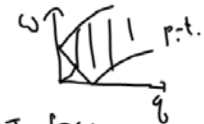
Résumé:

↑ ↑ ↑ ↑
↓ ↓ ↓

État ordonné



À partir de N

χ_{zz} ou χ_{+-}

$$\mathrm{Im}\left[\frac{\chi^0}{1-\frac{U}{2}\chi^0}\right] \propto \frac{c\,\omega/v_F q}{(1-UN(E_F))^2+\left(\frac{\omega}{v_F q}\right)^2}$$

$$\mathrm{Im}\left[\chi^{+-}+\chi^{-+}\right]$$

$$\chi^{R}_{-+}+\chi^{R}_{+-} = \frac{\Delta/U}{\omega+i\eta+Dq^2} - \frac{\Delta/U}{\omega+i\eta-Dq^2}$$

$$\mathrm{Im}\left(\chi^{+-}+\chi^{-+}\right)^R = -\frac{\Delta}{U}\pi\,\delta(\omega+Dq^2) + \frac{\Delta}{U}\pi\,\delta(\omega-Dq^2)$$

$$\chi^{+-}(q,\omega=0)+\chi^{-+}(q,\omega=0) = \int\frac{d\omega'}{\pi}\frac{\mathrm{Im}(\quad)}{\omega'}$$

$$= +\frac{\Delta}{U}\frac{2}{Dq^2}$$

$$\langle S_x^2\rangle$$

$$T\sum_{iq_n}\sum_q\frac{1}{N}\chi^{-+}(q,iq_n)\,e^{iq_n\eta}$$

$$\frac{1}{N}\sum_q \underbrace{T\sum_{iq_n} e^{iq_n\eta}\frac{\Delta}{U}\frac{1}{iq_n+Dq^2}}$$

$$= \frac{\Delta}{U}\frac{1}{N}\sum_q\frac{-1}{e^{-\beta Dq^2}-1}$$

$$= \frac{\Delta}{U}\int^{\Lambda}\frac{d^dq}{(2\pi)^d}\frac{T}{Dq^2} \leftarrow$$

$$Z = \int dS_x\, e^{-\beta C(\vec{\nabla}S_x)^2} \rightarrow \int dS_x\, e^{-\beta C q^2|S_x|^2}$$

$$\rightarrow \langle|S_x(q)|^2\rangle C q^2 = \frac{k_B T}{2}$$

$$\langle S_x^2\rangle = \int\frac{d^dq}{(2\pi)^d}|S_x(q)|^2 =$$

nov. 22 - 08:55

$$\langle S_z^2 \rangle = \langle (n_\uparrow - n_\downarrow)^2 \rangle$$

$$= \langle n_\uparrow^2 \rangle + \langle n_\downarrow^2 \rangle - 2\langle n_\uparrow n_\downarrow \rangle$$

$$T \sum_n \frac{1}{N} \sum_q \frac{\chi_0}{1 - \frac{U_{sp}}{2}\chi_0} = n - 2\langle n_\uparrow n_\downarrow \rangle$$

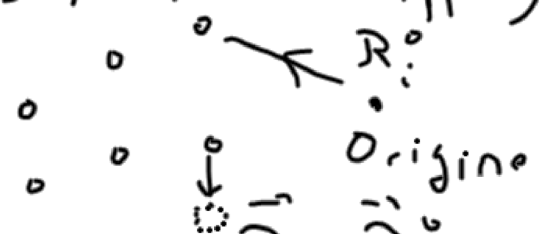
$$T \sum_n \frac{1}{N} \sum_q \frac{\chi_0}{1 + \frac{U_{ch}}{2}\chi_0} = n + 2\langle n_\uparrow n_\downarrow \rangle - n^2$$

$$U_{sp} = \frac{U\langle n_\uparrow n_\downarrow \rangle}{\langle n_\uparrow \rangle \langle n_\downarrow \rangle}$$

Interactions électron-phonon

1. Phonons
2. Int. él. phonon "nue" 
3. Int. effective entre électrons
4. Phonon renormalisés. 
5. Th. de Migdal
6. Int. e-e effective (avec écrantage)
7. m^* venant des phonons

① Phonons : (Rappel)



$$\vec{R}_i - \vec{R}_i^{0} = \vec{u}_i$$

$$\vec{u}_i = \frac{1}{\sqrt{NM}} \sum_{\vec{k}\lambda} Q_\lambda(\vec{k})\, \vec{e}^{\,\lambda}(\vec{k})\, e^{-i\omega_\lambda(k)t}$$

$$e^{i\vec{k}\cdot\vec{R}_i^{0}}$$

Modes propres

$$Q_\lambda(\vec{k}) = \frac{1}{\sqrt{2\omega_\lambda(k)}} \left(a_\lambda(\vec{k}) + a_\lambda^{+}(\vec{k}) \right)$$

$$\sum_i \left(\frac{p_i^2}{2M} + \frac{(u_i - u_j)^2}{2} k \right) \longrightarrow a^{+} a$$

$$[a, a^{+}] = 1$$

$$\mathcal{D}(\vec{k}, \tau-\tau') \equiv -2\omega_k \langle T_\tau Q_k(\tau) Q_{-k}(\tau') \rangle$$

$$\boxed{\mathcal{D}(\vec{k}, \omega_n) = \frac{-2\omega_k}{\omega_n^2 + \omega_k^2}}$$

Devoir # 2

② Int. e-ph

$$H = K_e + V_{ee} + \underbrace{K_i + V_{ii}} + V_{ie}$$

$$\omega_{ip}^2 = \frac{(Ze)^2 n 4\pi}{M \epsilon_i}$$

$$V_{e-i} = \text{statique} + \sum_{i\sigma} \int d^3r\, \Psi^\dagger_\sigma(r) \Psi_\sigma(r)\, \vec{u} \cdot \nabla_{R_i} V(\vec{r} - \vec{R}_i) \Big|_{\vec{R}_i^0}$$

$$\Psi_\sigma(r) = \frac{1}{\sqrt{\Omega}} \sum_k c_{k\sigma}\, e^{ik\cdot r}$$

$r + R_i^0$

$$= \frac{1}{\Omega} \sum_{i\sigma} \sum_k \sum_{k'} \int d^3r\; e^{-ik\cdot r + ik'\cdot r}\, c^+_{k\sigma} c_{k'\sigma}\; \vec{u}_i \cdot \vec{\nabla}_{R_i} V(r - R_i)\Big|_{R_i^0}$$

$r + R_i^0$

$$\boxed{\frac{1}{\Omega} \sum_i e^{-i(k-k')\cdot R_i^0}\, u_i = \frac{1}{\sqrt{NM}}\, Q(k'-k)\, \vec{e}\,(k'-k)}$$

$$\vec{W}(k,k') = -\int d^3r\; e^{i(k-k')\cdot r}\, \nabla_r V(r)$$

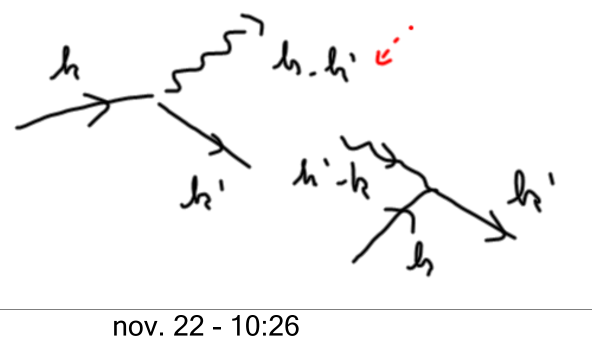
$$= i(\vec{k}-\vec{k}')\left[\frac{-4\pi Z e^2 n}{|k-k'|^2 \epsilon_i}\right]$$

$\sqrt{2\omega_k}$

$$= \sum_{\sigma k k'} \frac{c^+_{k'\sigma} c_{k\sigma}}{\sqrt{\Omega}} \left[\frac{\vec{W}(k,k')\cdot \vec{e}\,(k-k')}{\sqrt{NM}\,\sqrt{2\omega}}\right] Q(k-k')$$

$\sqrt{\Omega}$

$$\sum_{\sigma k k'} \frac{[M_{kk'}]}{\sqrt{\Omega}}\, c^+_{k'\sigma} c_{k\sigma} \left[a_{k'-k} + a^+_{k-k'}\right]$$



$$M_{k'k} = \frac{i\,4\pi\, Z e^2 n_0}{\sqrt{\rho_M\, 2\omega_{ip}}\;|k-k'|\,\epsilon_i}$$

13 Interaction e-e <u>médiée</u> par les phonons :

$$\langle \langle \Psi \Psi^{+} \rangle_e \rangle_i \qquad \langle a \rangle_i = 0$$

$$\langle a^{+} \rangle_i = 0$$

$$Z = \mathrm{Tr}_{ie}\left[T_\tau \left(e^{-\beta(K_e + V_{ee})} e^{-\beta(K_i + V_{ii})} e^{-\int_0^\beta V_{ei}(\tau)\,d\tau} \right)\right] \frac{Z_i^\circ Z_e^\circ}{Z_i^\circ Z_e^\circ}$$

$$Z = Z_e^\circ Z_i^\circ \left\langle \left\langle T_\tau e^{-\int_0^\beta d\tau\, V_{ei}(\tau)} \right\rangle_i \right\rangle_e$$

$$= Z_e^\circ Z_i^\circ \left\langle T_\tau e^{\sum_{n=1}^{\infty} \frac{1}{n!} \left\langle \left(-\int_0^\beta d\tau\, V_{ei}(\tau)\right)^n \right\rangle_{i\,c}} \right\rangle_e$$

$$\ln \langle e^x \rangle = \langle e^x \rangle - 1$$

$$\langle e^x \rangle = e^{[\langle e^x \rangle_c - 1]^c}$$

$$= Z_e^\circ Z_i^\circ \left\langle T_\tau e^{\boxed{\frac{1}{2}\int_0^\beta d\tau \int_0^\beta d\tau' \langle V_{ei}(\tau) V_{ei}(\tau') \rangle_i^c}} \right\rangle_e$$

$$\frac{1}{2\Omega_3} \sum_{\substack{kq\\ \sigma}} \sum_{\substack{k'q'\\ \sigma'}} \int_0^\beta d\tau \int_0^\beta d\tau'\, M_q M_{q'}$$

$$c^+_{k\sigma}(\tau) c_{k+q\sigma}(\tau) \boxed{\sqrt{2\omega_q}\, \langle Q(q,\tau) Q(q',\tau') \rangle \sqrt{2\omega_{q'}}}\; c^+_{k'\sigma} c_{k'+q\sigma'}$$

$$= \frac{1}{2\Omega} \sum_{\substack{kqk'q'\\ \sigma\sigma'}} M_q M_{-q} \int d\tau\, d\tau'\, c^+_{k\sigma}(\tau) c_{k+q\sigma}(\tau)$$

$$\mathcal{D}(q, \tau-\tau')\, c^+_{k'\sigma'}(\tau') c_{k'+q,\sigma'}(\tau')$$

$$\boxed{V_{eff} = V_q\, \delta(\tau-\tau') + |M_q|^2 \mathcal{D}(q, \tau-\tau')}$$

nov. 22 - 10:41

$$\text{Interaction} = \underset{\text{Coulomb direct}}{V_c} + V_p \leftarrow \text{avec phonons}$$

$$V_{eff} = \frac{V_c+V_p}{1+(V_c+V_p)\chi}$$

$$= \frac{V_c}{1+V_c\chi} + \left[\frac{V_c+V_p}{1+(V_c+V_p)\chi} - \frac{V_c}{1+V_c\chi}\right]$$

$$= V_c^e + \left[\frac{V_p}{1+(V_c+V_p)\chi} - \frac{V_c V_p \chi}{[1+(V_c+V_p)\chi][1+V_c\chi]}\right]$$

écrante

$$= V_c^e + \frac{V_p}{1+(V_c+V_p)\chi}\left[1 - \frac{V_c\chi}{1+V_c\chi}\right]$$

$$= V_c^e + \frac{V_p}{1+(V_c+V_p)\chi}\left[\frac{1}{1+V_c\chi}\right]$$

$$V_c^e = \frac{V_c}{1+V_c\chi} = \frac{V_c}{\epsilon}$$

$$= V_c^e + \frac{1}{1+V_c\chi}\left[\frac{V_p}{1+\frac{V_p\chi}{1+V_c\chi}}\right]\frac{1}{1+V_c\chi}$$

$$V_p \Rightarrow M_q \mathcal{D}^0(q,iq_n) M_{-q}$$

$$[\;] = M_q \frac{\mathcal{D}_0(q,iq_n)}{1+\frac{|M_q|^2}{1+V_c\chi}\chi\mathcal{D}_0(q,iq_n)} M_{-q}$$

$$\omega_{ip}^2 V_c$$

$$= M_q \frac{-2\omega_{ip}}{q_n^2+\omega_{ip}^2+|M_q|^2\frac{\chi}{1+V_c\chi}(-2\omega_{ip})}$$

$$|M_q|^2\, 2\omega_{ip} = \left(\frac{4\pi Z e^2 n_i}{\epsilon_i}\right)^2 \frac{1}{q^2}\, \frac{(\hbar\, \cancel{2\omega_{ip}})}{\cancel{2}\rho\, \cancel{\omega_{ip}}}$$

$$= \left(\frac{4\pi Z^2 e^2 n_i}{\epsilon_i M}\right)\left(\frac{4\pi e^2}{q^2 \epsilon_i}\right)$$

$$= \omega_{ip}^2 V_c$$

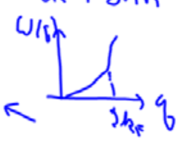
$$\mathcal{D}^e = \frac{-2\omega_{ip}}{q_n^2 + \omega_{ip}^2 - \omega_{ip}^2 V_c \frac{\chi}{1+V_c\chi}}$$

$$= \frac{-2\omega_{ip}}{q_n^2 + \omega_{ip}^2\left[1 - \frac{V_c\chi}{1+V_c\chi}\right]}$$

$$= \frac{-2\omega_{ip}}{q_n^2 + \frac{\omega_{ip}^2}{1+V_c\chi}} \rightsquigarrow 2k_F$$

Anomalies de Kohn

$$= \frac{-2\omega_{ip}}{q_n^2 + \frac{\omega_{ip}^2}{1+\frac{q_{TF}^2}{q^2}}}$$



$$\frac{\omega_{ip}^2}{1+\frac{q_{TF}^2}{q^2}} \longrightarrow \frac{\omega_{ip}^2}{q_{TF}^2}\, q^2 = \omega^2(q)$$

$$C_s = \frac{\omega_{ip}}{q_{TF}}$$

$$C_s^2 = \frac{\cancel{4\pi} Z^2 \cancel{e^2} n_i}{\cancel{M}\, \frac{\cancel{6\pi} n \cancel{e^2}}{\epsilon_F}} \quad n_i = \frac{n}{Z}$$

$$C_s^2 = \frac{2}{3}\,\frac{Z}{M}\,\frac{m v_F^2}{2} = \left[\frac{Zm}{3M}\right] v_F^2$$

$$\boxed{C_s = \sqrt{\frac{Zm}{3M}}\; v_F}$$

Bohm-Staver

Mahan p.1-14 phonons

Rickayzen p.169-171 ph.

Int. el-ph

Mahan p. 566-576

Rickayzen p. 181-193

Diag. Feynman: Mahan p.156

Rickayzen p. 185-186

Écrantage Mahan (6.3.16)

(6.3.12)

Ashcroft + Mermin

$$V_{\text{eff}}^{\text{écranté}} = \frac{V(q)}{1 + \frac{q_{TF}^2}{q^2} - \frac{\omega_{ip}^2}{\omega^2}}$$

$$= \frac{V(q)}{1 + (\epsilon - 1)_{el} + (\epsilon - 1)_{ph}}$$