2.2 Réponse linéaire

- Schroedinger vs Heisenberg
- Représentation interaction
- Réponse linéaire

2.3 Propriétés générales

- Déf.
- Prop. de symétrie
- Kramers-Kronig
- Dissipatif $\omega \chi''$
- Fluct. dissipation
- Règles de somme

2.2 Rép. linéaire

$$\mathcal{H}(t) = H_0 + \delta\mathcal{H}(t)$$

$$\delta\mathcal{H}(t) = -\int d^3r\, A_i(\vec{r})\, a_i(\vec{r},t)$$

$$\begin{array}{cc} \uparrow & \uparrow \\ S_z & h_z \\ \vec{j} & \frac{e\vec{A}}{c} \\ \rho & \varphi \end{array}$$

On cherche:

$$\delta\langle B(\vec{r},t)\rangle = \frac{i}{\hbar}\int_{t_0}^{t} dt'\, d^3r'\, \langle[B^0(\vec{r},t), A_i^0(\vec{r}',t')]\rangle\, a_i(\vec{r}',t')$$

$$\rightarrow \chi^R_{BA}(\vec{r},t,\vec{r}',t') = \frac{i}{\hbar}\langle[B^0(\vec{r},t), A^0(\vec{r}',t')]\rangle\,\theta(t-t')$$

– Schrödinger vs Heisenberg.

$$\boxed{i\hbar \frac{\partial \Psi_s}{\partial t} = \mathcal{H}(t)\, \Psi_s}$$

$\frac{\partial}{\partial t} \langle \Psi_s | \Psi_s \rangle = 0$ avec $\mathcal{H}(t)$ hermitien

$$\to \Psi_s(t) = U(t,0)\, \Psi_s$$
$$= U(t,t_0)\, \Psi_s(t_0)$$

$$U(t,t) = 1$$

$$\longrightarrow \quad U^{+}(t,t_0)\,U(t,t_0) = 1 \qquad \text{unitaire}$$

$$\longrightarrow \quad U(t_0,t)\,U(t,t_0) = 1 \qquad \text{inv. } t \to -t$$

$$U^{-1}(t,t_0) = U^{+}(t,t_0) = U(t_0,t)$$

$$\longrightarrow \langle \Psi_S(\underline{t}) | \, O_S \, | \Psi_S \underline{(t)} \rangle = \langle \Psi_H | \, O_H(t) \, | \Psi_H \rangle$$

$$O_S = O_H \quad \text{à } t = 0$$

$$\Psi_S = \Psi_H \quad \text{à } t = 0$$

$\rightarrow O_H(t) = U^+(t,0)\, O_S\, U(t,0)$

Si H_0 est indép. de t :

$$U(t,t_0) = e^{-iH_0(t-t_0)/\hbar}$$

- Repr. d'interaction

$$\mathcal{H}(t) = H_0 + \delta H(t)$$

$$\rightarrow \boxed{U(t,0) = e^{-iH_0 t/\hbar}\, U_I(t,0)}$$

$$U_I^{-1}(t,t_0) = U^\dagger(t,t_0)$$

$$U(t,t_0) = U(t,0)\,U(\underline{0},t_0)$$

$$= e^{-iH_0 t/\hbar}\, U_I(t,0)\, U_I(0,t_0)\, e^{+\frac{iH_0 t_0}{\hbar}}$$

$$\boxed{U(t,t_0) = e^{-iH_0 t/\hbar}\, U_I(t,t_0)\, e^{+iH_0 t_0/\hbar}}$$

$$U^\dagger(t,t_0) = e^{-iH_0 t_0/\hbar}\, U_I^\dagger(t,t_0)\, e^{+iH_0 t/\hbar}$$

$$\langle \Psi_s | U^\dagger(t,0)\, O_s\, U(t,0) | \Psi_s \rangle$$

$$\langle \Psi_s | U_I^\dagger(t,0)\, e^{iH_0 t/\hbar}\, O_s\, e^{-iH_0 t/\hbar}\, U_I(t,0) | \Psi_s \rangle$$

$$\equiv \langle \Psi_I(t) | O_I(t) | \Psi_I(t) \rangle$$

$$|\Psi_I(t)\rangle = U_I(t,0) |\Psi_s\rangle$$

$$O_I(t) = e^{iH_0 t/\hbar}\, O_s\, e^{-iH_0 t/\hbar}$$

Eq. mvt pour U_I :

$t_0=0$ $\quad i\hbar \dfrac{\partial U(t,t_0)}{\partial t} = \mathcal{H}(t)\, U(t,t_0)$

$$\Psi_s(t) = U(t,0)\,\Psi_s$$

$$U(0,t_0)$$

$$U(t,0) = e^{-iH_0 t/\hbar}\, U_I(t,0)$$

$$\cancel{H_0 e^{-iH_0 t/\hbar}}\; U_I(t,0) \qquad \left(e^{-iH_0 t/\hbar}\, U_I(t,0)\right)$$

$$+i\hbar\, e^{-iH_0 t/\hbar} \frac{\partial}{\partial t} U_I(t,0) = (\cancel{H_0} + \delta H(t))$$

$$i\hbar \frac{\partial U_I}{\partial t}(t,0) = \left[e^{iH_0 t/\hbar}\, \delta H(t)\, e^{-iH_0 t/\hbar}\right] U_I(t,0)$$

$$\boxed{i\hbar \frac{\partial U_I}{\partial t}(t,0) = \delta H_I(t)\, U_I(t,0)}$$

$$U(0,t_0)$$

$$i\hbar\left[U_I(t,0) - U_I(0,0)\right] = \int_0^t \delta H_I(t')\, U_I(t',0)\, dt'$$

$$\boxed{U_I(t,t_0) = 1 - \frac{i}{\hbar}\int_{t_0}^{t} \delta H_I(t')\, dt'}$$

- Réponse linéaire

$$\delta H_I(t) = -\int d^3r \cdot A_i^{\circ}(\vec{r},t)\, a_i(\vec{r},t)$$

$$\uparrow$$

$$\langle B(\vec{r},t)\rangle = \langle U^{+}(t,t_0)\,\underline{B(\vec{r})}\,U(t,t_0)\rangle$$

$$= \langle \cancel{e^{-iH_0t_0/\hbar}}\; U_I^{+}(t,t_0)\, e^{+iH_0t/\hbar}$$

$$B(\vec{r})\, e^{-iH_0t/\hbar}\; U_I(t,t_0)\, \cancel{e^{+iH_0t_0/\hbar}}\rangle$$

$$\langle x\rangle = \frac{T_r[\rho x]}{T_r[\rho]}$$

$$\langle B(\vec{r},t)\rangle = \langle U_I^{\dagger}(t,t_0)\, B^{0}(r,t)\, U_I(t,t_0)\rangle$$

$$U_I(t,t_0) = 1 + \frac{i}{\hbar}\int_{t_0}^{t} dt' \int d^3r'\, A_i^{0}(\vec{r}\,',t')\, a_i(\vec{r}\,',t')$$

$$\boxed{\delta\langle B(\vec{r},t)\rangle = \frac{i}{\hbar}\int_{t_0}^{t} dt' \int d^3r'\, \langle [B^{0}(\vec{r},t), A_i^{0}(\vec{r}\,',t')]\rangle\, a_i(\vec{r}\,',t')}$$

$t_0 = -\infty$

$$\delta\langle B(\vec{r},t)\rangle = \int d^3r' \int_{-\infty}^{\infty} dt'\, \chi^{R}_{BA_i}(\vec{r},t;\vec{r}\,',t')\, a_i(r',t')$$

$$\boxed{\chi^{R}_{BA}(r,t;r',t') = \frac{i}{\hbar}\langle [B^{0}(\vec{r},t), A^{0}(\vec{r}\,',t')]\rangle\, \theta(t-t')}$$

$$\delta\langle B(\vec{q},\omega)\rangle = \chi^{B}_{BA_i}(\vec{q},\omega)\, a_i(\vec{q},\omega)$$

Relation de réciprocité d'Onsager

$$\vec{j}_e = \;\ldots\; \vec{\nabla} T$$

$$\vec{j}_s = \;\ldots\; \vec{\nabla}\varphi$$

$$e E \underbrace{v t} \lesssim k_B T$$

$$t \sim 10^{-6}\,s$$

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- Déf. + notation

$$\chi^R_{BA}(\vec{r},t;\vec{r}',t') = \frac{i}{\hbar}\langle[B(r,t),A(r',t')]\rangle\,\theta(t-t')$$

$$\chi''_{BA}(r,t;r',t') = \frac{1}{2\hbar}\langle[B(r,t),A(r',t')]\rangle$$

↗ fonction spectrale

$$\boxed{\chi^R_{A_iA_j}(t-t') = \frac{i}{\hbar}\langle[A_i(t),A_j(t')]\rangle\,\theta(t-t')}$$

$$\boxed{\chi^R_{A_iA_j}(t-t') = 2i\,\chi''_{A_iA_j}(t-t')\,\theta(t-t')}$$

– Symétries de X"

S est une symétrie si :

$$[S, H] = 0 \qquad [S, \rho] = 0$$

$$S\rho - \rho S = 0 \Rightarrow \boxed{S^{-1}\rho S = \rho}$$

$$\frac{Tr[\rho O]}{Tr\rho} = \frac{Tr[S^{-1}\rho S O]}{Tr\rho} = \langle S O S^{-1}\rangle$$

$$= \langle O \rangle$$

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· Inv. sous translation.

$$\chi''_{BA}(r,t;r',t') = \chi''_{BA}(r-r';t-t')$$

· Parité :

$$P^{-1}\,O(\vec{r})\,P = \epsilon^{P}\,O(-r)$$

$$\epsilon^{P} = \pm 1$$

$$\rho(\vec{r}) = \sum_\alpha \delta(\vec{r} - \vec{r}_\alpha)$$

$$\mathcal{P}^{-1} \rho(\vec{r}) \mathcal{P} = \sum_\alpha \delta(\vec{r} + \vec{r}_\alpha)$$

$$= \sum_\alpha \delta(-\vec{r} - \vec{r}_\alpha) = \rho(-\vec{r})$$

$$\mathcal{P}^{-1} \vec{p}(\vec{r}) \mathcal{P} = \sum_\alpha - \frac{\hbar}{i} \nabla_{r_\alpha} \delta(-\vec{r} - \vec{r}_\alpha)$$

$$= -p(\vec{r})$$

$$\chi''_{BA}(\vec{r},t;\vec{r}',t') = \epsilon^P_B \epsilon^P_A \chi''(-\vec{r},t;-\vec{r}',t')$$

$$P^{-1} B A P = P^{-1} B P P^{-1} A P$$

- Sym. sous inversion de t :

$$-i\hbar \frac{\partial \psi_s^*}{\partial t} = H \psi_s^*(t)$$

$$\mathcal{T}_t = \underset{\longrightarrow}{K}\, U \qquad \underset{\rightarrow}{K} = \text{complexe conjugué de ce qui est à droite}$$

$$U = 1$$

Notation de Dirac ne fonctionne pas!

$$\langle\alpha|X|\beta\rangle = \left(\langle\alpha|X\right)|\beta\rangle$$

$$= \langle\alpha|\left(X|\beta\rangle\right)$$

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$\langle \alpha | K | \beta \rangle$

Antiunitaire !

$$T_t \left(a_1 |\psi_1\rangle + a_2 |\psi_2\rangle \right) = a_1^* T_t |\psi_1\rangle + a_2^* T_t |\psi_2\rangle$$

$\langle \alpha | \beta \rangle$

$$\boxed{\langle \alpha | U^\dagger U | \beta \rangle = \langle \alpha | \beta \rangle}$$

$\langle \alpha | \overleftarrow{K} \, \overrightarrow{K} | \beta \rangle$

$$\langle \alpha | \beta \rangle$$

$$\left| \langle \alpha | \underleftarrow{K} \, \underrightarrow{K} | \beta \rangle \right| = \left| \langle \beta | \alpha \rangle \right|$$

$$\left| \langle \alpha' | \beta' \rangle \right| = \left| \langle \alpha | \beta \rangle \right|$$

$$\langle i | \underleftarrow{K} \; O \; \underrightarrow{K} | j \rangle$$

$$= \langle i | \underleftarrow{K} \, \underrightarrow{K} \, O^* | j \rangle = \left(\langle i | O^* | j \rangle \right)^*$$

$$\boxed{\begin{aligned} \langle \psi | \varphi \rangle^* &= \langle \varphi | \psi \rangle \\ \langle \psi | O | \varphi \rangle^* &= \langle \varphi | O^\dagger | \psi \rangle \end{aligned}}$$

$$\langle i | \underleftarrow{K}\, O\, \underrightarrow{K} | j \rangle = \langle j | O^{*\dagger} | i \rangle$$

$$\langle \underleftarrow{K}\, O\, \underrightarrow{K} \rangle = \langle O^{\dagger *} \rangle$$

$$= \epsilon^t \langle O^\dagger \rangle$$

$$\langle \overleftarrow{K} A(t) B \overrightarrow{K} \rangle = \langle B^{*+} A^{*+}(t) \rangle$$

$$= \epsilon_B^t \epsilon_A^t \langle B A(-t) \rangle$$

$$A^{+}(t) = A(t)$$

$$A^{+*}(t) = A^{*}(t) = e^{-iHt/\hbar} \underset{\approx}{A^{*}} e^{iHt/\hbar}$$

$$= A(-t) \, \epsilon_A^t$$

$$\chi''_{A_iA_j}(t-t') = \epsilon^{t}_{A_i}\,\epsilon^{t}_{A_j}\,\chi''_{A_jA_i}(-t'-(-t))$$

$$\boxed{\chi''_{A_iA_j}(\omega) = \epsilon^{t}_{A_i}\,\epsilon^{t}_{A_j}\,\chi''_{A_jA_i}(\omega)}$$

Spin

$$\vec{r} \times \vec{p} \to -\vec{r} \times \vec{p}$$

$$\vec{S} = -\vec{S}$$

$$T_t = \underset{\to}{K} U = \underset{\to}{K} e^{i\delta} \sigma_y$$

$$T_t |\uparrow\rangle = -i e^{-i\delta} |\downarrow\rangle \to |\downarrow\rangle$$

$$T_t |\downarrow\rangle = i e^{-i\delta} |\uparrow\rangle \xrightarrow[e^{-i\delta} = i]{} -|\uparrow\rangle$$

Neutron

$$\langle [S^z(q,\omega), S^z(-q,-\omega)] \rangle$$

$$= \chi''_{S^z_q S^z_{-q}}(\omega)$$

Avec T_t et χ'' commutateur

$$\longrightarrow \chi''_{S^z_q S^z_{-q}}(\omega) = \chi''^{*}_{S^z_q S^z_{-q}}(\omega) = -\chi''_{S^z_q S^z_{-q}}(-\omega)$$