# Ferromagnétisme :

7.1 Modèle de Hubbard

1. Sol. $U=0$ sans int.
2. Sol. $t=0$ élec. libres

7.2 $U \ll t$ bande faiblement remplie
Ferrom.

1. Stoner
2. $|\Psi\rangle$ variationnel
3. $-T\ln Z < -T\ln Z_{\tilde{0}} + \langle H - H_{\tilde{0}} \rangle_{\tilde{0}}$
4. Éq. du gap. Div. Landau (L.-G.)
5. G

7.1. Modèle de Hubbard:    64

Sur réseau        Hubbard Gutzwiller
Kanamori

$$H = -\sum_{\substack{ij\\\sigma}} t_{ij}\, c^{+}_{i\sigma} c_{j\sigma} + \frac{U}{2}\sum_{i\sigma} c^{+}_{i\sigma} c^{+}_{i-\sigma} c_{i-\sigma} c_{i\sigma}$$

$t_{ij} = t^{*}_{ji}$        $c_{i\sigma} c_{i\sigma} = 0$

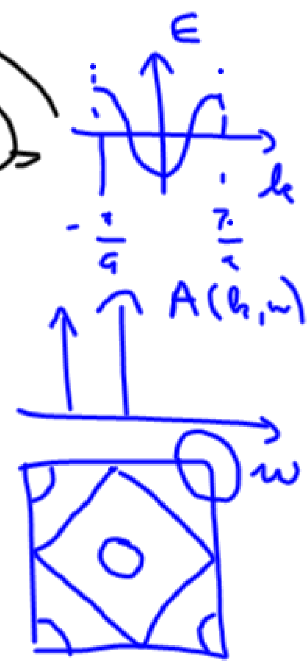
1 bande.    $i$ dénote site sur lequel l'état de Wannier est "localisé"

$$= K + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$U = 0$, $t_{ij} \neq 0$ i et j premiers voisins.

$$\mathcal{H}_0 = -t \sum_{\substack{\langle ij \rangle \\ \sigma}} \left( c^{\dagger}_{i\sigma} c_{j\sigma} + c^{\dagger}_{j\sigma} c_{i\sigma} \right)$$

$$c_{i\sigma} = \frac{1}{\sqrt{N}} \sum_{k} e^{-ik \cdot r_i} c_{k\sigma}$$




$$\sum_i e^{ik \cdot r_i} = N \delta_{k,0}$$

$$\boxed{\mathcal{H}_0 = -2t \sum_{k\sigma} \epsilon_k c^{\dagger}_{k\sigma} c_{k\sigma}}$$

$$\epsilon_k = -2t \left( \cos k_x a + \cos k_y a \right)$$

$$\mathcal{G}(k, ik_n) = \frac{1}{ik_n - \epsilon_k + \mu}$$

2. Limite atomique : $t = 0$

$$H_1 = N U n_\uparrow n_\downarrow$$

$\epsilon_0 = 0$ $\begin{cases} \cdot\ n_\uparrow = n_\downarrow = 0 \\ \cdot\ n_\uparrow = 0,\ n_\downarrow = 1 \\ \cdot\ n_\downarrow = 1,\ n_\uparrow = 0 \end{cases}$

$\epsilon_1 = U$ $\quad n_\uparrow = n_\downarrow = 1$

$$\rightarrow Z = 1 + e^{-\beta(-\mu)} + e^{-\beta(-\mu)} + e^{-\beta(U - 2\mu)}$$

Sans. int :

$$Z = \left(1 + e^{\beta\mu}\right)\left(1 + e^{\beta\mu}\right)$$

$$\mathcal{G}(\tau) = -\langle T_\tau c_\sigma(\tau) \underset{\uparrow}{\,} c_\sigma^\dagger \rangle$$

$$\sum_n |n\rangle\langle n|$$

Eqs. du mvt:

$K = H_1 - \mu N$

$$\boxed{\frac{\partial G_\sigma(\tau)}{\partial \tau} = -\delta(\tau) - \langle T_\tau \left[ K, c_\sigma(\tau) \right] c_\sigma^+ \rangle = -\delta(\tau) + \mu G_\sigma(\tau) + U\, G_{2,\sigma}(\tau)}$$

$\longrightarrow (i\omega_n + \mu)\, G_\sigma(i\omega_n) = 1 + U\, G_{2,\sigma}(i\omega_n)$

$$G_{2,\sigma} = \langle T_\tau\, c_\sigma\, n_{-\sigma}\, c_\sigma^+(-\tau) \rangle$$

$$\boxed{\frac{\partial}{\partial \tau} G_{2,\sigma}(\tau) = \delta(\tau) \langle n_{-\sigma} \rangle + \mu G_{2,\sigma}(\tau) + U\, G_{2,\sigma}(\tau)}$$

$$(i\omega_n + \mu)\, G_{2,\sigma} = \langle n_{-\sigma} \rangle + U\, G_{2,\sigma}$$

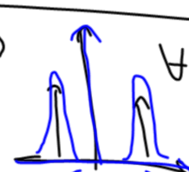
$$G_{2,\sigma} = \frac{\langle n_{-\sigma} \rangle}{i\omega_n + \mu - U}$$

$$G_\sigma(\omega_n) = \frac{1}{(i\omega_n + \mu)} \left[ 1 + \frac{U \langle n_{-\sigma} \rangle}{i\omega_n + \mu - U} \right]$$

$$\frac{1}{i\omega_n + \mu - U}\, \frac{1}{i\omega_n + \mu} = \frac{1}{U} \left[ \frac{1}{i\omega_n + \mu - U} - \frac{1}{i\omega_n + \mu} \right]$$

$$\boxed{G_\sigma(\omega_n) = \frac{1 - \langle n_{-\sigma} \rangle}{i\omega_n + \mu} + \frac{\langle n_{-\sigma} \rangle}{i\omega_n + \mu - U}}$$

$G_\sigma^R(\omega) \longrightarrow A(k, \omega)$ $\quad \forall k$



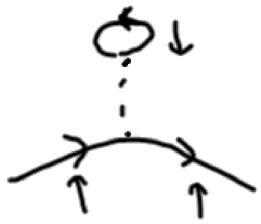

$\langle n_{-\sigma} \rangle = 1/2\ ,\ \mu = U/2$

$$G_\sigma^R(\omega) = \frac{1}{2} \left[ \frac{1}{\omega + i\eta + \frac{U}{2}} + \frac{1}{\omega + i\eta - \frac{U}{2}} \right]$$

$$= \frac{(\omega + i\eta)}{(\omega + i\eta)^2 - \left(\frac{U}{2}\right)^2} = \frac{1}{(\omega + i\eta) - \frac{U^2}{4(\omega + i\eta)}}$$

$$\Sigma^R(\omega) = \frac{U^2}{4(\omega + i\eta)}$$

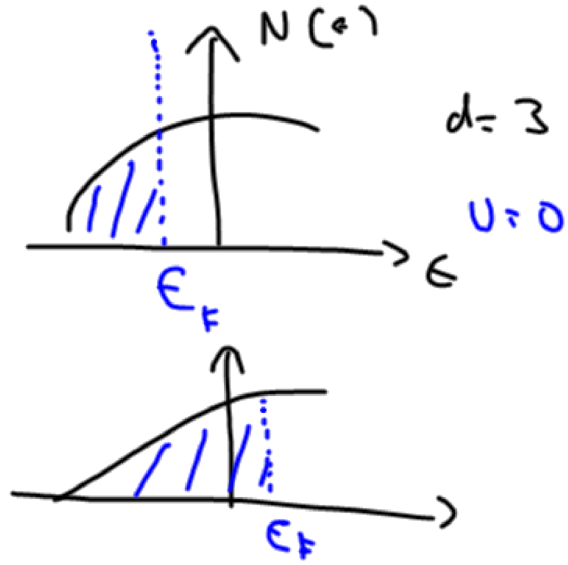
## 7.2 Modèle de Stoner. Ferromagn.



$$\epsilon_{k\uparrow} = \epsilon_k + U\langle n_\downarrow \rangle$$

$$\epsilon_{k\downarrow} = \epsilon_k + U\langle n_\uparrow \rangle$$

"Weiss"




$N \to \infty$

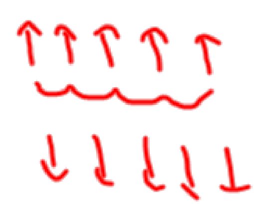
$$H_{S.B.} = H + h\left(\langle n_\uparrow \rangle - \langle n_\downarrow \rangle\right)\tilde{N}$$

infinitésimal

$h \to 0$

$h \to 0$ en premier

$N \to \infty$ et $h \to 0$



9. Fonction d'onde:

$$|\Psi\rangle = \prod_{k\uparrow} \theta(k_{F\uparrow} - |k|) \prod_{k\downarrow} \theta(k_{F\downarrow} - |k|)\, c^{+}_{k\uparrow} c^{+}_{k\downarrow} |0\rangle$$

$$k_{F\uparrow} \neq k_{F\downarrow}$$

$$\boxed{K = H - \mu N}$$

$$\langle n_\uparrow \rangle \neq \langle n_\downarrow \rangle$$

Ritz:

$$\frac{\langle \Psi | K | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$ doit être minimisé


$$|\Psi\rangle = |FS_\uparrow\rangle |FS_\downarrow\rangle$$

## 7.9.3 Princ. variationnel de Feynmann

$$\boxed{-T\ln Z \leq -T\ln Z_{\tilde{0}} + \langle H - H_{\tilde{0}} \rangle_{\tilde{0}}}$$

$$\tilde{H}_0 = \sum_{k\sigma} \tilde{\epsilon}_{k\sigma} c^{+}_{k\sigma} c_{k\sigma}$$

$$\tilde{H}_0 = \sum_{k\sigma} \epsilon_k c^{+}_{k\sigma} c_{k\sigma} + U \sum_i \langle n_{i\downarrow} \rangle c^{+}_{i\uparrow} c_{i\uparrow} + U \sum_i \langle n_{i\uparrow} \rangle c^{+}_{i\downarrow} c_{i\downarrow}$$

$$\tilde{\epsilon}_{k\uparrow} = \epsilon_k + U \langle n_\downarrow \rangle$$

$$\tilde{\epsilon}_{k\downarrow} = \epsilon_k + U \langle n_\uparrow \rangle$$

$$\sum_i c^{+}_{i\uparrow} c_{i\uparrow} = \sum_k c^{+}_{k\uparrow} c_{k\uparrow}$$

$$-T\ln Z_{\tilde{0}} + \langle H - \tilde{H}_0 \rangle_{\tilde{0}}$$

$$= -T\ln\Big[\prod_k \left(1+e^{-\beta(\epsilon_k + U\langle n_\downarrow\rangle - \mu)}\right) \prod_k \left(1+e^{-\beta(\epsilon_k + U\langle n_\uparrow\rangle - \mu)}\right)\Big]$$

$$+\, \underbrace{U\langle n_\uparrow\rangle_{\tilde{0}}\langle n_\downarrow\rangle_{\tilde{0}} N - 2U\langle n_\uparrow\rangle_{\tilde{0}}\langle n_\downarrow\rangle_{\tilde{0}} N}$$

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$$-T\sum_k \ln\left(1+e^{-\beta(\xi_k+U\langle n_\downarrow\rangle)}\right)$$

$$-T\sum_k \ln\left(1+e^{-\beta(\xi_k+U\langle n_\uparrow\rangle)}\right)$$

$\boxed{\xi_k \equiv \epsilon_k-\mu}$

$$-NU\langle n_\uparrow\rangle\langle n_\downarrow\rangle$$

déf : $m = \langle n_\uparrow\rangle - \langle n_\downarrow\rangle$

$n = \langle n_\uparrow\rangle + \langle n_\downarrow\rangle$

$$\cancel{+T}\sum_k \frac{e^{-\beta(\xi_k+U\langle n_\downarrow\rangle)}}{1+e^{-\beta(\xi_k+U\langle n_\downarrow\rangle)}}\left(\cancel{-\beta U}\frac{\partial\langle n_\downarrow\rangle}{\partial m}\right)$$

$$+\ \langle n_\downarrow\rangle \longleftrightarrow \langle n_\uparrow\rangle$$

$$-\cancel{U}N\frac{\partial\langle n_\uparrow\rangle}{\partial m}\langle n_\downarrow\rangle-\cancel{U}N\langle n_\uparrow\rangle\frac{\partial\langle n_\downarrow\rangle}{\partial m}=0$$

$$=-\sum_k f(\xi_k+U\langle n_\downarrow\rangle)+\sum_k f(\xi_k+U\langle n_\uparrow\rangle)$$

$$-N\left(\langle n_\downarrow\rangle-\langle n_\uparrow\rangle\right)=0$$

$$\langle n_\uparrow\rangle-\langle n_\downarrow\rangle=\frac{1}{N}\sum_k f(\xi_k+U\langle n_\downarrow\rangle)-f(\xi_k+U\langle n_\uparrow\rangle)$$

$$\langle n_\uparrow\rangle+\langle n_\downarrow\rangle=n= \qquad +$$

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## 7.2.4 Equation au gap (Landau)

$$\boxed{m = \frac{1}{N}\sum_k \left[ f\left(\xi_k + U\left(\frac{n}{2} - \frac{m}{2}\right)\right) - f\left(\xi_k + U\left(\frac{n}{2} + \frac{m}{2}\right)\right)\right]}$$

$n = \langle n_\uparrow \rangle + \langle n_\downarrow \rangle$

$m = \langle n_\uparrow \rangle - \langle n_\downarrow \rangle$

Soit $m \ll n$

$$m = \frac{1}{N}\sum_k \frac{\partial f}{\partial \xi_k}(-Um) + O(m^3)$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{\partial f}{\partial \xi_k}(-Um) + O(m^3)$$

$$m = N(E_F)\,U\,m + b\,m^3$$

$$\boxed{m^2 = \frac{1}{b}\left(1 - U N(E_F)\right)}$$

$b < 0$ Si $\boxed{1 - U N(E_F) < 0}$

$\boxed{1 = U N(E_F)}$ $m \neq 0$

où m devient $\neq 0$ $\boxed{\text{Critère de Stoner}}$

Si calcule $-T\ln Z$ essai en puissances de m:

$$\boxed{F = -\frac{1}{2}\left(1 - N(E_F)U\right)m^2 + \frac{b}{4}m^4}$$

m = paramètre d'ordre

Champ moyen $\boxed{F = -\frac{1}{2}a m^2 + \frac{b}{4}m^4}$ $\frac{\partial F}{\partial m} = 0$

Ginzburg-Landau

Argument simple pour Stoner:

$$\left.\begin{array}{l} \epsilon_{k_{F\uparrow}} + U\langle n_\downarrow\rangle = \mu \\ \epsilon_{k_{F\downarrow}} + U\langle n_\downarrow\rangle = \mu \end{array}\right\} \text{ Normal.}$$

$$\longrightarrow \epsilon_{k_{F\uparrow}} - \epsilon_{k_{F\downarrow}} = -U\left(\langle n_\downarrow\rangle - \langle n_\uparrow\rangle\right)$$

$$\frac{\partial \epsilon_{k_F}}{\partial k_F}\frac{\partial k_F}{\partial n}\cancel{\left(\langle n_\uparrow\rangle - \langle n_\downarrow\rangle\right)} = U\cancel{\left(\langle n_\uparrow\rangle - \langle n_\downarrow\rangle\right)}$$

$$\frac{\partial \epsilon_{k_F}}{\partial n} = U$$

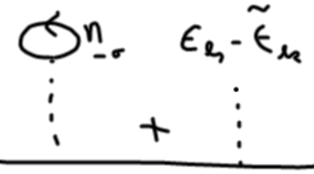
Critère de Stoner

$$1 = U\frac{\partial n}{\partial \epsilon_{k_F}} = U N(E_F)$$

## 7.9.5 Fonctions de Green "Milieu effectif"

$$\tilde{H}_0 = \sum_{k\sigma} \tilde{\epsilon}_{k\sigma}\, c^{+}_{k\sigma} c_{k\sigma}$$

$$\tilde{\Sigma}_\sigma =$$




$$\boxed{\tilde{\Sigma}_\sigma = U\langle n_{-\sigma}\rangle + \epsilon_k - \tilde{\epsilon}_{k\sigma} = 0}$$

$$\tilde{\epsilon}_{k\sigma} = \epsilon_{k\sigma} + U\langle n_{-\sigma}\rangle$$

$$\mathcal{G}_\sigma(k, ik_n) = \frac{1}{ik_n - \tilde{\epsilon}_{k\sigma} + \mu}$$

$$\langle n_\sigma\rangle = \sum_k \langle c^{+}_{k\sigma} c_{k\sigma}\rangle$$

$$= \sum_k -\langle T_\tau\, c_{k\sigma}(\tau = 0^-)\, c^{+}_{k\sigma}\rangle$$

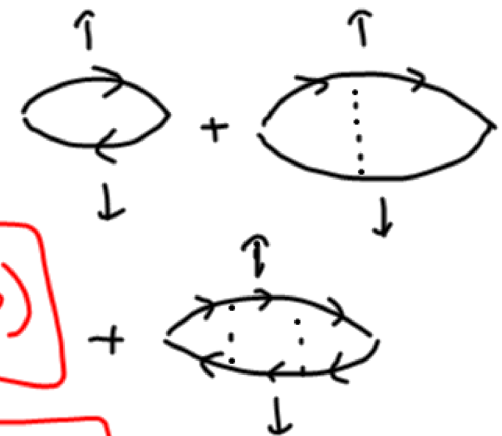
$$0^- \equiv \eta$$

$$= \sum_k T \sum_n \frac{e^{ik_n\eta}}{ik_n - \tilde{\epsilon}_{k\sigma} + \mu}$$

$$\boxed{\langle n_\sigma\rangle = \sum_k f(\tilde{\epsilon}_{k\sigma} - \mu)}$$

$\chi_{zz}$

$\chi_{+-} =$

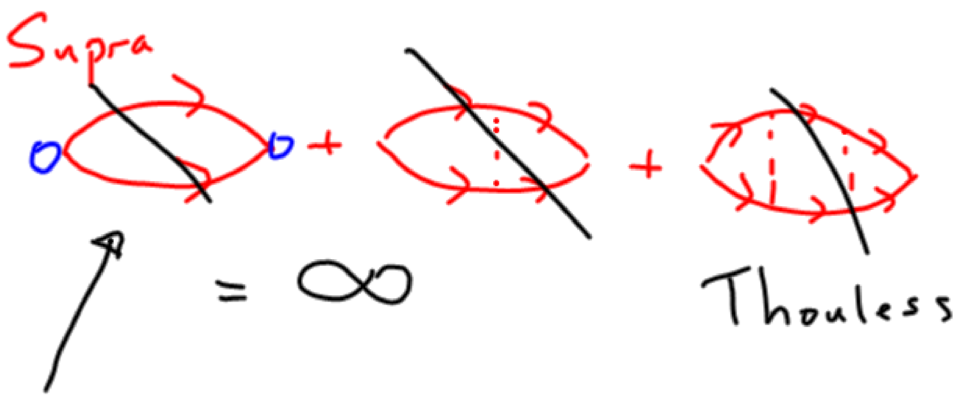


$$\boxed{1 = \frac{U}{2}\chi^0(q,0)}$$

$$\boxed{\frac{\chi^0}{1 - \frac{U}{2}\chi^0}}$$

$$1 - \frac{U}{2}\chi^0 = 0$$

$$\chi^0(q=0,\omega) = 2N(E_F)$$




$$(u + v b^{+})|0\rangle$$

$$\boxed{|BCS\rangle = \prod_k \left(u_k + v_k a^{+}_{k\uparrow} a^{+}_{-k\downarrow}\right)|0\rangle}$$

$$\sum_{\substack{k\,k' \\ \sigma\,\sigma'}} c^+_{k\sigma}\, c^+_{k'\sigma'}\; c_{k'+q\,\sigma'}\, c_{k\sigma}\; U$$

$$q = 0$$

$$U \sum_{k'} \langle c^+_{k'\sigma'}\, c_{k'\sigma'} \rangle = \langle n_{\sigma'} \rangle\, U$$

Supra

$$\sum_{k} U \langle c^+_{k\sigma}\, c^+_{-k\,-\sigma} \rangle \neq 0$$

$$k' + q = -k \qquad \sigma' = -\sigma$$

$$\tilde{H}_0 = c^+ c + \underset{2}{c^+ c^+} + \underset{\kappa}{c\,c}$$

$$(\Psi_k)_{\text{nambu}} = \begin{pmatrix} c_{k\uparrow} \\ c^+_{-k\downarrow} \end{pmatrix}$$

$$\rightarrow \left\{ \Psi_{k\alpha} , \Psi^+_{k'\beta} \right\} = \delta_{kk'} \delta_{\alpha\beta}$$

$$\tilde{H}_0 = \Psi^+ M \Psi$$