- $\chi^{R}_{A_j A_i}(\omega) = \int \frac{d\omega'}{\pi} \frac{\chi''_{A_j A_i}(\omega')}{\omega' - \omega - i\eta}$

$z \to \omega + i\eta$ Retardée

$z \to \omega - i\eta$ Avancée.

- $\omega \chi''_{A_j A_i}(\omega)$ positive déf.

- $S_{A_i A_j}(\omega) = \frac{2\hbar}{1 - e^{-\beta\hbar\omega}} \chi''_{A_i A_j}(\omega)$

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$$S_{A_j A_i}(-t) = S_{A_i A_j}(t - i\hbar\beta)$$

$$S_{A_j A_i}(-\omega) = e^{-\beta\hbar\omega} S_{A_i A_j}(\omega)$$

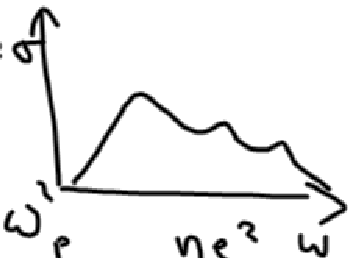
sept. 13 - 09:06

Menu du jour!

- Règles de somme
- Kubo pour conductivité

Règles de somme.

- Relier exp. les unes aux autres Re σ



$$\int \operatorname{Re}\sigma \, \frac{d\omega}{2\pi} = \frac{\omega_p^2}{8\pi} = \frac{ne^2}{2m}$$

- Limites $\omega \to \infty$ fcts de rép.
- Contraintes sur phénoménologie

Règle de somme T.D.

$$\chi_{A_iA_j} \equiv \frac{\partial \langle A_i \rangle}{\partial a_j} = \lim_{q\to 0} \chi^R_{A_iA_j}(q,0)$$

$$= \lim_{q\to 0} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\chi''_{A_iA_j}(\omega)}{\omega}$$

$$\delta \langle A_i \rangle_\omega = \chi^R_{A_iA_j}(\omega)\, \delta a_j(\omega)$$

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$$\lim_{q\to 0} \chi^{R}_{nn}(q,\omega=0)$$

$$= \lim_{q\to 0} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\chi''_{nn}(q,\omega)}{\omega} = \left(\frac{\partial n}{\partial \mu}\right)_{T,V}$$

Autre preuve

$$\langle NN \rangle - \langle N \rangle^2 = \frac{1}{\beta}\left(\frac{\partial n}{\partial \mu}\right)_{T,V}$$

$$= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S_{NN}(\omega)$$

F.D.

$$\int dt\, e^{i\omega t} \langle N(t) N \rangle$$

$$\propto \delta(\omega)$$

$$S_{NN}(\omega) = \lim_{\omega \to 0} \frac{2\hbar}{1 - e^{-\beta\hbar\omega}} \chi''_{NN}(\omega)$$

$$= \frac{2}{\beta\omega} \chi''_{NN}(\omega)$$

Moments, règles de somme et développement $\omega \to \infty$

$$\chi^R_{A_i A_j}(q,\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \frac{\chi''_{A_i A_j}(\omega')}{\omega' - \omega - i\eta}$$

$\omega \to \infty$

$$\simeq \sum_{n=1}^{\infty} -\frac{1}{\omega^{2n}} \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} (\omega')^{2n-1} \chi''_{A_i A_j}(q,\omega')$$

Calcul des moments.

$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi}\, \omega^n \chi''_{A_i A_j}(\omega)$$

$$= \left(i \frac{\partial}{\partial t}\right)^n \left[\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t}\, 2\chi''_{A_i A_j}(\omega)\right]$$

$$= \frac{1}{\hbar} \left\langle \left[\left(i\frac{\partial}{\partial t}\right)^n A_i(t), A_j(0)\right] \right\rangle \qquad t=0$$

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$$= \frac{1}{\hbar} \left\langle \left[\left[\left[A_i(t), \frac{H}{\hbar} \right], \frac{H}{\hbar} \right] \ldots A_j(0) \right] \right|_{t=0}$$

$$i \frac{\partial}{\partial t} A_i(t) = \left[A_i(t), \frac{H}{\hbar} \right]$$

$$A_i(t) = e^{iHt/\hbar} A_i e^{-iHt/\hbar}$$

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Exemple : règle de somme f

$$\boxed{\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\,\omega\,\chi''_{nn}(q,\omega)=\frac{nq^2}{m}}$$

à prouver

$$= \frac{i}{\hbar V}\left\langle\left[\frac{\partial}{\partial t}n_q(t)\,,\,n_{-q}(0)\right]\right\rangle_{t=0}$$

$$= -\frac{1}{\hbar^2 V}\left\langle\left[[H,n_q],n_{-q}\right]\right\rangle \quad (1)$$

V ?

$$\int d^3(r-r') \langle n(r) n(r') \rangle e^{-iq\cdot(r-r')}$$

$$= \frac{1}{V} \int d^3r \; e^{-iq\cdot r} \int d^3r' e^{iq\cdot r'} \langle n(r) n(r') \rangle$$

$$\frac{r+r'}{2}, \; r-r' \longrightarrow r, \; r'$$

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$$[p^x_\beta, n_q] = \frac{\hbar}{i}\left[\frac{\partial}{\partial x_\beta}, \sum_\alpha e^{-iq r_\alpha}\right]$$

$$n_q = \int dr\, e^{-iqr} \underbrace{\sum_\alpha \delta(r - r_\alpha)}_{n(r)}$$

$$= \sum_\alpha e^{-iqr_\alpha}$$

$$= -\hbar q^x e^{-iqr_\beta}$$

$$\left[\sum_\beta \frac{p_\beta^2}{2m}, n_q\right] = \frac{2}{2m}\sum_\beta \vec{p}_\beta \cdot \left(-\hbar\vec{q}\, e^{-iq\cdot r_\beta}\right)$$

$$[p\cdot p, n] = p\cdot \underbrace{[p, n]} + \underbrace{[p, n]\cdot p}$$

$$\frac{1}{m}\sum_\beta \left(-\hbar\vec{q}\, e^{-iq\cdot r_\beta}\cdot \left[\vec{p}_\beta, n_{-q}\right]\right)$$

$$= -\frac{1}{m}\sum_{\beta=1}^{N} \hbar^2 q^2 e^{-iq\cdot r_\beta}\, e^{iq\cdot r_\beta} \underset{(2)}{=} -\frac{\hbar^2 q^2}{m} N$$

Formule de Kubo

$E_y(q_x, \omega) = i(\omega + i\eta) A_y / c$

diamagnétique

À prouver

$$\boxed{\delta \langle j_\mu (q,\omega) \rangle = \left[\chi^R_{j_\mu j_\nu} (q,\omega) - \frac{ne^2}{m} \delta_{\mu\nu} \right] \frac{A_\nu(q,\omega)}{c} - \chi^R_{j_\mu \rho} (q,\omega)\, \phi(q,\omega)}$$

$$E = -\frac{1}{c} \frac{\partial A}{\partial t} - \nabla \varphi \qquad B = \nabla \times A$$

$$A(t) = \int \frac{d\omega}{2\pi} e^{-i(\omega + i\eta) t} A(\omega)$$

Inv. de jauge

$$A \to A + \nabla\Lambda$$

$$\varphi \to \varphi - \frac{1}{c}\frac{\partial\Lambda}{\partial t}$$

$$\Psi \to e^{ie\Lambda/\hbar c}\,\Psi$$

$$\vec{p}_\alpha = \frac{\hbar}{i}\vec{\nabla}_\alpha \longrightarrow \left(\frac{\hbar}{i}\vec{\nabla}_\alpha - \frac{e}{c}\vec{A}(r_\alpha)\right)$$

Inv. de jauge.

sept. 13 - 10:02

$$\delta H(t) = \int d^3r\, \varphi(r,t)\, \rho(r)$$

$$-\frac{\hbar^2}{2m}\nabla_\alpha^2 \longrightarrow -\frac{\hbar^2}{2m}\nabla_\alpha^2 - \frac{e\hbar}{2mci}\Big(A(r_\alpha)\cdot\nabla_\alpha$$

$$+\nabla_\alpha\cdot A(r_\alpha)\Big) + \frac{e^2}{2mc^2}A^2(r_\alpha)$$

$$\delta H(t) = -\sum_\alpha \frac{e\hbar}{2mci}\Big(A(r_\alpha)\cdot\nabla_\alpha + \nabla_\alpha\cdot A_\alpha(r_\alpha)\Big)$$

$$= -\frac{1}{c}\int d^3r\, A(r,t)\, j(\vec{r}) \longleftarrow$$

$$j(r) = \frac{e}{2m}\sum_\alpha \Big(\delta(r-r_\alpha)p_\alpha + p_\alpha\delta(r-r_\alpha)\Big)$$

$$\boxed{j^{A}(\vec{r}) = j(r) - \frac{e}{mc} A(r)\,\rho(r)}$$

$$p_\alpha \to \frac{\hbar}{i}\nabla_\alpha - \frac{e}{c} A(r_\alpha)$$

$$\rho(r) = \sum_\alpha \delta(r - r_\alpha)$$

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Conductivité transversale

transversale

$$\boxed{\delta \langle j_y(q_x,\omega)\rangle = \sigma_{yy}(q_x,\omega)\, E_y(q_x,\omega)}$$

déf.

<u>Transversal</u> $\vec{\nabla}\cdot\vec{C} = 0$ $\vec{q}\cdot\vec{C} = 0$

<u>Longitudinal</u> $\vec{\nabla}\times\vec{C} = 0$ $\vec{q}\times\vec{C} = 0$

$$\vec{A}^L = (\hat{q}\cdot\vec{A})\,\hat{q}$$

$$\vec{A}^T = (\overleftrightarrow{I} - \hat{q}\hat{q})\cdot\vec{A}$$

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$$\overleftrightarrow{\sigma}^{T}(q,\omega) = (\overleftrightarrow{I} - \hat{q}\hat{q}) \cdot \overleftrightarrow{\sigma} \, (\overleftrightarrow{I} - \hat{q}\hat{q})$$

$$\overleftrightarrow{\sigma}^{L}(q,\omega) = \hat{q}\,\hat{q} \cdot \overleftrightarrow{\sigma} \cdot \hat{q}\,\hat{q}$$

$$\boxed{\sigma_{yy}(q_x,\omega) = \frac{1}{i(\omega + i\eta)} \left[\chi^{R}_{j_y j_y}(q_x,\omega) - \frac{ne^2}{m} \right]}$$

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Conductivité longitudinale

À prouver :

$$\delta\langle j_x(q_x,\omega)\rangle = \frac{1}{i(\omega+i\eta)}\left[\chi^R_{j_x j_x}(q_x,\omega) - \frac{n e^2}{m}\right]\left(\frac{i(\omega+i\eta)}{c}A_x(q_x,\omega) - q_x\phi(q_x,\omega)\right)$$

$$= \frac{1}{iq_x}\chi^R_{j_x\rho}(q_x,\omega)\left(\quad \text{idem} \quad\right)$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot j$$

$$\boxed{\frac{\partial \rho(q,t)}{\partial t} = -iq \cdot j_q(q,t)}$$

$$\frac{\partial}{\partial t} \chi^R_{j_x \rho}(q_x, t) = \frac{\partial}{\partial t}\left(\frac{i}{\hbar}\theta(t)\langle[j(q_x,t), \rho(-q_x,0)]\rangle\right.$$

$$\delta(t)\frac{i}{\hbar V}\langle[j_x(q_x,0), \rho(-q_x,0)]\rangle$$

$$+\theta(t)\frac{i}{\hbar V}(-iq_x)\langle[j_x(q_x,0), j_x(-q_x,-t)]\rangle$$

$$\boxed{-i(\omega+i\eta)\chi^R_{j_x\rho}(q,\omega) = iq_x\frac{ne^2}{m} - iq_x\chi^R_{j_x j_x}(q,\omega)}$$

$$\underline{\text{Car}}\quad \frac{i}{\hbar V}\langle[\dot{j}_x(q_x,0),\rho(-q_x,0)]\rangle = \frac{i q_x n e^2}{m}$$

$$= i\int\frac{d\omega}{\pi}\chi''_{\dot{j}_x\rho}(q_x,\omega) = i\int\frac{d\omega}{\pi}\frac{\omega}{q_x}\chi''_{\rho\rho}(q,\omega)$$

$$\frac{\partial\rho}{\partial t} = -iq_x\cdot\dot{j}_x \qquad = \frac{i}{q_x}\frac{ne^2}{m}q_x^2$$

$$-i\omega\,\rho(q,\omega) = -iq_x\cdot\dot{j}_x(q_x,\omega)$$

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$$\boxed{\begin{aligned}\sigma_{xx}(q_x,\omega) &= \frac{1}{i(\omega+i\eta)}\left[\chi^R_{j_x j_x}(q_x,\omega)-\frac{ne^2}{m}\right]\\ &= \frac{1}{iq_x}\,\chi_{j_x\rho}(q_x,\omega)\end{aligned}}$$

$\varphi=0$ $\quad A$ indép. de t $\Rightarrow$

$$\left[\chi_{j_x j_x}(q_x,0)-\frac{ne^2}{m}\right]=0$$

Règle de somme f

$A = 0 \qquad \delta\langle j_x(q,\omega)\rangle = -\chi^R_{j_x\rho}(q,\omega)\phi(q,\omega)$

Λ indép. de x, dép. de t.

$$A \to A + \nabla\Lambda = 0$$

$$\varphi \to \varphi - \frac{1}{c}\frac{\partial\Lambda}{\partial t} \neq 0$$

$$\Rightarrow \boxed{\chi_{j_x\rho}(0,\omega) = 0}$$

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	D	D_s
Métal	D	0
Isolant	0	0
S.C.	D	D_s

$$D_s = \pi \lim_{q_x \to 0} \left[\frac{ne^2}{m} - \chi^R_{j_y j_y}(q_x, 0) \right]$$

$$D = \pi \lim_{\omega \to 0} \left[\frac{ne^2}{m} - \mathrm{Re}\, \chi^R_{j_x j_x}(0,\omega) \right]$$

0

si pas d'interaction