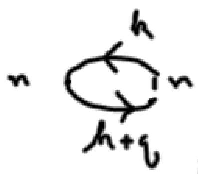
5.2 Modes coll. et ϵ

1. Déf. + prolongement analytique.
2. χ^0_{nn} Lindhard
3. Paramètre de développement r_s
4. Approches élémentaires
   - écrantage - plasma.
5. χ_{nn} en présence d'interactions

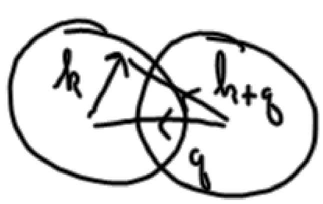


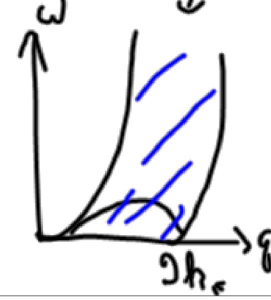
$$-2\,T\sum_{ik_n}\int\frac{d^3k}{(2\pi)^3}\,\mathcal{G}^0(k+q,\,ik_n+iq_n)\,\mathcal{G}^0(k,\,ik_n)$$

$$\frac{1}{ik_n+iq_n-\xi_{k+q}}\qquad\frac{1}{ik_n-\xi_k}$$

$$\chi^{0R}_{nn}(q,\omega)=-2\int\frac{d^3k}{(2\pi)^3}\,\frac{f(\xi_k)-f(\xi_{k+q})}{\omega+i\eta+\xi_k-\xi_{k+q}}$$

$\mathrm{Im}\,\chi^R_0\neq 0$





5.2.3 Paramètre de développement

a_0 = rayon de Bohr

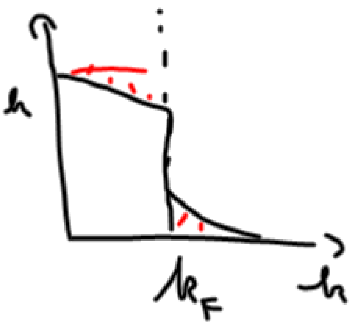
$$\frac{e^2}{a_0} = \frac{\hbar^2}{m a_0^2} \qquad \boxed{a_0 = \frac{1}{me^2} = 0.5\ \text{Å}}$$

$$n_0 = \frac{\#\ \text{él.}}{V} \equiv \frac{1}{\frac{4\pi}{3}(a_0 r_s)^3}$$ déf. de r_s

$$\boxed{n_0 a_0^3 = \frac{1}{\frac{4\pi}{3} r_s^3}}$$

$$\boxed{n_0 = \frac{k_F^3}{3\pi^2}} \qquad N = V\int_0^{k_F} \frac{d^3k}{(2\pi)^3}\, 2$$

$$\frac{k_F^3 a_0^3}{3\pi^2} = \frac{1}{\frac{4\pi}{3} r_s^3}$$



$$\boxed{r_s = C\,\frac{1}{k_F a_0}}$$

Paramètre de dév: $\dfrac{\text{Pot.}}{\text{Cin.}} \sim \dfrac{e^2 k_F}{\frac{k_F^2}{m}}$

$$\frac{e^2 m}{k_F} = \frac{1}{k_F a_0} \propto r_s$$

5.2.4 Écrantage + osc. plasma (simple)

$$-\nabla^2\phi(\vec r) = 4\pi\left[\rho_i(\vec r) + \delta\rho(\vec r)\right]$$

$$\delta\rho = -e\left[n(r) - n_0\right]$$

Thomas Fermi

$$\frac{n(r)}{n(0)} = \frac{k_F^3(r)}{k_F^3}$$

$$\boxed{\frac{k_F^2(r)}{2m} + (-e\phi(r)) = \mu = E_F}$$

$$\frac{n(r)}{n_0} = \left[\frac{k_F^2(r)/2m}{k_F^2/2m}\right]^{3/2} = \left[\frac{E_F + e\phi(r)}{E_F}\right]^{3/2}$$

$$= \left[1 - \frac{(-e\phi(r))}{E_F}\right]^{3/2}$$

$$= \left[1 - \frac{3}{2}\left(\frac{-e\phi(r)}{E_F}\right)\right] \quad \text{linéaire}$$

$$-\nabla^2\phi(r) = 4\pi\left[\rho_i - e n_0\left(1 - \frac{3}{2}\left(\frac{-e\phi(r)}{E_F}\right) - 1\right)\right]$$

$$= \left[4\pi\rho_i - \frac{6\pi n_0 e^2}{E_F}\phi(r)\right]$$

$$= 4\pi\left[\rho_i - e\frac{\partial n}{\partial\mu}(e\phi)\right]$$

$$\boxed{\begin{gathered} q^2\phi(q) = 4\pi\rho_i - q_{TF}^2\phi(q) \\ q_{TF}^2 \equiv \frac{6\pi n_0 e^2}{E_F} = 4\pi e^2\frac{\partial n}{\partial\mu}\end{gathered}}$$

$$\phi(q) = \frac{4\pi\rho_i}{q^2 + q_{TF}^2} = \frac{4\pi\rho_i}{\epsilon_1(q)\, q^2}$$

$$\epsilon_1(q) = 1 + \frac{q_{TF}^2}{q^2} \qquad \boxed{\lambda_{TF} \propto a_0\sqrt{r_s}}$$

$$q_{TF}^2 \propto \frac{1}{\lambda_{TF}^2} = \frac{6\pi n_0 e^2}{E_F} \propto \frac{k_F^3 e^2}{k_F^2/m} \propto k_F m e^2$$

$$\propto \frac{k_F}{a_0} \propto \frac{k_F a_0}{a_0^2} \propto \frac{1}{r_s a_0^2}$$

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Plasma:

$$\vec{j} = -e n_0 \vec{v}$$

$$\frac{\partial \vec{j}}{\partial t} = -e n_0 \frac{\partial \vec{v}}{\partial t} = -e n_0 \frac{(-e\vec{E})}{m}$$

$$\frac{\partial \vec{j}}{\partial t} = \frac{n_0 e^2}{m} \vec{E}$$

$$\frac{\partial \nabla \cdot j}{\partial t} = \frac{n_0 e^2}{m} \vec{\nabla} \cdot \vec{E} \rightarrow \nabla^2 \phi$$

$$-\frac{\partial^2 \rho}{\partial t} = \frac{n e^2}{m}(4\pi \rho)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$$

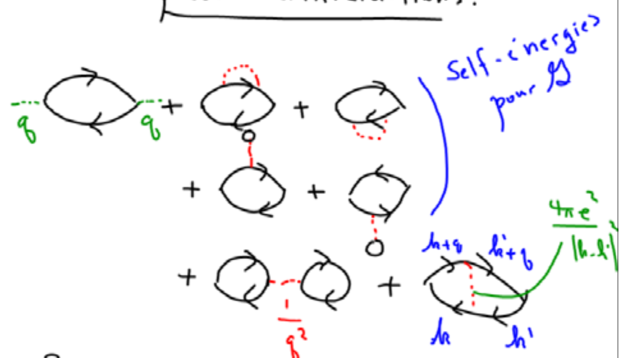
$$\frac{\partial^2 \rho}{\partial t^2} = -\frac{4\pi n e^2}{m} \rho$$

$$\boxed{\omega_p^2 = \frac{4\pi n e^2}{m}}$$ fréqu. plasma

$$\lim_{\omega \to \omega_p} \epsilon^L(q=0, \omega) = a(\omega - \omega_p)$$

$$\lim_{q \to 0} \epsilon^L(q, 0) = 1 + \frac{q_{TF}^2}{q^2}$$

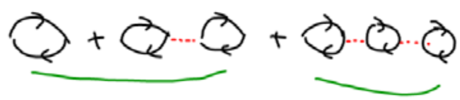
5.9.5 Réponse de densité en présence d'interactions.

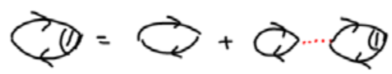


Sommer diagrammes en bulle

Approx. RPA

χ_{nn} =






= Polarisation irréductible

Diagrammes qui ne peuvent pas être coupés en deux en enlevant

$$\chi_{nn}(q,iq_n) = \chi^0_{nn}(q,iq_n) - V_q\, \chi^0_{nn}(q,iq_n)\, \chi_n(q,iq_n)$$

$$\left[1 + V_q\, \chi^0_{nn}(q,iq_n)\right] \chi_{nn}(q,iq_n) = \chi^0_{nn}(q,iq_n)$$

$$\boxed{\chi_{nn}(q,iq_n) = \frac{\chi^0_{nn}(q,iq_n)}{1 + V_q\, \chi^0_{nn}(q,iq_n)}}$$

$$\frac{1}{\epsilon_L(q,iq_n)} \equiv 1 - \frac{4\pi e^2}{q^2}\chi_{nn}(q,iq_n)$$

$$= 1 - \frac{V_q\, \chi^0_{nn}}{1 + V_q\, \chi^0_{nn}}$$

$$\boxed{\frac{1}{\epsilon_L} = \frac{1}{1 + V_q\, \chi^0_{nn}}}$$

oct. 25 - 09:43

Continuum particule-trou

$$\boxed{\mathrm{Im}\,\frac{\chi^{0R}_{nn}}{1+V_q\chi^{0R}_{nn}}}$$

$$= \frac{\mathrm{Im}\,\chi^{0R}_{nn}}{\left(1+V_q\,\mathrm{Re}\,\chi^{0R}_{nn}\right)^2+\left(\mathrm{Im}\,\chi^{0R}_{nn}\right)^2}$$

$$\boxed{\frac{1}{V_q}+\mathrm{Re}\,\chi^{0R}_{nn}=0}$$

Position des nouveaux pôles

$$\mathrm{Im}\left(\frac{x+iy}{1+(x+iy)}\right)$$

$$=\mathrm{Im}\left(\frac{(x+iy)(1+x-iy)}{(1+x)^2+y^2}\right)$$

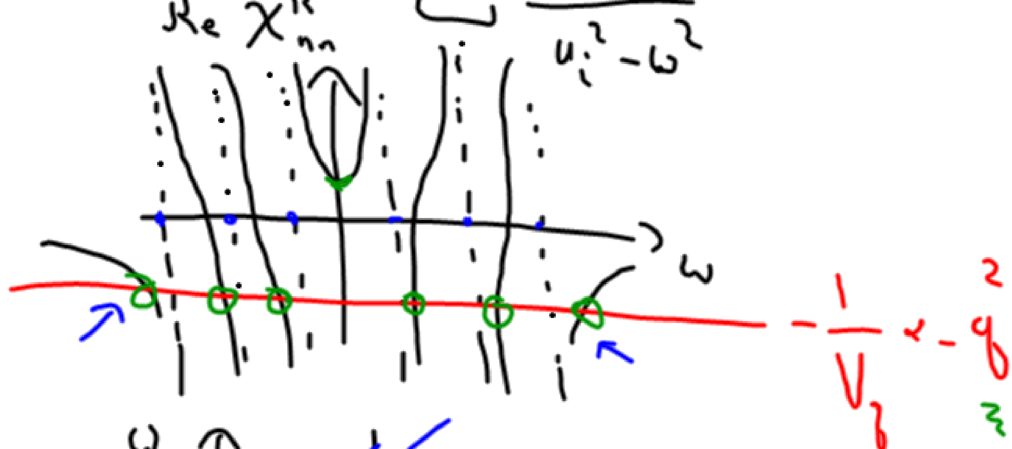
$$=\frac{y}{(1+x)^2+y^2}$$

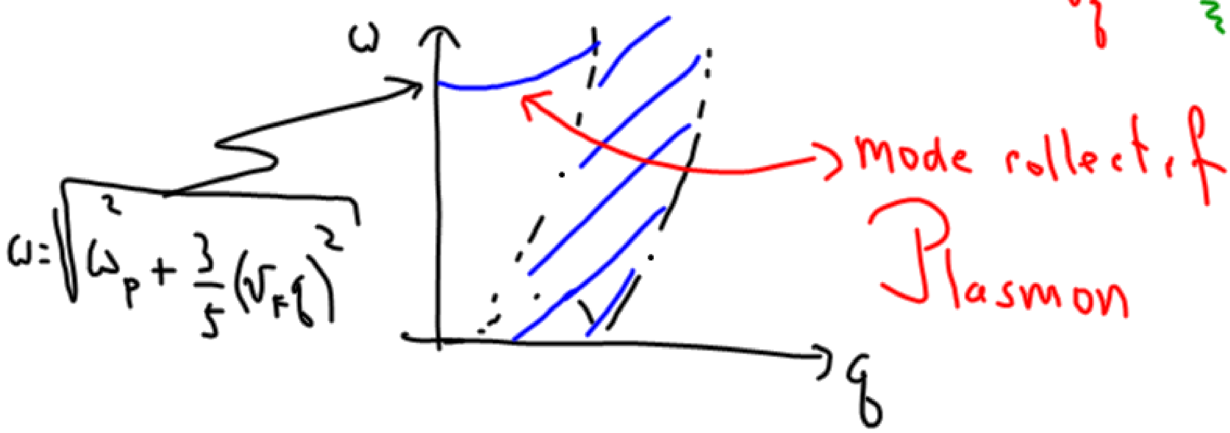
$$\boxed{\frac{1}{V_q} + \mathrm{Re}\,\chi^{0R}_{nn}(q,\omega) = 0}$$

$$\mathrm{Re}\,\chi^{0R}_{nn}(q,\omega) = \mathcal{P}\int \frac{d\omega'}{\pi} \frac{\mathrm{Im}\,\chi^{0R}_{nn}(\omega')(\omega'+\omega)}{(\omega'-\omega)(\omega'+\omega)}$$

$$= \mathcal{P}\int \frac{d\omega'}{\pi} \frac{\omega'\,\mathrm{Im}\,\chi^{0R}_{nn}(\omega')}{\omega'^2-\omega^2}$$

$$\propto \mathcal{P}\sum \frac{A_i}{\omega_i^2-\omega^2}$$





Écrantage:

$$\mathrm{Re}\,\epsilon^L = \epsilon_1^L(q, \omega = 0)$$

$$\epsilon_1^L = 1 + V_q \,\mathrm{Re}\, \chi^{0R}_{nn}(q, \omega = 0)$$

$$\boxed{\epsilon_1^L(q,0) = 1 + \frac{q_{TF}^2}{q^2}\left[\frac{1}{2} + \frac{k_F}{q}\left(1 - \frac{q^2}{(2k_F)^2}\right) \ln\left|\frac{q+2k_F}{q-2k_F}\right|\right]}$$

$$\lim_{q\to 0} \epsilon_1(q,0) = \lim_{q\to 0}\left[1 + \right.$$

$$\left. -2V_q \int \frac{d^3k}{(2\pi)^3} \frac{f(\xi_k) - f(\xi_{k+q})}{\xi_k - \xi_{k+q}}\right]$$

$$= \lim_{q\to 0}\left[1 + V_q(-2)\int \frac{d^3k}{(2\pi)^3} \frac{-\partial f(\xi_k)}{\partial \mu}\right]$$

$$= \lim_{q\to 0}\left[1 + V_q \frac{\partial}{\partial \mu} n\right]$$

$$= \lim_{q\to 0}\left[1 + \frac{4\pi e^2 \frac{\partial n}{\partial \mu}}{q^2}\right] \qquad \to q_{TF}^2$$

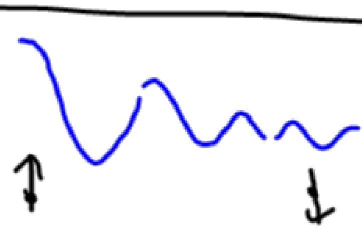
Oscillations de Friedel

$$\frac{4\pi e^2}{\epsilon^L(q,0)\,q^2} = \frac{4\pi e^2}{q_{TF}^2 + q^2} \longrightarrow \frac{4\pi e^2}{r} e^{-r q_{TF}}$$

$$\boxed{V(r) \propto \frac{\cos(2k_F r)}{r^3}}$$

$$e^{-r/\xi_{th}}$$

$$\xi_{th} = \frac{v_F}{T}$$



RKKY

$$\Delta E = k_B T$$

$$\hbar \Delta k\, v_F = k_B T$$

$$\frac{\hbar}{\xi_{th}} v_F = k_B T$$

$$\xi_t = \frac{\hbar v_F}{k_B T}$$

Plasmon:

$$\lim_{q\to 0}\lim_{\omega\gg v_F q} \epsilon^1(q,\omega) = \lim_{q\to 0}\lim_{\omega\gg v_F q}$$

$$\Big[1 - 2V_q \int \frac{d^3k}{(2\pi)^3} \cancel{\frac{f(\xi_k)-f(\xi_{k+q})}{\omega}} \to 0$$

$$+ 2V_q \int \frac{d^3k}{(2\pi)^3} \frac{(f(\xi_k)-f(\xi_{k+q}))(\xi_k-\xi_{k+q})}{\omega^2}$$

$$\xi_k - \xi_{k+q} = \frac{k^2}{2m} - \left(\frac{k^2}{2m} + \frac{\vec{k}\cdot\vec{q}}{m} + \frac{q^2}{2m}\right)$$

$$k' = -k-q \qquad f(\xi_{k+q}) \to f(\xi_{-k'})$$

$$k = -k'-q \qquad \xi_k \to \xi_{-k'-q}$$

$$\xi_{-k} = \xi_k \qquad \xi_{k+q} \to \xi_{-k'}$$

$$1 + 4V_q \int \frac{d^3k}{(2\pi)^3} \frac{f(\varepsilon_k)(\varepsilon_k - \varepsilon_{k+q})}{\omega^2}$$

$$1 + \cancel{2}V_q \left(-\frac{q^2}{\cancel{2}m}\right)\frac{2}{\omega^2} \int \frac{d^3k}{(2\pi)^3} f(\varepsilon_k)$$

$$= 1 - \frac{V_q q^2}{m\,\omega^2} n \qquad \boxed{V_q q^2 = 4\pi e^2}$$

$$= 1 - \frac{4\pi n e^2}{m\,\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} = \epsilon^L(q,\omega)$$

$\omega \gg v_F q$

$$\epsilon^L(q,\omega) = \frac{\omega^2 - \omega_p^2}{\omega^2} = \frac{(\omega - \omega_p)(\omega + \omega_p)}{\omega^2}$$

$$\approx \frac{2(\omega - \omega_p)}{\omega_p}$$

$$\epsilon_1^L = \left[1 - \frac{\omega_p^2}{\omega^2} - \frac{3}{5}\,\frac{\omega_p^2 (v_F q)^2}{\omega^4} + \cdots \right]$$

$\omega \to \infty$ alors $\epsilon_1^L = 1$

$$\boxed{\epsilon_1^L = 0}$$

$$0 = 1 - \frac{\omega_p^2}{\omega^2} - \frac{3}{5}\,\frac{\omega_p^2 (v_F q)^2}{\omega^4}$$

$$0 = \omega^2 - \omega_p^2 - \frac{3}{5}\,\frac{\omega_p^2}{\omega^2} (v_F q)^2$$

$$0 = \omega^2 - \omega_q^2$$

$$\boxed{\omega_q^2 = \omega_p^2 + \frac{3}{5} (v_F q)^2}$$