q}}$$

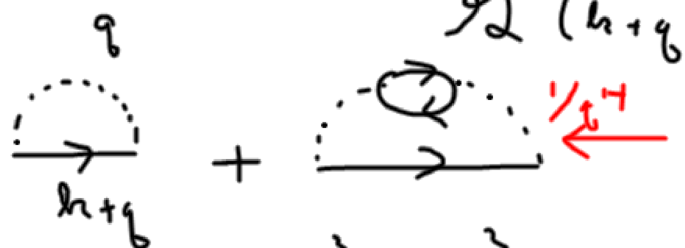
$$\int_1 \Sigma(1,1')\, G(1',1^+)$$

$$= T \Sum_{ik_n} \int \frac{d^3k}{(2\pi)^3} \Sigma(k, ik_n)\, G(k, ik_n)\, e^{ik_n \eta}$$

$$= \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V_q \left[T \Sum_{iq_n} \chi_{nn}(q, iq_n) - n_0 \right]$$


RPA
+
+
+
+
+
+ . . .
1/q¹
1/q²
q'
q

$$\Sigma_{RPA}(k, ik_n) = -\int \frac{d^3q}{(2\pi)^3} T \sum_{iq_n} \frac{V_q}{1+V_q \chi^0_{nn}(q, iq_n)} \mathcal{G}^0(k+q, ik_n+iq_n)$$



$$-V_q \mathcal{G}_0 \qquad (-)^2 \mathcal{G}_0 V_q^2 \chi^0_{nn}$$

$$-V_q \mathcal{G}_0 \left[1 - V_q \chi^0_{nn} \right]$$

$$= -\int \frac{d^3q}{(2\pi)^3} T \sum_{iq_n} \frac{V_q}{\epsilon(q, iq_n)} \mathcal{G}^0(k+q, ik_n+iq_n)$$

$$A(k,\omega) = -2\,\mathrm{Im}\, G^R(k,\omega)$$

$$= \frac{-2\,\mathrm{Im}\,\Sigma^R(k,\omega)}{\left(\omega - \epsilon_k + \mu - \mathrm{Re}\,\Sigma^R(k,\omega)\right)^2 + \left(\mathrm{Im}\,\Sigma^R\right)^2}$$

$\operatorname{Im}\Sigma^R$

$$\Sigma_{RPA}(k, ik_n) = -\int \frac{d^3q}{(2\pi)^3} T \sum_{iq_n} V_q \left[\frac{1}{\varepsilon} - \frac{V_q \chi^0_{nn}(q,iq_n)}{1+V_q \chi^0_{nn}(q,iq_n)} \right] \mathcal{G}^0(k+q, ik_n+iq_n)$$

$$= \Sigma'_{HF}(k) + \int \frac{d^3q}{(2\pi)^3} T \sum_{iq_n} \left[V_q \chi^{RPA}_{nn} V_q \right] \mathcal{G}^0(k+q, ik_n+iq_n)$$

$\operatorname{Im}\Sigma_{HF} = 0$

$$= \Sigma_{HF} + \int \frac{d^3q}{(2\pi)^3} \int \frac{d\omega'}{\pi} V_q \chi''_{nn}(q,\omega') V_q \left[n_B(\omega') + f(\xi_{k+q}) \right] \frac{1}{ik_n+\omega'-\xi_{k+q}}$$

$$-T \sum_{iq_n} \frac{1}{iq_n-\omega'} \frac{1}{ik_n+iq_n-\xi_{k+q}}$$

$$= -T \sum_{iq_n} \left[\frac{1}{iq_n-\omega'} - \frac{1}{ik_n+iq_n-\xi_{k+q}} \right] \frac{1}{ik_n+\omega'-\xi_{k+q}}$$

$$= \left[n_B(\omega') + f(\xi_{k+q}) \right] \frac{1}{ik_n+\omega'-\xi_{k+q}}$$

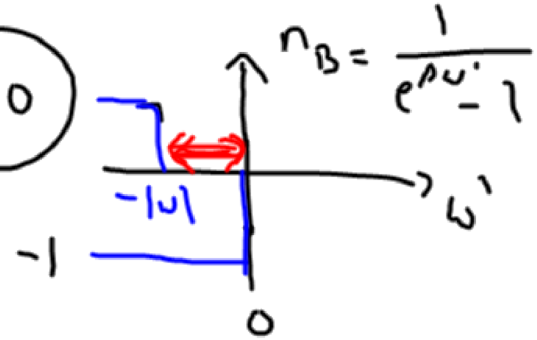
$$\Sigma^R(k,\omega) = \Sigma_{HF}$$

$$+ \int \frac{d^3q}{(2\pi)^3}\int \frac{d\omega'}{\pi}\, \frac{V_q\, \chi''_{nn}(q,\omega')\, V_q}{\omega + i\eta + \omega' - \xi_{k+q}} \left[n_B(\omega') + f(\xi_{k+q})\right]$$

$$\mathrm{Im}\,\Sigma^R(k,\omega) =$$

$$-\pi \int \frac{d^3q}{(2\pi)^3}\int \frac{d\omega'}{\pi} \left[n_B(\omega') + f(\omega+\omega')\right] V_q\, \chi''_{nn}(q,\omega')\, V_q\, \delta(\omega+\omega'-\xi_{k+q})$$

T=0



$\omega > 0$

$\omega = 0 \quad \forall k$

$\mathrm{Im}\,\Sigma^R(k,0) = 0$

pour q petit. $\chi''(q,\omega') = f(q)\,\omega'$

$$\int_{-\omega}^{0} d\omega' \left[n_B(\omega') + f(\omega+\omega')\right]\omega'$$

$$\propto \left[\omega^2 + (\pi T)^2\right]$$