$(\omega + i\eta - H_0 - V) G^R = 1$

$(G_0^{R-1} - V) G^R = 1$

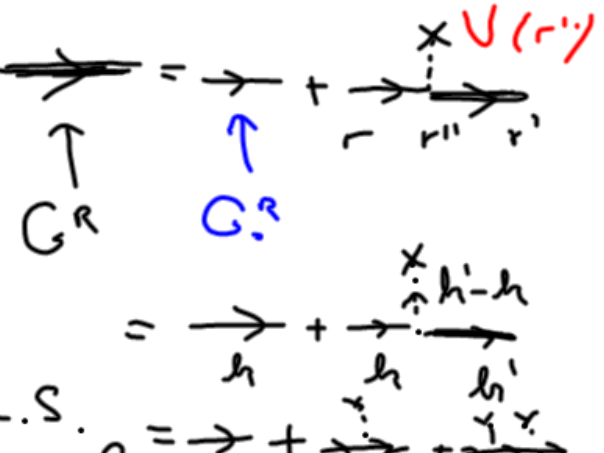
$G_0^{R-1} G^R = 1 + V G^R$

$$\boxed{G^R = G_0^R + G_0^R V G^R}$$ L.S.

$\langle r | G^R | r' \rangle = \langle r | G_0^R | r' \rangle + \int \langle r | G_0^R | r'' \rangle V(r'') \langle r'' | G^R | r' \rangle dr''$

$$\boxed{G^R = G_0^R + G_0^R \Sigma^R G^R}$$

↖ Self-énergie irréductible



$$G^R = \frac{1}{G_0^{-1\,R} - \Sigma^R} = \frac{1}{\omega + i\eta - \frac{k^2}{2m} - \cfrac{b_0}{\omega + i\eta - a_0 - \cfrac{b_1}{\omega + i\eta - a_1 - \ddots}}}$$

↑ Poles déplacés.

$$\langle r|G^R|r'\rangle = \sum_n^N \frac{\varphi_n(r)\,\varphi_n^*(r')}{\omega + i\eta - E_n}$$

$$\Sigma^R = a - \frac{i}{\tau}$$

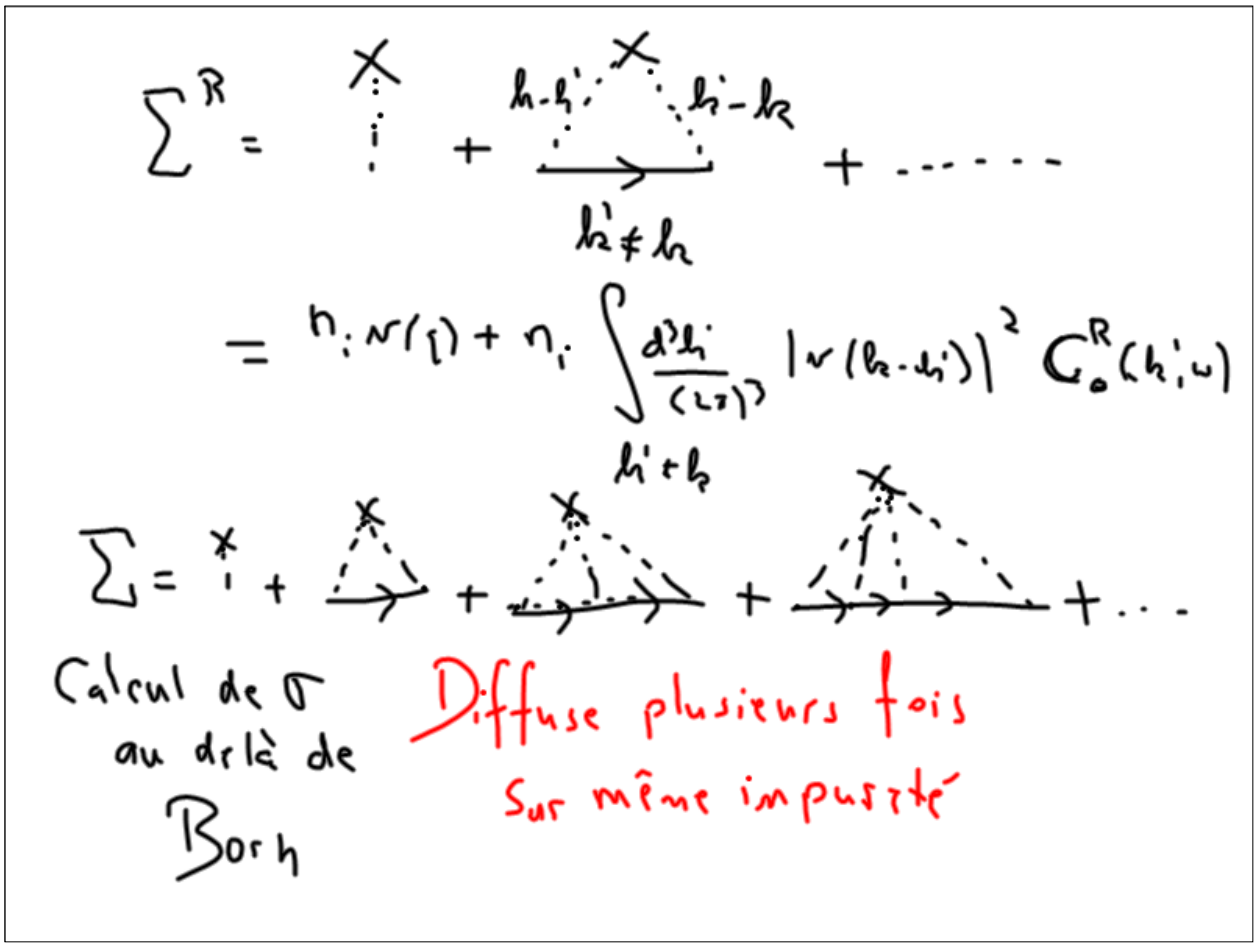
Moyenne sur impuretés:

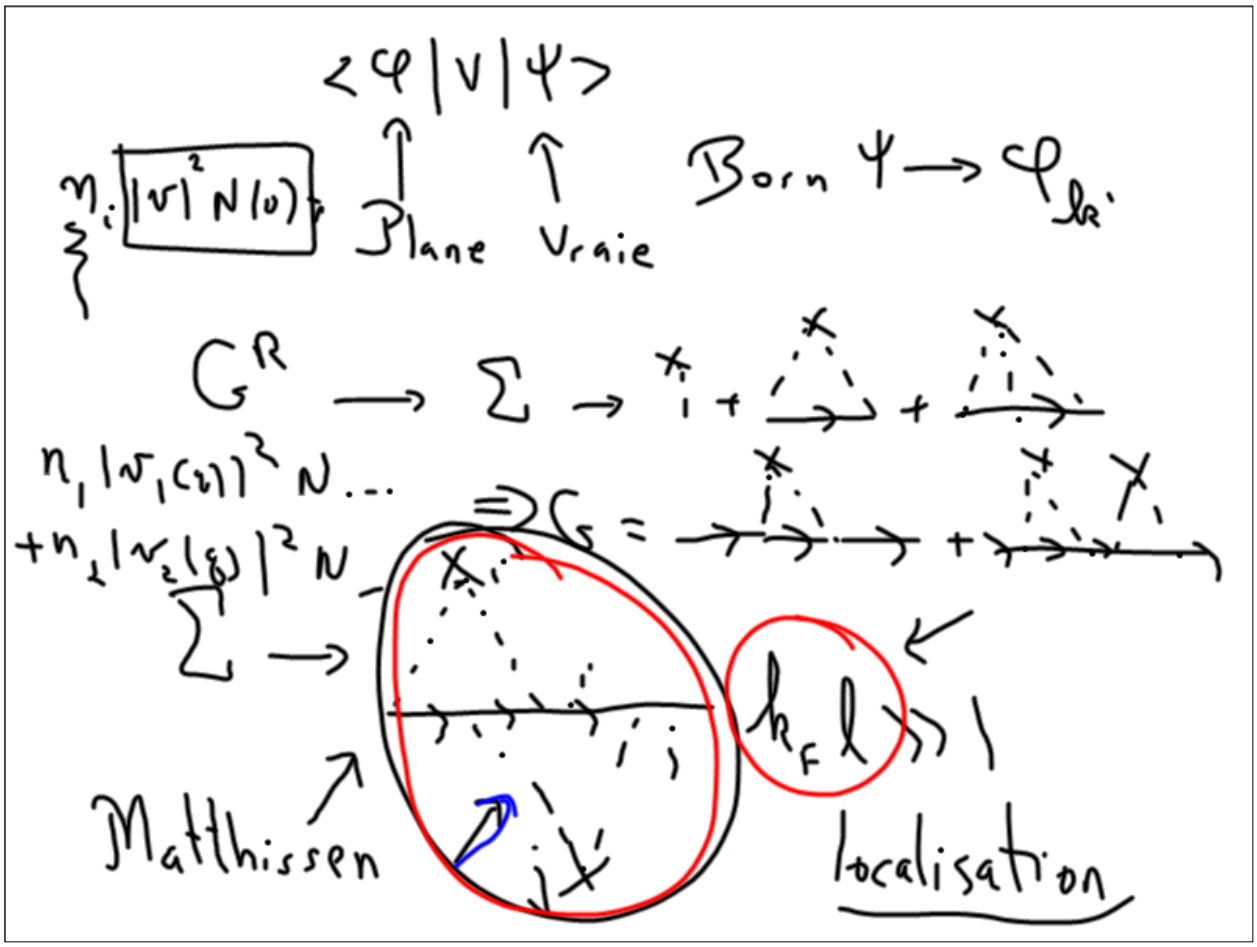
$$v_c(q) = \sum_{i=1}^{N_i} e^{-i q \cdot R_i} v(q)$$

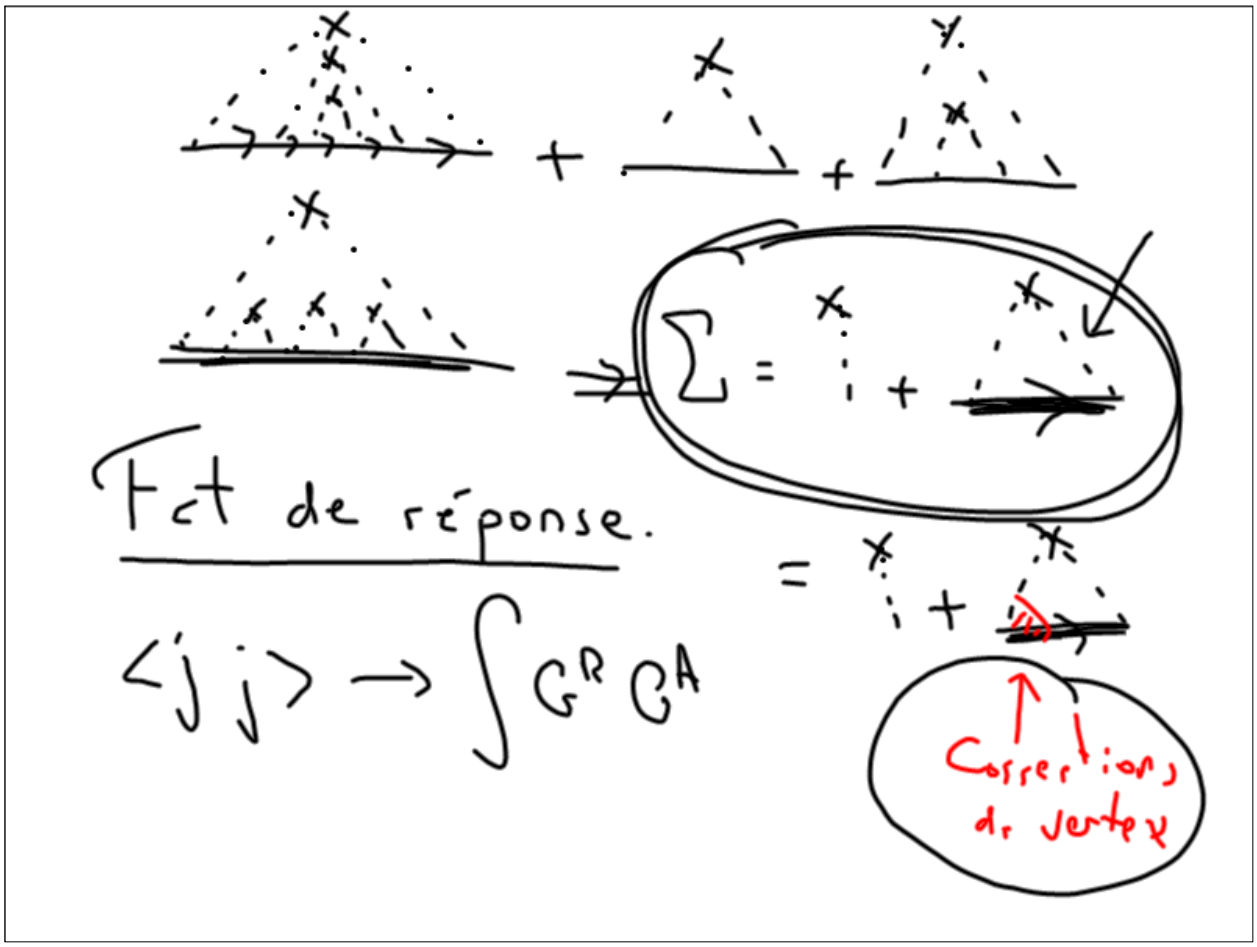
$$\overline{v_c(q)} = v(q) (2\pi)^3 \delta(q) n_i$$

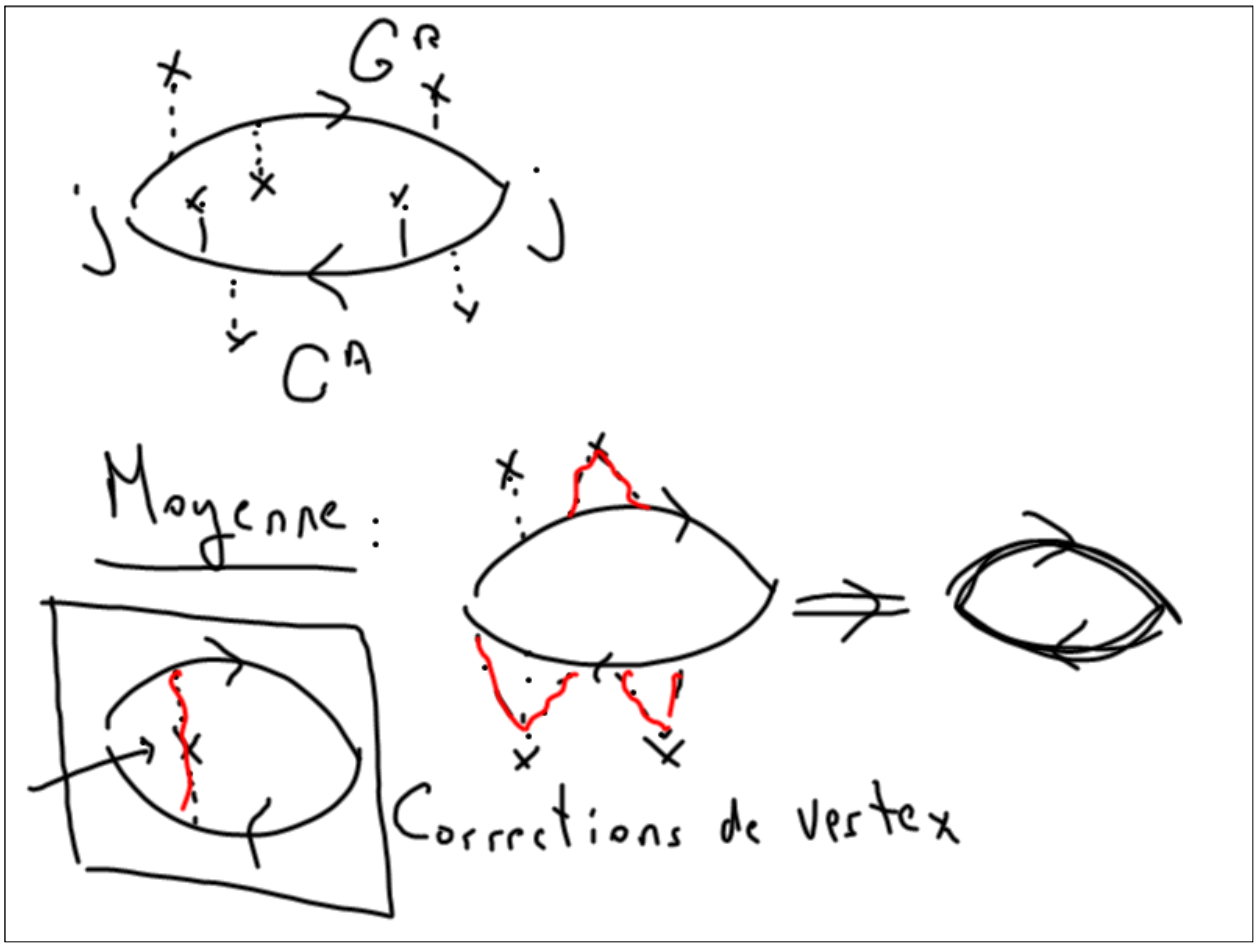
$$\overline{v_c(q) v_c(q')} = n_i v(q) v(-q) \delta^3(q+q') (2\pi)^3$$

$$= n_i |v(q)|^2 \delta^3(q+q') (2\pi)^3$$


Σ^R =
+
$k-k'$
$k'-k$
$k' \neq k$
+ ------
$= n_i V(\ell) + n_i \int \frac{d^3k'}{(2\pi)^3} |V(k-k')|^2 G_0^R(k';\omega)$
$k' \neq k$
Σ =
+
+
+
+ . . .
Calcul de Σ au delà de Born
Diffuse plusieurs fois
Sur même impureté

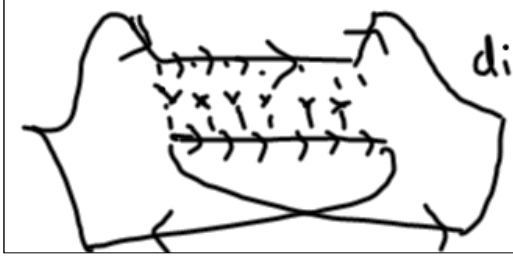

$\langle \varphi | V | \psi \rangle$
$\eta_i |V|^2 N(0)$
Plane
Vraie
Born $\psi \to \varphi_{k_i}$
$G^R \to \Sigma \to$
$\eta_1 |v_1(0)|^2 N \ldots$
$+\eta_2 |v_2(0)|^2 N$
$\Sigma \to$
$\Rightarrow \Sigma =$
$k_F l \gg 1$
Matthissen
localisation


Fct de réponse.
$\langle j\, j \rangle \rightarrow \int G^R G^A$
Corrections de vertex


G^R
J
J
C^A
Moyenne :
Corrrections de Vertex

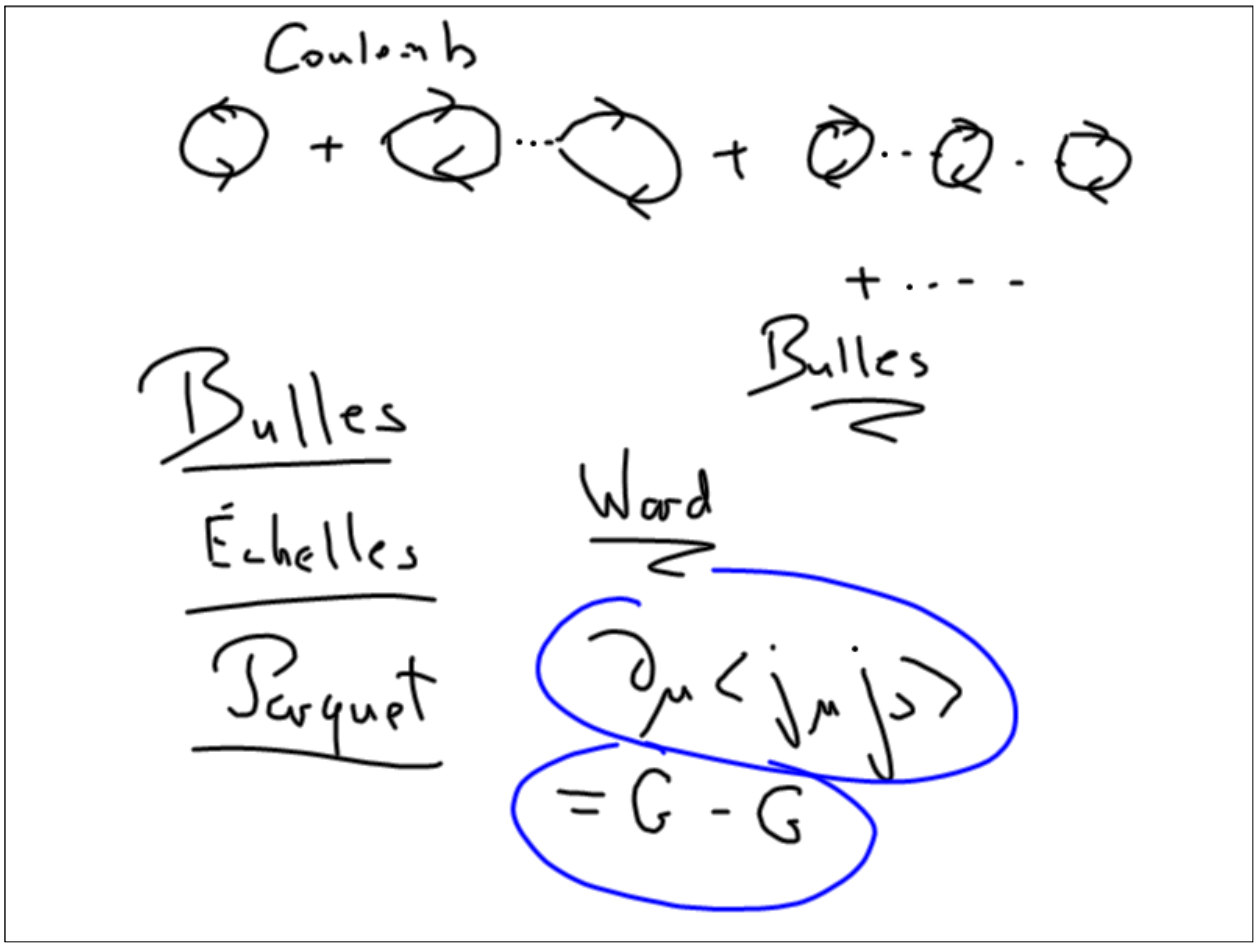
Pour satisfaire les lois de conservation : il faut choisir les corrections de vertex de telle sorte qu'elles soient cohérentes avec la self-énergie retenue. Formel: Identités de Ward.

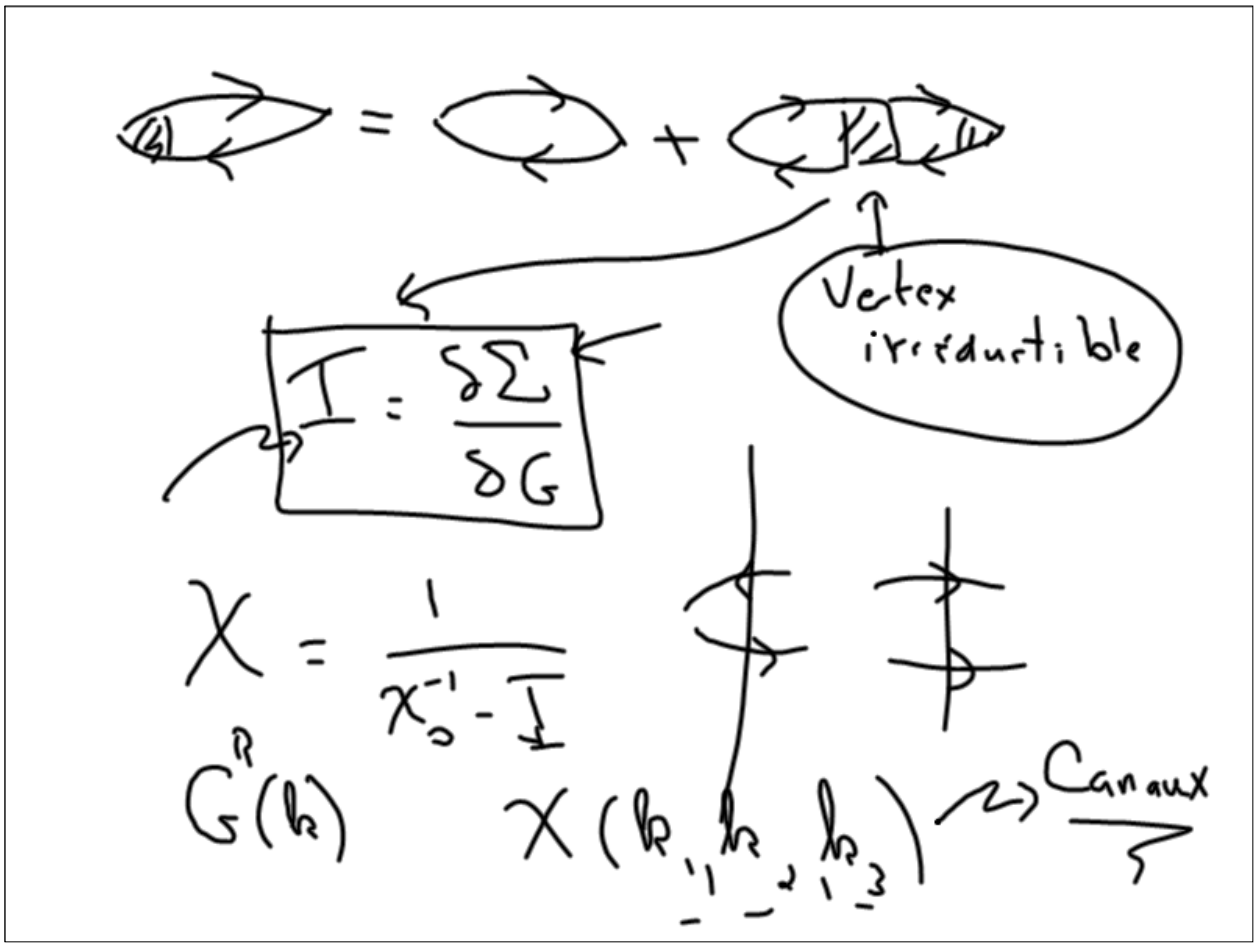
Corrections de vertex



diagrammes en échelle

(approx. de matrice T)


Coulomb
+
+
+ ...
Bulles
Bulles
Échelles
Parquet
Ward
$\partial_\mu \langle j_\mu j \rangle$
$= G - G$


=
+
Vertex
irréductible
$\Gamma = \frac{\delta\Sigma}{\delta G}$
$\chi = \frac{1}{\chi_0^{-1} - \Gamma}$
$G^R(k)$
$\chi(k_1, k_2, k_3)$
Canaux

$\underline{I}_1, \underline{I}_2, \underline{I}_3$

{ Relations de croisement
{ Crossing symmetry

Chapitre 4

4.1 - Seconde quantification
- 1 corps
- 2 corps
- Repr. Heisenberg

4.2. Motivation pour déf. de G^R
- Exemple H quadratique.

4.3 - Repr. int.

4.4 - K.B + K.S. (contours)

4.5 - Matsubara.

4.6 - Signif. phys. 4.7 Théorèmes.

$$\Psi(x_1^3, x_2^4, x_3) = \pm \Psi(x_2^4, x_1^3, x_3)$$

indiscernables

Symétriques: bosons

Antisymétriques: fermions.

Base à 1 particule.

$$A_1 |\alpha_1\rangle |\alpha_2\rangle |\alpha_3\rangle$$
$$+ A_2 |\beta_1\rangle |\beta_2\rangle |\beta_3\rangle$$

$$\Psi(x,y) = (x-y) N e^{\frac{-|x-y|}{a}}$$

Seconde quantification (Negele-Orland)

$|\alpha_1 \alpha_2 \ldots \alpha_N\}$ pas normalisé

$|\alpha_1 \ldots \alpha_N\rangle = \frac{1}{\sqrt{\prod_\alpha n_\alpha!}} |\alpha_1 \alpha_2 \ldots \alpha_N\}$

$|\alpha\rangle \ldots$ états à 1 corps.

Normalisé et (anti) symétrisé

$$|\alpha_1 \ldots \alpha_N\} \equiv a^+_{\alpha_1} a^+_{\alpha_2} \ldots a^+_{\alpha_N} |0\rangle$$

$a^+_{\alpha_1}$ rajoute (crée) une particule dans l'état α_1 et symétrise $\left\{\begin{matrix}\\ \\ \\ \end{matrix}\right.$ (ou antisymétrise) avec les autres particules.

Exemple:

$$\langle \vec{r}_1 | \langle \vec{r}_2 | a^+_{\alpha_1} a^+_{\alpha_2} |0\rangle = \varphi_{\alpha_1,\alpha_2}(\vec{r}_1, \vec{r}_2)$$

Fermions: ζ = zeta

$$a_\alpha^+ a_\beta^+ - \zeta a_\beta^+ a_\alpha^+ = 0 \qquad \zeta \quad \xi$$

$$a_\alpha a_\beta - \zeta a_\beta a_\alpha = 0$$

$$a_\alpha a_\beta^+ - \zeta a_\beta^+ a_\alpha = \delta_{\alpha\beta}$$

ζ ξ

où $\zeta = -1$ fermions

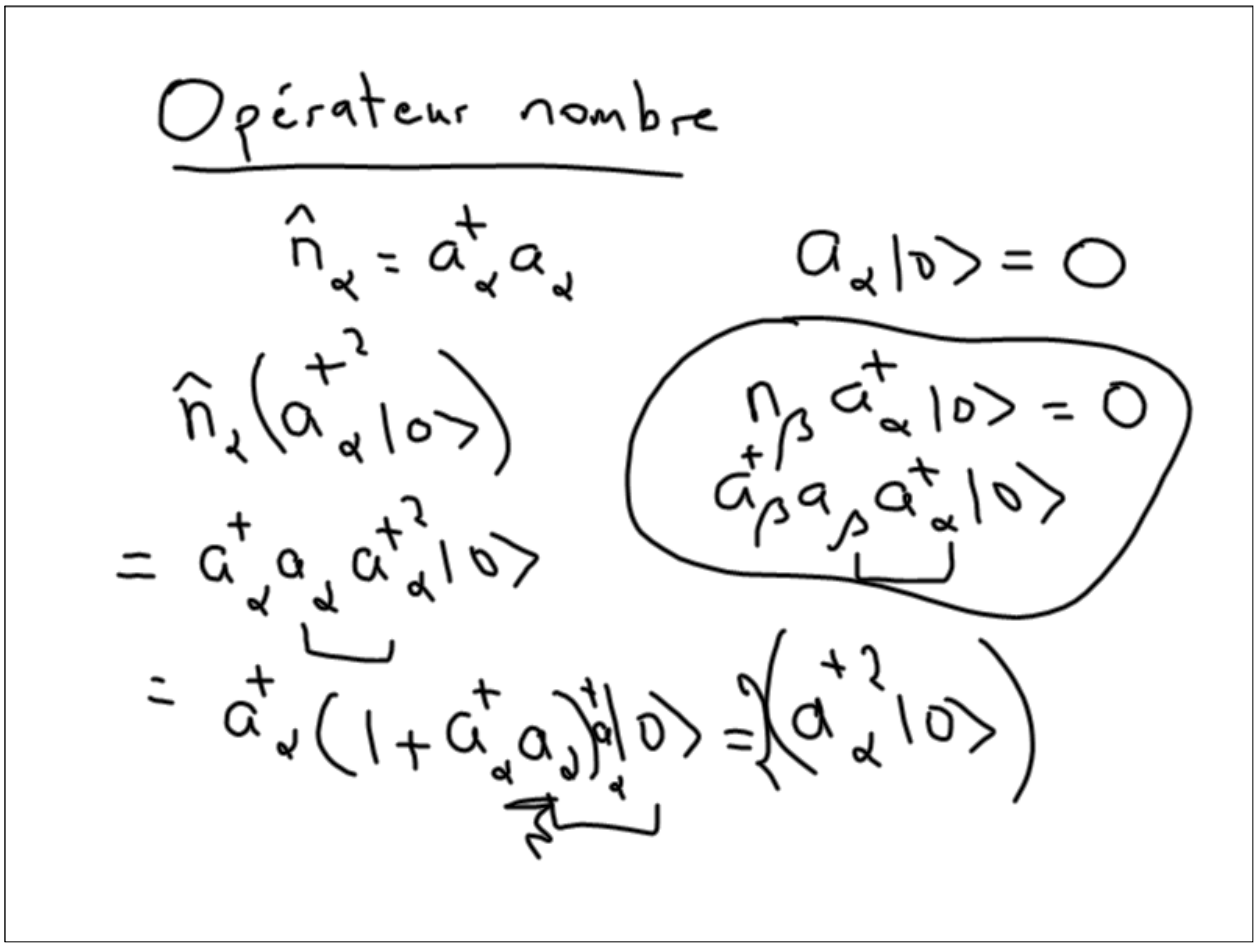
$+1$ bosons

Formule de changement de base

$$a_\alpha^+ = \sum_\mu c_\mu^+ \langle \mu | \alpha \rangle$$

$$|\alpha\rangle = \sum_\mu |\mu\rangle \langle \mu | \alpha \rangle$$

$$a_\alpha = \sum_\mu \langle \alpha | \mu \rangle c_\mu$$


Opérateur nombre
$\hat{n}_\alpha = a^+_\alpha a_\alpha$
$a_\alpha|0\rangle = 0$
$\hat{n}_\alpha (a^{+2}_\alpha |0\rangle)$
$= a^+_\alpha a_\alpha a^{+2}_\alpha |0\rangle$
$= a^+_\alpha (1 + a^+_\alpha a_\alpha) a^+_\alpha |0\rangle = 2(a^{+2}_\alpha |0\rangle)$
$n_\beta a^+_\alpha |0\rangle = 0$
$a^+_\beta a_\beta a^+_\alpha |0\rangle$

Opérateurs à 1 corps.

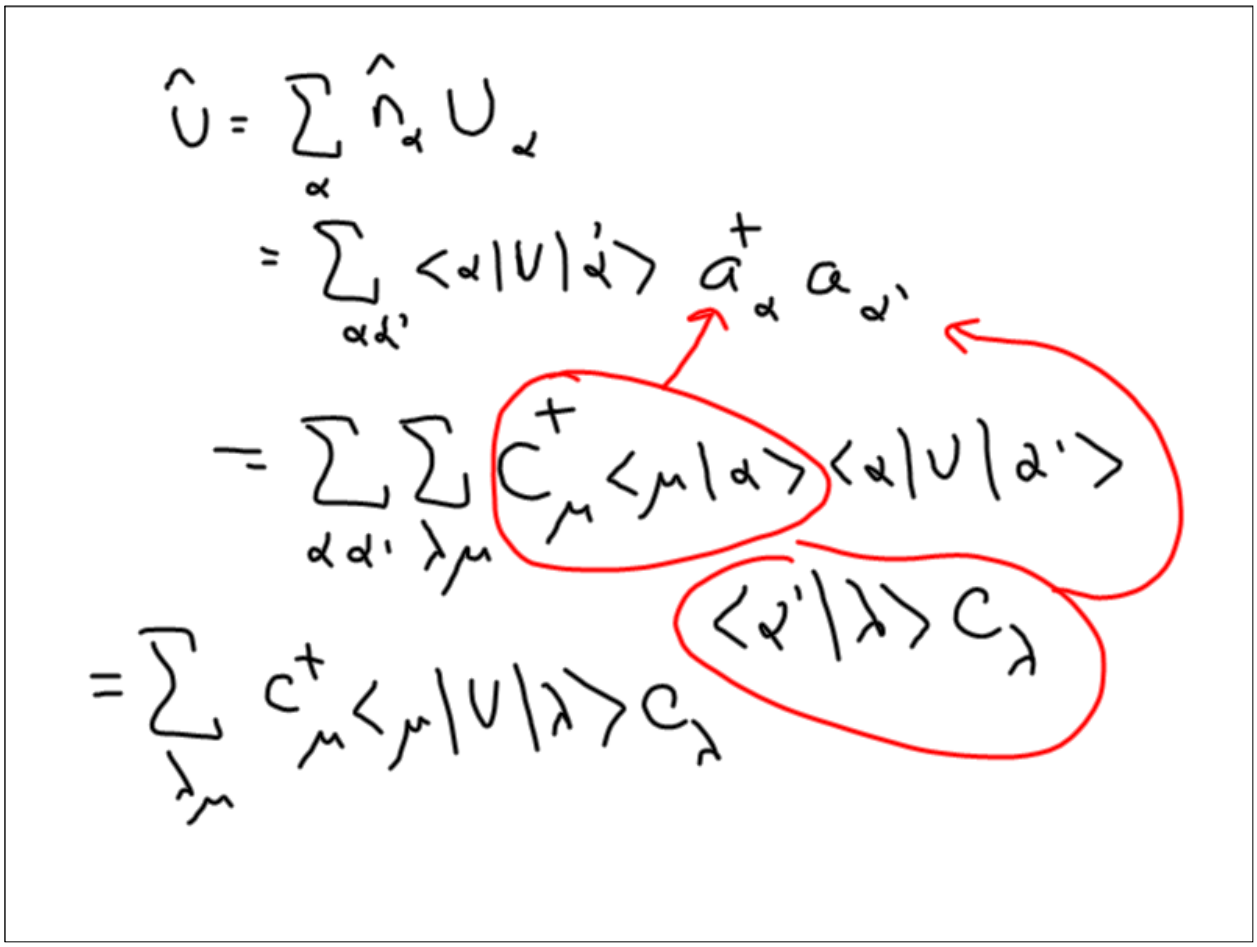
Dans la base propre

$$\hat{U}|\alpha\rangle = U_\alpha |\alpha\rangle = \langle\alpha|U|\alpha\rangle\,|\alpha\rangle$$

U_α : Valeur propre.

$$\hat{U}\,\{|\alpha_1 \ldots \ldots \alpha_N\}\} = \left(\sum_{i=1}^{N} U_{\alpha_i}\right) |\alpha_1 \ldots \ldots \alpha_N\}$$

$$= \left(\sum_{\alpha} U_\alpha \hat{n}_\alpha\right) |\alpha_1 \ldots \alpha_N\}$$


$\hat{U} = \sum_{\alpha} \hat{n}_\alpha U_\alpha$
$= \sum_{\alpha\alpha'} \langle\alpha|U|\alpha'\rangle\, a^+_\alpha a_{\alpha'}$
$= \sum_{\alpha\alpha'}\sum_{\lambda\mu} c^+_\mu \langle\mu|\alpha\rangle\langle\alpha|U|\alpha'\rangle\langle\alpha'|\lambda\rangle c_\lambda$
$= \sum_{\lambda\mu} c^+_\mu \langle\mu|U|\lambda\rangle c_\lambda$

Base utilisée souvent

$$\sum_k |k\rangle\langle k| = 1 = \int dr\, |r\rangle\langle r|$$

$$- \langle r|k\rangle = \frac{1}{\sqrt{V}} e^{ik\cdot r}$$

$$- \langle k|r\rangle = \frac{1}{\sqrt{V}} e^{-ik\cdot r}$$

$$\langle r|r'\rangle = \sum_k \langle r|k\rangle\langle k|r'\rangle$$

$$= \frac{1}{V}\sum_k e^{ik(r-r')} = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(r-r')} = \delta(r-r')$$

$$\langle k|k'\rangle = \int d^3r\, \langle k|r\rangle\langle r|k'\rangle$$

$$= \frac{1}{V}\int d^3r\, e^{-i(k-k')\cdot r} = \delta_{kk'}$$

$$\boxed{\psi(r) = \sum_k \langle r|k\rangle c_k}$$

$$\rightsquigarrow \{c_k, c_{k'}^\dagger\} = \delta_{kk'}$$

$$\boxed{\{\psi(r), \psi^\dagger(r')\} = \delta^3(r-r')}$$

$$\{\psi(r), \psi^\dagger(r')\} = \sum_{kk'} \langle r|k\rangle \{c_k, c_{k'}^\dagger\} \langle k'|r'\rangle$$

$$= \sum_k \langle r|k\rangle\langle k|r'\rangle = \langle r|r'\rangle = \delta(r-r')$$

$$\hat{T} = \sum_{k} \langle k | \frac{\hbar^2}{2m} | k \rangle c_k^+ c_k$$

$$= -\frac{1}{2m} \int d^3 r \, \psi^+(\vec{r}) \left(\nabla^2 \psi(r) \right)$$

$$= \frac{1}{2m} \int d^3 r \left(\nabla \psi^+(r) \right) \cdot \left(\nabla \psi(r) \right)$$

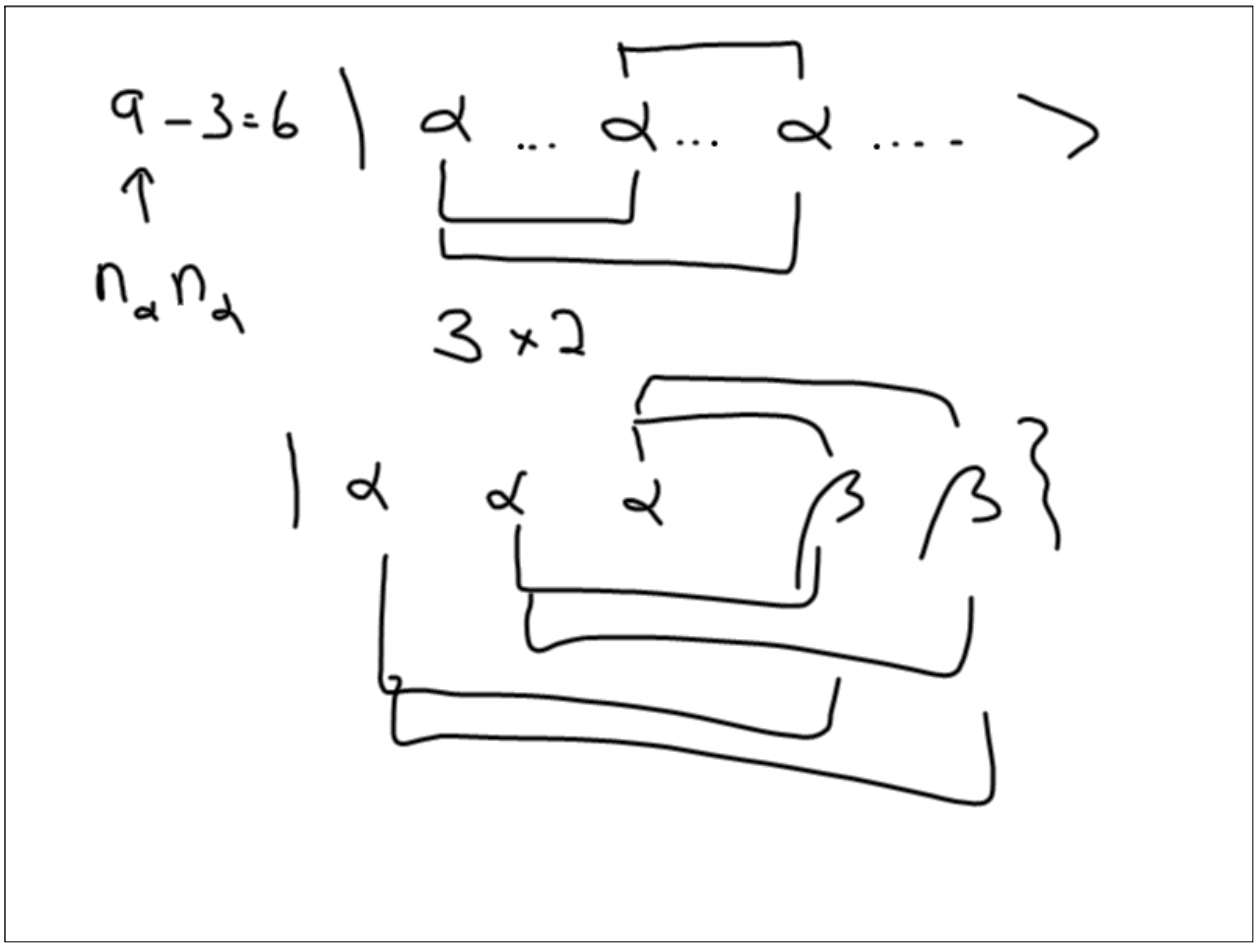
Opérateurs à 2 corps

$$\hat{V}\,|\alpha\rangle \otimes |\beta\rangle = V_{\alpha\beta}\,|\alpha\rangle \otimes |\beta\rangle$$

Base propre.

$$\hat{V}\,|\alpha_1 \dots \alpha_N\} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}^{N} V_{\alpha_i \alpha_j}\,|\alpha_1 \dots \alpha_N\}$$

$$= \frac{1}{2}\sum_{\alpha\beta} V_{\alpha\beta}\left(\hat{n}_\alpha \hat{n}_\beta - \delta_{\alpha\beta}\hat{n}_\alpha\right)|\alpha_1 \dots \alpha_N\}$$


9 - 3 = 6
$n_\alpha n_\alpha$
3 × 2

$n_\alpha \quad n_\beta$
$\uparrow \quad \uparrow$
$3 \quad 2$

$$\hat{n}_\alpha \hat{n}_\beta - \delta_{\alpha\beta} \hat{n}_\alpha$$

$$= a^\dagger_\alpha a_\alpha a^\dagger_\beta a_\beta - \delta_{\alpha\beta} a^\dagger_\alpha a_\alpha$$

$$= a^\dagger_\alpha \left(\delta_{\alpha\beta} - a^\dagger_\beta a_\alpha \right) a_\beta - \delta_{\alpha\beta} a^\dagger_\alpha a_\alpha$$

$$= -a^\dagger_\alpha a^\dagger_\beta a_\alpha a_\beta = a^\dagger_\alpha a^\dagger_\beta a_\beta a_\alpha \leftarrow$$

Destruction à droite de création par déf. $=$

Ordre normal

$$\hat{V} = \frac{1}{2}\int d^3x\, d^3y\; v(x-y)\,\psi^+(x)\,\psi^+(y)\,\psi(y)\,\psi(x)$$

Heisenberg

$$c_k(t) = e^{i\hat{H}t}\, c_k\, e^{-i\hat{H}t}$$

$$c_k^+(t) = e^{i\hat{H}t}\, c_k^+\, e^{-i\hat{H}t}$$

Cas particulier

$$\hat{H} = \sum_k \epsilon_k c_k^+ c_k$$

$$i\frac{\partial}{\partial t} c_k(t) = -\left[\hat{H}, c_k(t)\right]$$

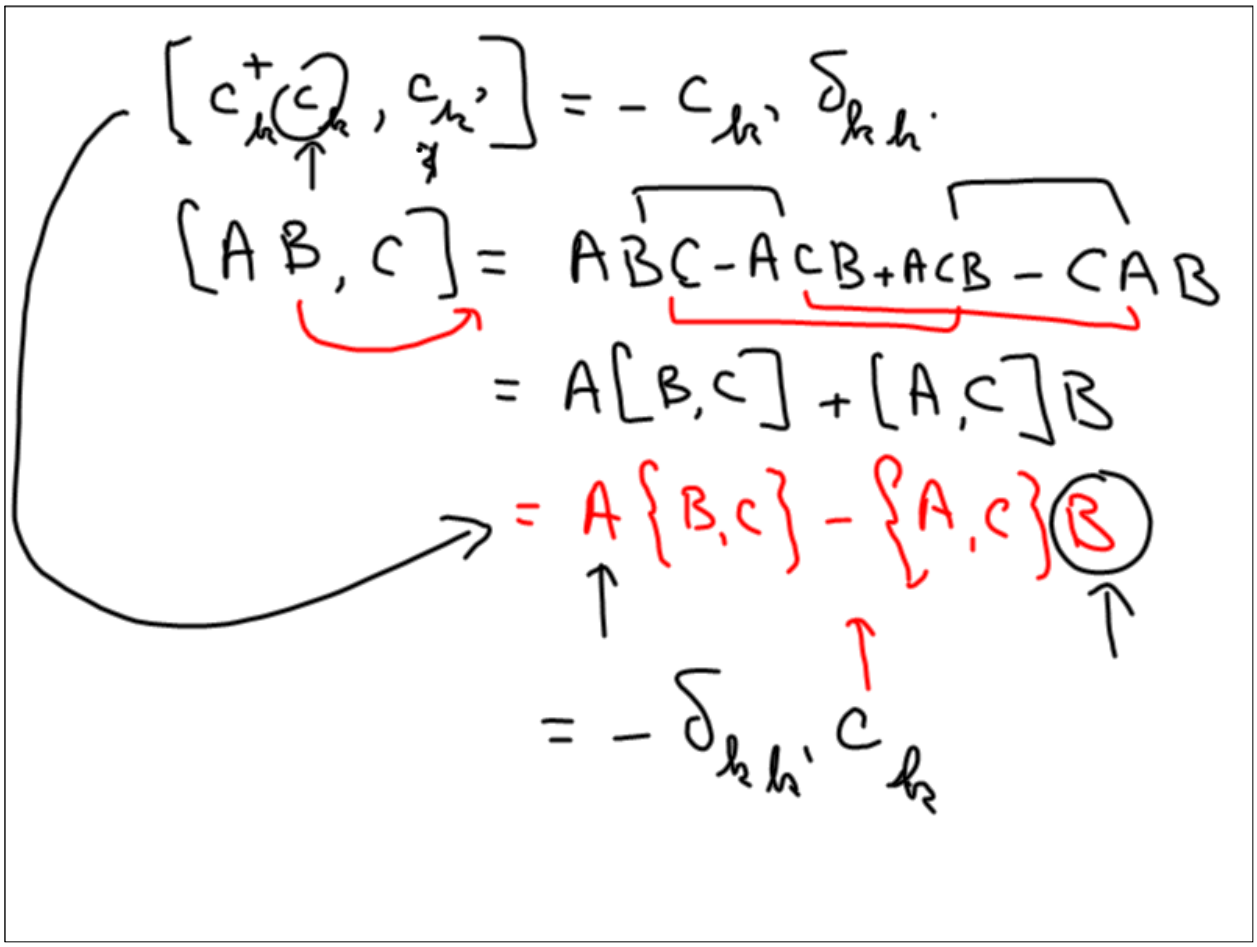
$$\hat{H}(t) = e^{i\hat{H}t}\,\hat{H}\,e^{-i\hat{H}t} = \sum_k \epsilon_k c_k^+(t) c_k(t)$$

$$\rightarrow\; i\frac{\partial}{\partial t} c_k(t) = -\sum_{k'} \delta_{kk'}\epsilon_{k'}(-)\, c_{k'}$$

$$= \epsilon_k\, c_k(t)$$

$$\frac{\partial}{\partial t} c_k(t) = -i\epsilon_k c_k(t)$$

$$\rightarrow \boxed{c_k(t) = e^{-i\epsilon_k t} c_k}$$


$[c^+_k c_k, c_{k'}] = -c_{k'}\delta_{kk'}$
$[AB, C] = ABC - ACB + ACB - CAB$
$= A[B,C] + [A,C]B$
$= A\{B,C\} - \{A,C\}B$
$= -\delta_{kk'} c_k$

sept. 27 - 10:51