Règle de somme f

- $$\int \frac{d\omega}{\pi}\, \omega\, \chi''_{\rho\rho}(\vec{q},\omega) = \frac{ne^2}{m} q^2$$

$\omega \to \infty$

$$\boxed{\chi^R(q,\omega) = \int \frac{d\omega'}{\pi} \frac{\chi''(q,\omega')}{\omega'-\omega-i\eta}}$$

$\omega\rho = \vec{q}\cdot\vec{j}$ $\quad q_x\,,\ j(q_x)$

- $$\int \frac{d\omega}{\pi} \frac{\overset{x}{\cancel{q}}}{\omega} \chi''_{jj}(q,\omega) = \frac{ne^2}{m} \overset{x}{\cancel{q}}$$

$$\to \boxed{\int \frac{d\omega}{\pi}\, \frac{\chi''_{jj}(q_x,\omega)}{\omega} = \frac{ne^2}{m} = \chi^R_{\underset{\uparrow}{j_x}\underset{\uparrow}{j_x}}(q_x,0)}$$

$$\delta H(t) = -\int d^3r\, \underbrace{j(\vec r)}\cdot \frac{\vec A(r,t)}{c} \quad \longleftarrow$$

$$= \int d^3r\, \varphi(\vec r,t)\,\rho(\vec r)$$

$$\to j^A(r) = j(r) - \frac{ne^2}{m}\frac{A}{c} = j(r) - \frac{e\rho}{mc}A$$

$$\longrightarrow \boxed{\frac{\hbar}{i}\nabla \to \frac{\hbar}{i}\nabla - \frac{eA}{c}} \quad j^A = \frac{\delta H}{\delta A}$$

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$$\sigma^{L}(q_x, \omega) = \frac{1}{i(\omega + i\eta)}\left[\underline{\chi^{R}_{j_x j_x}(q_x, \omega) - \frac{ne^2}{m}}\right]$$

$$= \frac{1}{i q_x}\,\chi^{R}_{j_x \rho}(q_x, \omega)$$

$$\varphi = 0 \qquad \boxed{\Lambda(x)} \qquad \varphi \to \varphi - \frac{1}{c}\frac{\partial \Lambda}{\partial t}$$

$$A = 0 \qquad \boxed{\Lambda(t)} \qquad A \to A - \nabla\Lambda$$

$$\chi^{R}_{j_x \rho}(0, \omega) = 0$$

$$\boxed{\sigma^{T}(q_x, \omega) = \frac{1}{i(\omega + i\eta)}\left[\chi^{R}_{j_y j_y}(q_x, \omega) - \frac{ne^2}{m}\right]}$$

Aujourd'hui

- Drude
- Métal, Isolant, Supracond.
- Règle de somme
- $\overset{\leftrightarrow}{\epsilon}$ vs $\overset{\leftrightarrow}{\sigma}$

Drude

$$\frac{1}{\omega+i\eta} \to \mathcal{P}\frac{1}{\omega} - i\pi\delta(\omega)$$

$$\sigma^L(q_x,\omega) = \frac{1}{i(\omega+i\eta)}\left[\chi^R_{\dot{j}_x\dot{j}_x}(q_x,\omega) - \frac{ne^2}{m}\right]$$

$$\boxed{\begin{aligned}\mathrm{Re}\,\sigma^L(q_x,\omega) &= \mathcal{P}\frac{\chi''_{j_xj_x}(q_x,\omega)}{\omega}\\ &\quad - \pi\delta(\omega)\left[\underbrace{\mathrm{Re}\,\chi^R_{\dot{j}_x\dot{j}_x}(q_x,\omega) - \frac{ne^2}{m}}\right]\end{aligned}}$$

$\underline{q_x \to 0}$ avant $\underline{\omega \to 0}$ Transport DC

$\omega = 0$

$$\mathrm{Re}\,\chi^R_{\dot{j}_x\dot{j}_x}(q,\omega) = \mathcal{P}\int\frac{d\omega'}{\pi}\,\frac{\chi''_{\dot{j}_x\dot{j}_x}(q,\omega')}{\omega'}$$

$$\boxed{\begin{aligned}&\delta(\omega)\left[\underbrace{\frac{ne^2}{m} - \mathrm{Re}}\,\chi^R_{\dot{j}_x\dot{j}_x}(0,\omega)\right]\\ &\equiv \delta(\omega)\,D\end{aligned}}$$

Electrons libres.

$$m \frac{d\vec{j}}{dt} = ne^2 \vec{E}$$

$$j = -nev$$

$$\frac{dj}{dt} = \frac{ne^2}{m} E$$

$$\frac{dj}{dt} = -ne \frac{dv}{dt}$$

$$= ne \frac{1}{m} eE$$

$$-i(\omega + i\eta)\, j = \frac{ne^2}{m} E$$

$$j(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} j(\omega)$$

$$j = \frac{1}{-i(\omega + i\eta)} \frac{ne^2}{m} E$$

$$= \sigma E \to \sigma = -\frac{1}{i(\omega + i\eta)} \frac{ne^2}{m}$$

$$\mathrm{Re}\,\sigma = \delta(\omega)\, ne^2/m$$

Isolant. $\lim_{\omega\to 0} \mathrm{Re}\,\sigma^{L}(q_x = 0, \omega) = 0$

$$\lim_{\omega\to 0} \mathrm{Re}\,\chi^{R}_{j_x j_x}(0,\omega) = \frac{ne^2}{m}$$

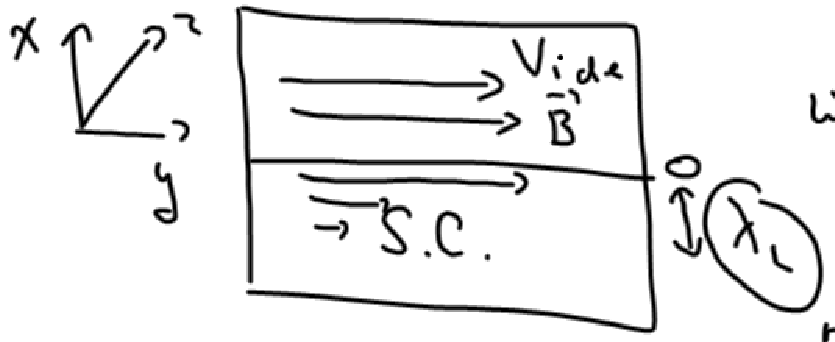
$$\lim_{\omega\to 0}\lim_{q_x\to 0} P\int \frac{d\omega'}{\pi}\frac{\chi''_{j_x j_x}(0,\omega')}{\omega'-\omega}$$

$$= \lim_{q_x\to 0} \int \frac{d\omega'}{\pi}\frac{\chi''_{j_x j_x}(q_x,\omega')}{\omega'}$$

Supraconducteur

$$\mathrm{Re}\,\sigma^{T}(q_x, \omega) = \text{ à venir}$$

Longueur de pénétration.



$\omega = 0$

Longueur de pénétration de London

$$\delta\langle j_y^T(q_x,\omega)\rangle = \left[\underbrace{\chi^R_{j_y j_y}(q_x,\omega) - \frac{n e^2}{m}}\right]\frac{A_y}{c}$$

$$\boxed{\delta\langle j_y^T(q_x,0)\rangle \cong -\frac{n_s e^2}{m}\frac{A_y}{c}}$$

$\neq 0$ à $\omega = 0$

Supra cond.

n_s = densité superfluide

$$\vec\nabla\times\delta\langle \vec j_y(q_x,0)\rangle = -\frac{n_s e^2}{m}\frac{\vec B}{c}$$

$$\nabla\times B = \frac{4\pi}{c}j - \frac{1}{c}\cancel{\frac{\partial E}{\partial t}} \to 0$$

$$j = \frac{\nabla\times B}{4\pi}c$$

$$\nabla\times(\nabla\times B) = \nabla\cancel{(\nabla\cdot B)}\to 0 - \nabla^2 B$$

$$\frac{c}{4\pi}\left(-\nabla^2 B\right) = -\frac{n_s e^2}{m}\frac{B}{c}$$

$$\nabla_x^2 B_y = +\frac{n_s e^2}{mc^2}4\pi\, B_y$$

$$\frac{\partial^2 B_y}{\partial x^2} = \frac{4\pi n_s e^2}{mc^2}B_y \qquad B_y = B_y^0 e^{-x/\lambda_L}$$

$$\frac{n_s e^2}{2m} = \frac{c^2}{4\pi c^2}\left(\frac{1}{\lambda_L}\right)^2 \qquad \boxed{\left(\frac{1}{\lambda_L}\right)^2 = \frac{4\pi n_s e^2}{mc^2}} \qquad n_s = n \qquad = \frac{\omega_p^2}{c^2}$$

Supracond:

$$\int \frac{d\omega'}{\pi} \frac{\chi''_{j_y j_y}(q_x, \omega')}{\omega'} - \frac{ne^2}{m}$$

$$= -\frac{n_s e^2}{m} \qquad \boxed{n_s < n}$$

$$= \chi^R_{j_y j_y}(q_x, 0) - \frac{ne^2}{m}$$

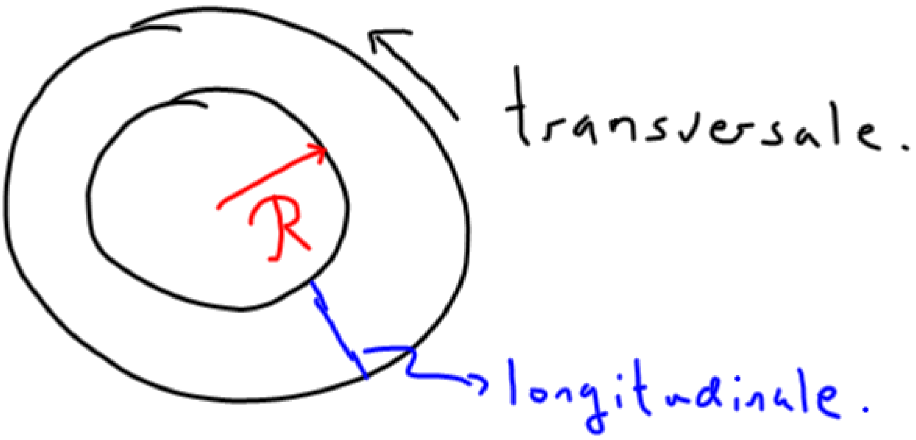
Vrai toujours

$$\lim_{q_x \to 0} \chi^R_{j_x j_x}(q_x, 0) = \frac{ne^2}{m}$$

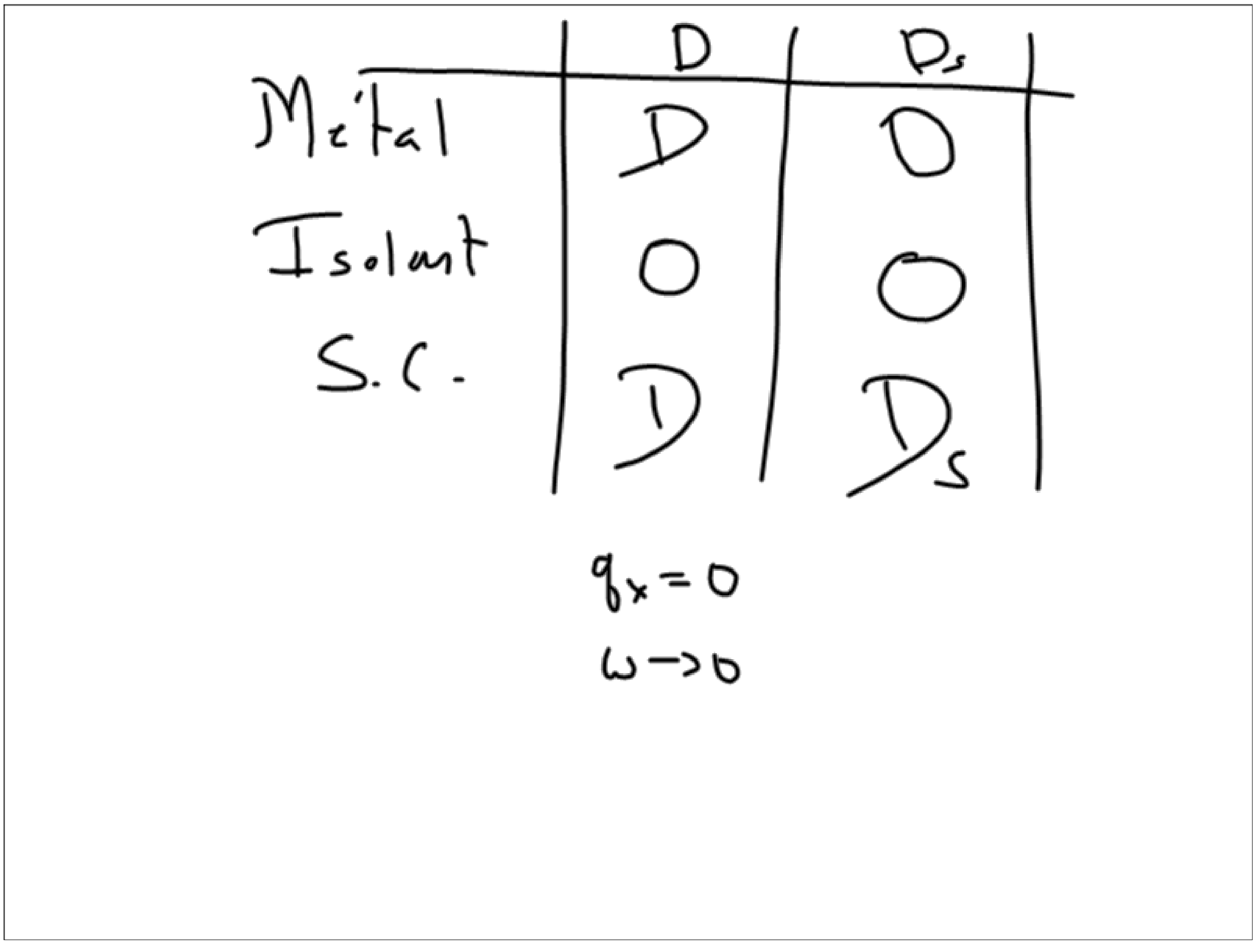
$$\boxed{\lim_{q_y \to 0} \chi^R_{j_x j_x}(q_y, 0) \neq \frac{ne^2}{m}}$$

$$\lim_{q_x \to 0} \chi^R_{j_x j_x}(q_x, 0) = \lim_{q_x \to 0} \int d^3r \, e^{-iq_x(r_x - r'_x)} \chi^R_{j_x j_x}(r - r', 0)$$

Ordre à longue portée



$R \to \infty$



	D	D_s
Métal	D	0
Isolant	0	0
S.C.	D	D_s

$q_x = 0$

$\omega \to 0$

$$\boxed{\text{Re}\,\sigma^{L\,T}(q_x,\omega) = \mathcal{P}\frac{\chi''_{\underset{y}{j_x}\underset{y}{j_x}}(q_x,\omega)}{\omega} - \pi\,\delta(\omega)\left[\text{Re}\,\chi^{R}_{\underset{y}{j_x}\underset{y}{j_x}}(q_x,\omega) - \frac{ne^2}{m}\right]}$$

① ②

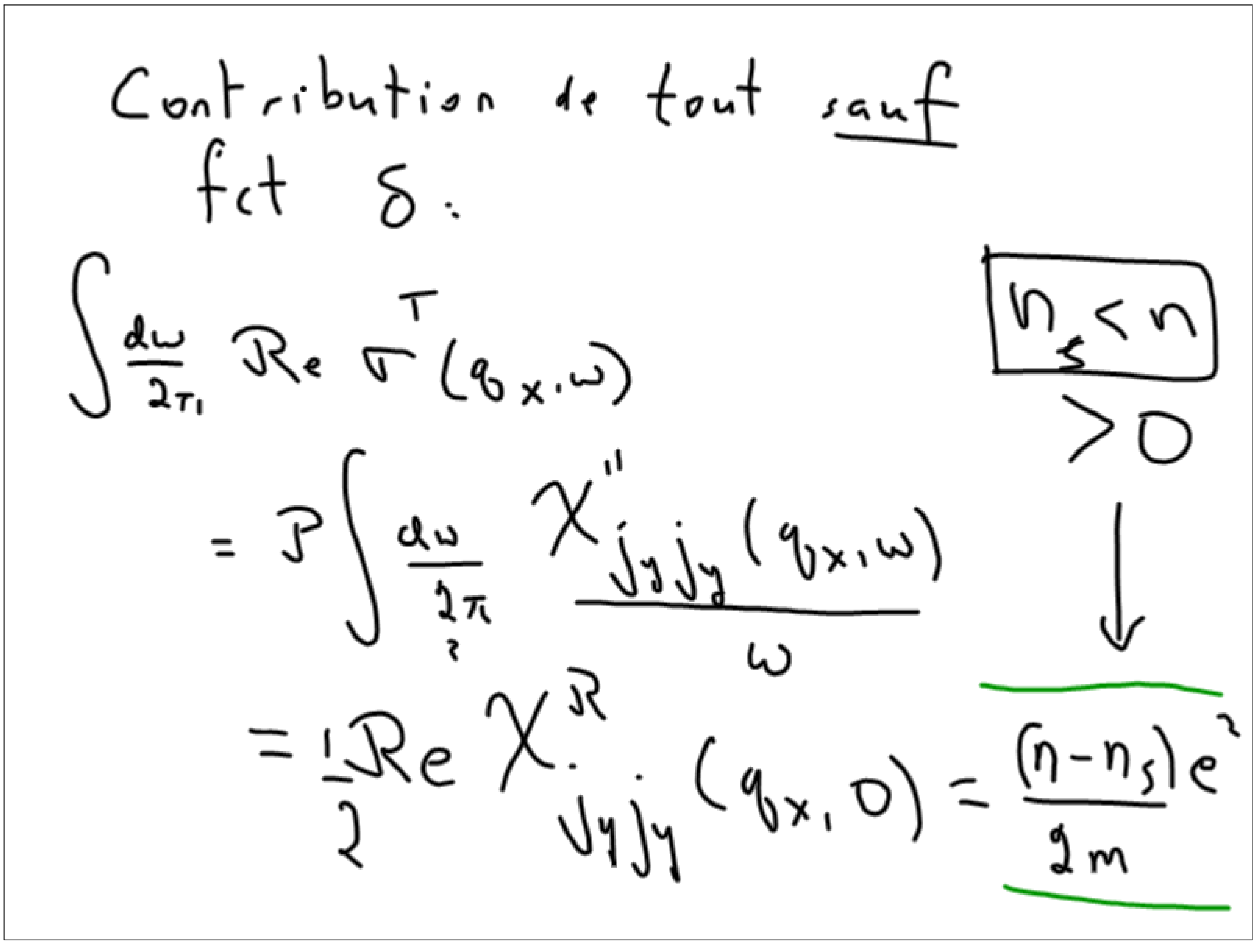
Rappel:

$$\nearrow\ \text{Re}\,\chi^{R}_{j_x j_x}(\underset{z}{\vec{q}},0) = \mathcal{P}\int\frac{d\omega'}{\pi}\,\frac{\overset{z}{\chi''_{j_x j_x}}(\vec{q},\omega')}{\omega'}$$

② ①

$$\boxed{\int\frac{d\omega}{2\pi}\,\text{Re}\,\sigma^{L\,T}(q_x,\omega) = \frac{ne^2}{2m}}$$

Règle de somme f



Contribution de tout <u>sauf</u>

fct δ :

$$\int \frac{d\omega}{2\pi} \, \mathcal{R}e \, \sigma^T(q_x, \omega)$$

$$\boxed{n_s < n}$$

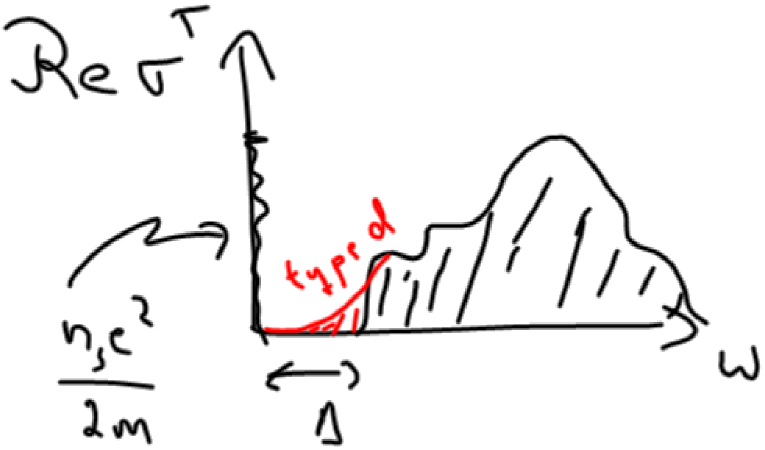
$$> 0$$

$$\downarrow$$

$$= \mathcal{P} \int \frac{d\omega}{2\pi} \, \frac{\chi''_{j_y j_y}(q_x, \omega)}{\omega}$$

$$= \frac{1}{2} \mathcal{R}e \, \chi^R_{j_y j_y}(q_x, 0) = \underline{\overline{\frac{(n-n_s)e^2}{2m}}}$$

À $\omega = 0$ fct. δ de poids $\frac{n_s e^2}{2m}$



$$\frac{ne^2}{2m}$$

$$\omega_p^2 = \frac{4\pi n e^2}{m}$$

Relation entre ϵ et σ

$$\vec{\nabla} \to i\vec{q} \qquad \frac{\partial}{\partial t} \to -i(\omega+i\eta)$$

$$i\vec{q}\cdot\vec{E} = 4\pi\rho$$

$$i\vec{q}\times\vec{E} = i(\omega+i\eta)\frac{\vec{B}}{c} \longleftarrow$$

$$i\vec{q}\cdot\vec{B} = 0$$

$$i\vec{q}\times\vec{B} = \frac{4\pi}{c}\vec{j} - i\frac{(\omega+i\eta)}{c}\vec{E}$$

$$i\vec{q}\times\vec{B} = \frac{4\pi}{c}\overleftrightarrow{\sigma}^T\cdot\vec{E} - i\frac{(\omega+i\eta)}{c}\vec{E}$$

$$\frac{c\,i\vec{q}\times\left(i\vec{q}\times\vec{E}\right)}{i(\omega+i\eta)} = \frac{q^2E}{i(\omega+i\eta)} = \nearrow$$

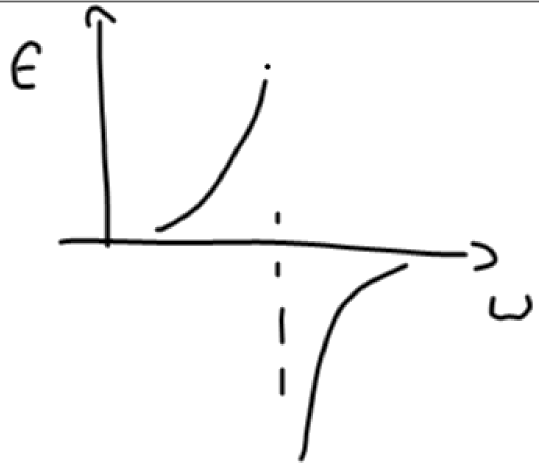
$$q^2E = i(\omega+i\eta)\frac{4\pi}{c^2}\overleftrightarrow{\sigma}^T\cdot\vec{E} + \frac{(\omega+i\eta)^2}{c^2}\vec{E}$$

$$= \frac{(\omega+i\eta)^2}{c^2}\overleftrightarrow{\epsilon}^T\vec{E}$$

$$\sqrt{\epsilon} = n+ik$$

$$\boxed{\overleftrightarrow{\epsilon}^T(\vec{q},\omega) = 1 + \frac{4\pi i}{(\omega+i\eta)}\overleftrightarrow{\sigma}^T}$$

$$\boxed{\begin{gathered}\overleftrightarrow{\epsilon}^T(q,\omega) = \left(1-\frac{\omega_p^2}{(\omega+i\eta)^2}\right)\overleftrightarrow{I} + \frac{4\pi}{(\omega+i\eta)^2}\chi^R_{jj}(q,\omega)\\ \omega_p^2 = \frac{4\pi ne^2}{m}\end{gathered}}$$



q fixe

Longitudinale

$$i\vec{q}\cdot\vec{E} = 4\pi\left(\rho_e + \delta\langle\rho\rangle\right)$$

$$i\vec{q}\cdot\underbrace{\overset{\leftrightarrow}{\epsilon}^L\cdot\vec{E}}_{\vec{D}} = 4\pi\rho_e$$

$$q_x E_x = 4\pi\left(\rho_e + \delta\langle\rho\rangle\right)$$

$$q_x E_y = \frac{4\pi\rho_e}{\epsilon_L}$$

$$\boxed{\frac{1}{\epsilon_L} = \frac{\rho_e + \delta\langle\rho\rangle}{\rho_e}}$$

$$\delta\langle\rho\rangle = -\chi^R_{\rho\rho}(q,\omega)\,\phi_e(q,\omega)$$

$$\phi_e(q,\omega) = \frac{4\pi}{q^2}\rho_e$$

$$\boxed{\frac{1}{\epsilon_L} = 1 - \frac{4\pi}{q^2}\chi^R_{\rho\rho}(q,\omega)}$$

$\epsilon_L = 0$

$\Rightarrow$

modes

coll.

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