Ch.7 Sym. Brisée

1. Modèle de Hubbard
2. Ferromagnétisme
   1. Stoner
   2. Variation Ψ
   3. Feynman
   4. Dév. de Landau $(1-UN(0))m = bm^2$

      $m = \langle n_\uparrow - n_\downarrow \rangle$
   5. Pt de vue milieu effectif $\tilde{\Sigma} = 0$

      $\epsilon_{k} - \epsilon_{k}$ + U
   6. Point de vue de phase <u>normale</u>

      à $T = T_c$ ou $U = U_c$ $\chi = \infty$

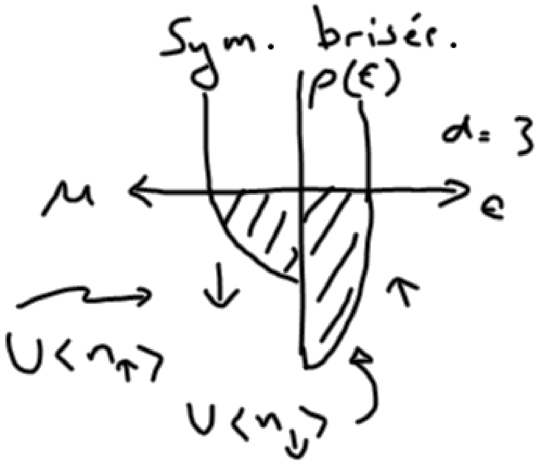
      $\xi \to \infty$
   7. Paramagnons : modes collectifs
   8. Modes collectifs "ferro."

      "Magnons" Théorème de Goldstone.

      $\exists$ sym. <u>continue</u> brisée

      $\Rightarrow \exists$ $\omega = 0$
   9. Subtilités :

      Kanamori-Brückner
3. Antiferro. à $n=1$
4. Au-delà du champ moyen



$$1 - UN(E_F) = 0$$

$$T = 0$$

$$1 - \frac{U}{2}\chi_0(T) = 0$$

à T transition

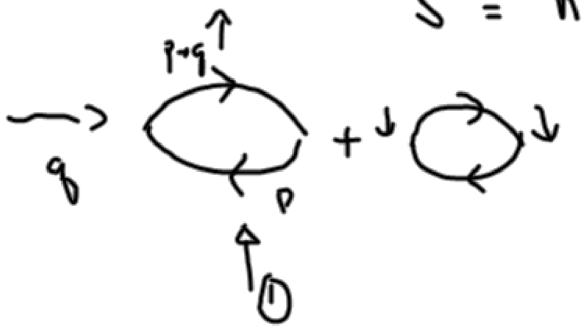
T_c = temp critique

⑥ Inst. de Stoner vue de la phase normale.

$\boxed{U=0}$ $\quad \dfrac{\partial S^z}{\partial h} \propto \langle T_\tau S^z S^z \rangle_{c\ \substack{\omega=0 \\ q\to 0}}$

$$\langle T_\tau S^z S^z \rangle_c = \underbrace{\langle n_\uparrow n_\uparrow \rangle_c}_{①} + \underbrace{\langle n_\downarrow n_\downarrow \rangle_c}_{①} - \langle n_\uparrow n_\downarrow \rangle_c - \langle n_\downarrow n_\uparrow \rangle_c$$

$$S^z = n_\uparrow - n_\downarrow$$



$$= -\frac{2}{N}\sum_{p,\sigma} T \sum_n \mathcal{G}(p+q, ip_n + iq_n)\,\mathcal{G}(p, ip_n)$$

$$= -\frac{2}{N}\sum_p \frac{f(\xi_p) - f(\xi_{p+q})}{iq_n + \xi_p - \xi_{p+q}} \equiv \chi_0(q, iq_n)$$

$$\frac{\partial}{\partial h} \frac{\mathrm{Tr}\, e^{-\beta(K - hS_z)} S_z}{\mathrm{Tr}\, e^{-\beta(K-hS_z)}} = \underline{\langle S_z S_z \rangle - \langle S_z \rangle \langle S_z \rangle}$$

$$\langle (n_\uparrow - n_\downarrow)(n_\uparrow - n_\downarrow) \rangle - \langle n_\uparrow - n_\downarrow \rangle \langle n_\uparrow - n_\downarrow \rangle$$

$$c^\dagger_\uparrow c_\uparrow \; c^\dagger_\uparrow c \qquad c^\dagger c\, c^\dagger c$$

Inv. sous rotation:

$$\langle S^+S^-\rangle + \langle S^-S^+\rangle = \langle S^z S^z\rangle$$

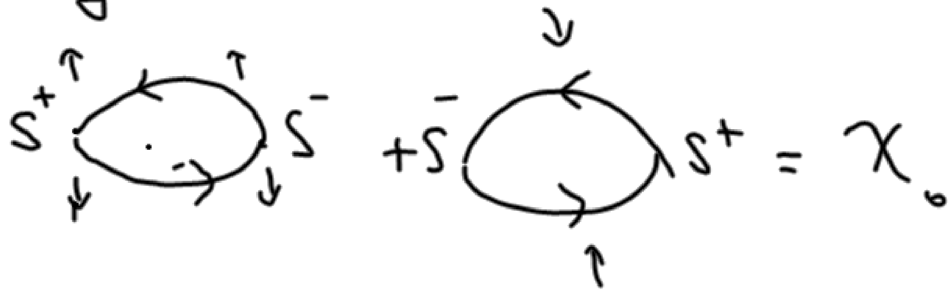
Preuve:

$$\frac{1}{2}\langle (S_x+iS_y)(S_x-iS_y)\rangle \frac{1}{2}$$

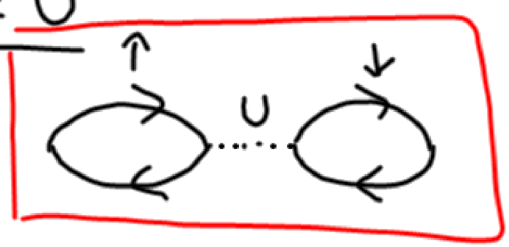
$$+\frac{1}{2}\langle (S_x-iS_y)(S_x+iS_y)\rangle \frac{1}{2}$$

$$=\frac{1}{4}\Big(\langle S_xS_x+S_yS_y\rangle + \langle S_xS_x+S_yS_y\rangle\Big)$$

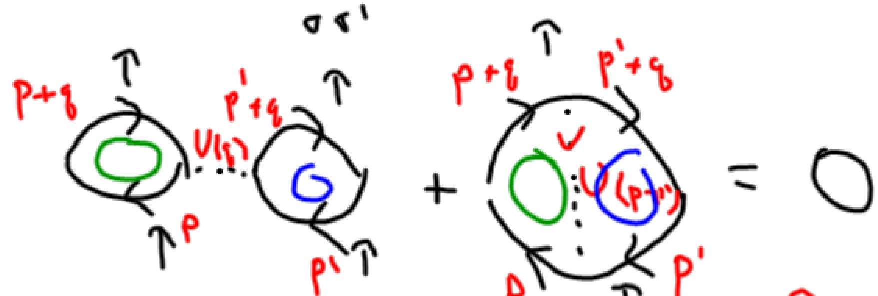
Diagrammes:

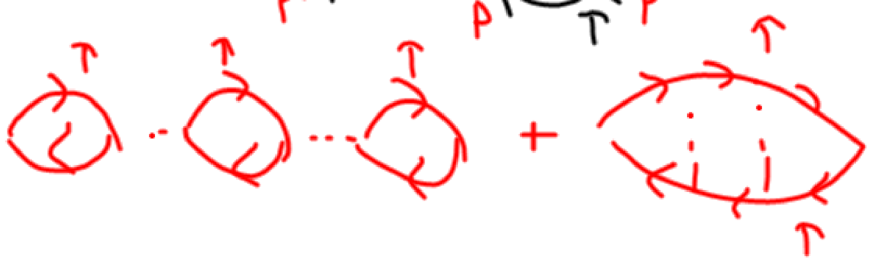


Effet de U

$S_z S_z \longrightarrow$  GRPA

$$\frac{U}{2} \sum_{\substack{i \\ \sigma\sigma'}} c^{+}_{i\sigma} c^{+}_{i\sigma'} c_{i\sigma'} c_{i\sigma}$$



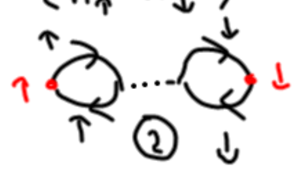


Susceptibilité de spin:

$\langle S_z S_z \rangle = \Big\{ \langle n_\uparrow n_\uparrow \rangle$

$+ \quad (\uparrow \leftrightarrow \downarrow)$

$- \Big\{ \langle n_\uparrow n_\downarrow \rangle$



$$\frac{\chi_0}{2} - (-U)\left(\frac{\chi_0}{2}\right)\left(\frac{\chi_0}{2}\right) + \left(\frac{\chi_0}{2}\right)^3 (-U)^2$$

$$\frac{\chi_0}{2}\left[1 + U\frac{\chi_0}{2} + \left(U\frac{\chi_0}{2}\right)^2 + \cdots\right]$$

$$= \frac{\chi_0/2}{1 - \frac{U}{2}\chi_0} \longrightarrow \chi = \frac{\chi_0}{1 - \frac{U}{2}\chi_0}$$

$\langle S^+ S^- \rangle = S^+$

$S^- + \ldots + \cdots$

$$= \frac{\chi_0}{2} - U \frac{\chi_0}{2}\left(-\frac{\chi_0}{2}\right) + (-U)^2 \frac{\chi_0}{2}\left(-\frac{\chi_0}{2}\right)^2$$

$$= \frac{\chi_0/2}{1 - \frac{U}{2}\chi_0} = \langle S^+ S^- \rangle$$

$$\chi(q, iq_n) = \frac{\chi_0(q, iq_n)}{1 - \frac{U}{2}\chi_0(q, iq_n)}$$

$$\chi^R(q, 0) = \frac{\chi_0^R(q, 0)}{1 - \frac{U}{2}\chi_0(q, 0)} = \frac{\partial \langle S_z \rangle}{\partial h}$$

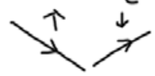
Instable

$$\frac{U}{2}\chi_0(q, 0) > 1$$

$$\lim_{q \to 0} \chi_0(q, 0) = 2N(E_F)$$

(7) Facteur de structure magnétique

(modes collectifs) $N_i \neq J$



$$S^{+-}(q,\omega) = \frac{2}{1-e^{-\beta\omega}} \chi''_{+-}(q,\omega)$$

$\beta \to \infty$

$$S^{+-} = 2\chi''_{+-} \qquad \omega > 0$$

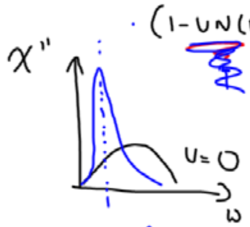
$$= 2\,\mathrm{Im}\left[\frac{\chi_0/2}{1-\frac{U}{2}\chi_0}\right] \qquad \chi_0 = \chi_0' + i\chi_0''$$

$$= \frac{\chi_0''}{\left(1-\frac{U}{2}\chi_0'\right)^2 + \left(\frac{U}{2}\chi_0''\right)^2}$$

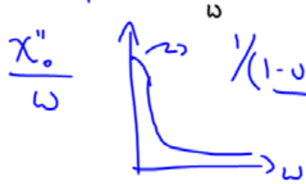
Modes $q \to 0$, $\omega \to 0$

$$= \frac{\chi_0''}{\left(1-UN(E_F)\right)^2 + \left(\chi_0''\frac{U}{2}\right)^2}$$

$$= \frac{C\omega/v_F q}{\left(1-UN(E_F)\right)^2 + \left(\frac{U}{2}C\frac{\omega}{v_F q}\right)^2}$$



q petit $\qquad UN(E_F) \sim 0.8$



$1/\left(1-UN(E_F)\right)^2$

Paramagnon

Il est suramorti

$$\chi(q,\omega=0) = \frac{\chi_0(q,\omega=0)}{\left(1-\frac{U}{2}\chi_0(0,0)\right) - \frac{U}{2}\frac{\partial^2\chi_0}{\partial q^2}q^2 + \dots}$$

q petit.

$$\propto \frac{A}{\xi^{-2}+q^2}$$

$$\boxed{\xi^{-2} \propto \left(1-UN(E_F)\right)}$$

$$\frac{\chi_0^R}{2}(\vec{q},\omega) = -\frac{1}{N}\sum_p \frac{f(\xi_p)-f(\xi_{p+q})}{\omega+i\eta+\xi_p-\xi_{p+q}}$$

q petit.

$$= -\frac{1}{N}\sum_p \frac{\partial f}{\partial \xi_p}\frac{(\xi_p-\xi_{p+q})}{\omega+i\eta+\xi_p-\xi_{p+q}}$$

$$\xi_p-\xi_{p+q} = \cancel{\frac{p^2}{2m}} - \left(\cancel{\frac{p^2}{2m}} + \frac{p\cdot q}{m} + \frac{q^2}{2m}\right)$$

$$= -\frac{p}{m}\cdot q - \cancel{\frac{q}{2m}q} \sim -\vec{v}_F\cdot\vec{q}$$

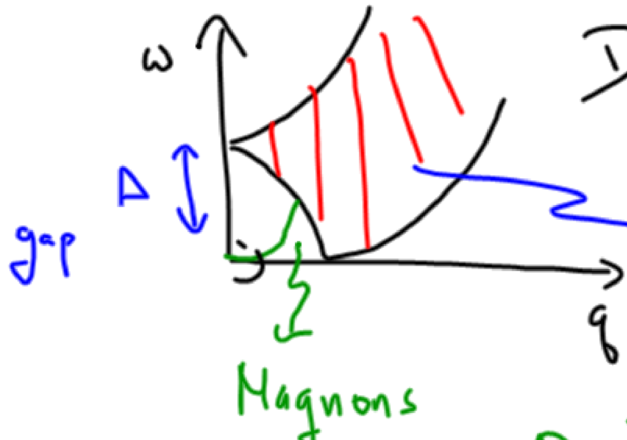
$$= -\int d\epsilon\, N(\epsilon)\int_{-1}^{1}\frac{d(\cos\theta)}{2}\frac{\partial f}{\partial\epsilon}\frac{(-v_F q\cos\theta)}{\omega+i\eta-v_F q\cos\theta}$$

$$\frac{\chi_0''}{2}(q,\omega) = \pi\int\frac{d\cos\theta}{2}\int d\epsilon\, N(\epsilon)\left(-\frac{\partial f}{\partial\epsilon}\right)\underline{v_F q\cos\theta}\;\delta(\omega-v_F q\cos\theta)$$

$$= \frac{\pi}{2}\frac{\omega}{v_F q}\int d\epsilon\, N(\epsilon)\left(-\frac{\partial f}{\partial\epsilon}\right)$$

$$\propto C\frac{\omega}{v_F q}$$

4. Modes collectifs dans l'état ordonné : modes de Goldstone.

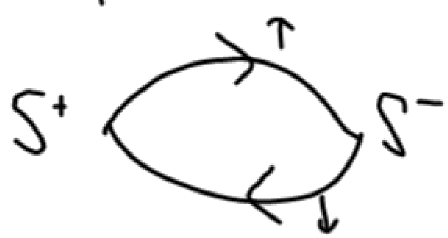


Domaine où $\chi''_{+-} \neq 0$ → Continuum particule-trou

Magnons $\omega = D q^2$ (Mode de Goldstone)

$$\tilde{\epsilon}_{k\sigma} = \epsilon_k + U \langle n_{-\sigma} \rangle$$

$$\tilde{\epsilon}_{k\downarrow} - \tilde{\epsilon}_{k\uparrow} = U \langle n_\uparrow - n_\downarrow \rangle = U m \equiv \Delta$$



$= \chi_{+-}$ — Dans l'état ordonné

$$\chi_0^{+-} = -\frac{1}{N}\sum_{p} \frac{f(\tilde{\xi}_{p+q\downarrow}) - f(\tilde{\xi}_{p\uparrow})}{\omega + i\eta + \underbrace{\tilde{\xi}_{p+q\downarrow} - \tilde{\xi}_{p\uparrow}}}$$

$$\tilde{\xi}_{p+q\downarrow} - \tilde{\xi}_{p\uparrow} = \vec{v}_F\cdot\vec{q} + U(\langle n_\uparrow\rangle - \langle n_\downarrow\rangle)$$

$$= \underbrace{\vec{v}_F\cdot\vec{q} + \Delta}$$

$$\vec{v}_F\cdot\vec{q} \ll \Delta$$

$$\chi_0^{+-} = -\frac{1}{N}\sum_p \left(f(\tilde{\xi}_{p+q\downarrow}) - f(\tilde{\xi}_p)\right)\frac{1}{\omega + i\eta + \Delta}\left(1 - \cancel{\frac{v_F\cdot q}{\omega + i\eta + \Delta}} + O(q^2)\right)$$

$$= -\left(\langle n_\downarrow\rangle - \langle n_\uparrow\rangle\right)\frac{1}{\omega + i\eta + \Delta} - Cq^2$$

$$= \frac{\Delta/U}{\omega + i\eta + \Delta} - Cq^2$$

$$\chi_{+-} = \frac{\chi_0^{+-}}{1 - \chi_0^{+-}U} = \frac{\frac{\Delta/U}{\omega + i\eta + \Delta}}{1 - \frac{\Delta}{\omega + i\eta + \Delta} + UCq^2}$$

$$= \frac{\Delta/U}{\omega + i\eta + UC\Delta q^2} \qquad \Delta \gg \omega$$

$$= \frac{\Delta/U}{\omega + i\eta + Dq^2} \qquad \underline{D \equiv UC\Delta}$$

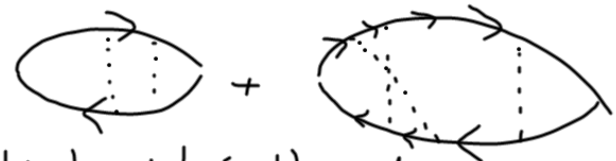
$$\rightarrow \chi''^{\,0}_{+-} = \frac{\Delta}{U}(-\pi)\,\delta(\omega + Dq^2)$$

$$\chi''^{\,0}_{-+} = -\frac{\Delta}{U}(-\pi)\,\delta(\omega - Dq^2)$$

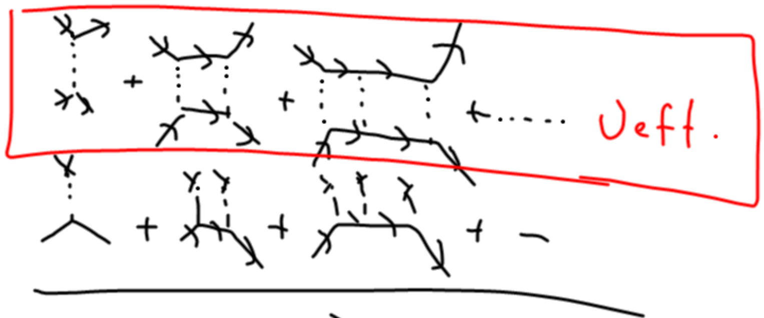
$$\boxed{\omega\left(\chi''_{-+} + \chi''_{+-}\right) > 0}$$ Positivité de la dissipation

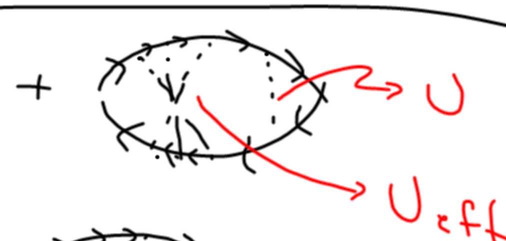
Stabilité $\Rightarrow D > 0$

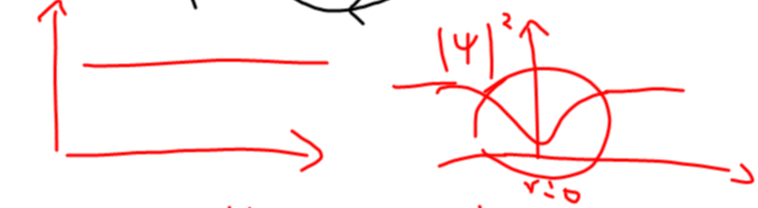
9. Renormalisation de U



Limite diluée, U grand





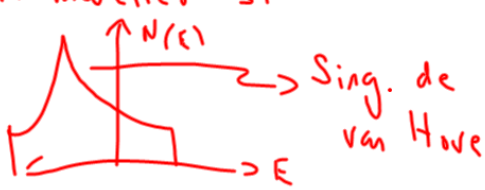


U_{eff} max $\sim W$

$N(E_F) \sim \frac{1}{W}$ $\quad N(E_F) \gg \frac{1}{W}$

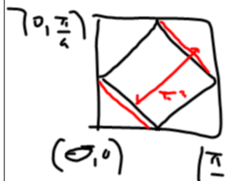
$1 - U_{eff} N(E_F) = 0$

Ça peut marcher si



$\boxed{3}$ Instabilité Antiferro : (à partir de N)

$n = 1$



$a = 1$

$$\chi^{+-}(\vec{q} = (\pi,\pi), 0)$$

$$= -\frac{1}{N}\sum_p \frac{f(\xi_{p+q}) - f(\xi_p)}{\xi_{p+q} - \xi_p}$$

$$\xi_{p+q} = -2t\left(\cos(k_x+\pi) + \cos(k_y+\pi)\right)$$

$$\xi_{p+q} = -\xi_p \quad \text{"Nesting"}$$

"Emboîtement"

$$= \frac{1}{N}\sum_p \frac{f(-\xi_p) - f(\xi_p)}{2\xi_p} \qquad f(-\omega) = 1 - f(\omega)$$

$$= \frac{1}{N}\sum_p \frac{1 - 2f(\xi_p)}{2\xi_p}$$

$$= \int_{-W/2}^{W/2} d\xi\, N(\xi)\, \frac{\tanh\left(\frac{\beta\xi}{2}\right)}{2\xi}$$

$$\propto N \ln\left(\frac{W}{T}\right)$$

$$1 - U\chi_0^{+-}(T) = 0 \qquad \exists\, T \text{ t.q. que satisfait.}$$

$$\Rightarrow T_c \propto e^{1/U}$$

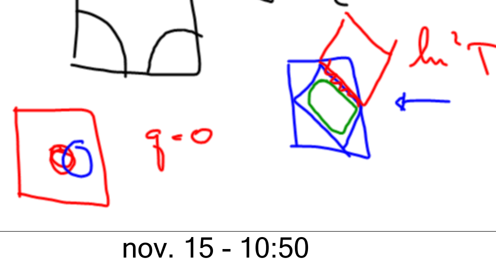
$d = 2$

$$\chi_0^{+-} \propto \ln^2 T$$

$$1 - UA\ln^2 T = 0 \qquad T_c \sim e^{1/\sqrt{U}}$$

Sans nesting :

$\exists\, U_c$



2 autres problèmes avec RPA.

① $\langle (n_\uparrow)^2 \rangle \neq \langle n_\uparrow \rangle$

Pauli

② Mermin-Wagner

$d = 2$

Sym. continue brisée

$\Rightarrow$ mode de Goldstone

Mode de Goldstone

$T \neq 0 \Rightarrow \not\exists$

ordre à longue portée

RPA viole

Mermin-Wagner