4.6 Signif. physique de $A(k,\omega)$

1. Sans int.
2. Repr. de Lehman
3. Interp. probabiliste
4. ARPES
5. Quasiparticules.
6. Liquide de Fermi et ARPES.
7. $n_k = \langle c_k^+ c_k \rangle$ dans liquide de Fermi:

Comportement asymptotique.

4.7 3 théorèmes.

1. Wick 2. Graphs connexes 3. Principe var.

$$A(k,\omega) = \delta(\omega - \xi_k)\, 2\pi$$

$$A(k,\omega) = e^{\beta\Omega} \sum_{mn} \left(e^{-\beta K_m} + e^{-\beta K_n} \right)$$

$$|\langle n | c_k | m \rangle|^2\, 2\pi\, \delta(\omega - (K_m - K_n))$$

$$|m\rangle = c^\dagger_{k_1} c^\dagger_{k_2} \cdots c^\dagger_{k_n} |0\rangle$$

$$\langle n| = \langle 0| c_{k_1} \cdots {}_{\wedge} \cdots c_{k_n}$$

$$|m\rangle = a_1 c^\dagger_{k_1} \cdots c^\dagger_{k_n} |0\rangle + a_2 c^\dagger_{k'_1} \cdots c^\dagger_{k'_n} |0\rangle + \cdots$$

$$\frac{d^2\sigma}{d\Omega\, d\omega} \propto f(\omega)\, A(k,\omega)$$

4.6.5 Quasiparticules.

$$\frac{a+b'-ib''}{a+b'-ib''}\;\frac{1}{a+b'+ib''}$$

$$\mathrm{Im} = \frac{-b''}{(a+b')^2+(b'')^2}$$

$$G^R(k,\omega) = \frac{1}{\omega+i\eta-\zeta_k-\Sigma^R(k,\omega)}$$

$$A(k,\omega) = -2\,\mathrm{Im}\,G^R(k,\omega)$$

$$G^R(k,\omega) = \int\frac{d\omega'}{2\pi}\frac{A(k,\omega')}{\omega+i\eta-\omega'}$$

$$\boxed{A(k,\omega) = \frac{-2\,\mathrm{Im}\,\Sigma^R(k,\omega)}{\left(\underbrace{\omega-\zeta_k-\mathrm{Re}\,\Sigma^R(k,\omega)}\right)^2+\left(\mathrm{Im}\,\Sigma^R(k,\omega)\right)^2}}$$

$$(E_k-\mu)-\zeta_k-\mathrm{Re}\,\Sigma^R(k,E_k-\mu)=0$$

$\omega = E_k-\mu$ Quasiparticule

$$E_k = \epsilon_k + \mathrm{Re}\,\Sigma^R(k)$$

Hartree-Fock Σ^R indép. de ω

Développe autour de $\omega = E_k-\mu$

$$\underbrace{\omega-\zeta_k-\mathrm{Re}\Sigma^R(k,\omega)}=0+\frac{\partial}{\partial\omega}\left[\omega-\zeta_k-\mathrm{Re}\Sigma^R\right](\omega-(E_k-\mu))$$

$$=\left(1-\frac{\partial\,\mathrm{Re}\Sigma^R(k,\omega)}{\partial\omega}\bigg|_{\omega=E_k-\mu}\right)(\omega-(E_k-\mu))$$

$$\equiv Z_k^{-1}\,(\omega-(E_k-\mu))$$

$$A(k,\omega)=\frac{-2\,\mathrm{Im}\Sigma^R}{\left(Z_k^{-1}\right)^2(\omega-(E_k-\mu))^2+\left(\mathrm{Im}\Sigma^R\right)^2}$$

$$=2\pi Z_k\frac{1}{\pi}\frac{-Z_k\,\mathrm{Im}\Sigma^R}{(\omega-(E_k-\mu))^2+\left(Z_k\,\mathrm{Im}\Sigma^R\right)^2}$$

$$\boxed{\Gamma_k\equiv-Z_k\,\mathrm{Im}\Sigma^R}\qquad\Gamma_k>0$$

$$\boxed{A(k,\omega)=Z_k\,2\pi\left[\frac{1}{\pi}\frac{\Gamma_k}{(\omega-(E_k-\mu))^2+\Gamma_k^2}\right]+\text{incohérent}}$$

Renormalisation de Ψ

Quasiparticule

$$\nabla_k\left[(E_k-\mu)-\zeta_k-\mathrm{Re}\Sigma^R(k,E_k-\mu)=0\right]$$

$$v_k=\nabla_k(\epsilon_k-\mu)\quad\text{Bandes}$$

$$v_k^*=\nabla_k(E_k-\mu)\ \text{= vitesse renormalisée par interaction,}$$

$\zeta_k\leftrightarrow k$

$$v_k^*-v_k-\nabla_k\mathrm{Re}\Sigma^R(k,(E_k-\mu))$$

$$-v_k^*\frac{\partial}{\partial\omega}\mathrm{Re}\Sigma^R(k,\omega)\bigg|_{\omega=E_k-\mu}=0$$

$$v_k^*\bigg(\underbrace{1-\frac{\partial}{\partial\omega}\mathrm{Re}\Sigma^R(k,\omega)}_{Z_k^{-1}}\bigg)_{\omega=E_k-\mu}-v_k\left(1+\frac{\partial}{\partial\zeta_k}\mathrm{Re}\Sigma^R\right)=0$$

$$\nabla_k\mathrm{Re}\Sigma^R(k,\omega)$$

$$=\nabla_k\zeta_k\frac{\partial}{\partial\zeta_k}\mathrm{Re}\Sigma^R(k,\omega)$$

$$\frac{m}{m^*}=\frac{v_k^*}{v_k}=\left(1+\frac{\partial}{\partial\zeta_k}\mathrm{Re}\Sigma^R(k,E_k-\mu)\right)Z_k$$

$$mv_k=\hbar k_F$$

k_F ne dépend pas des interactions

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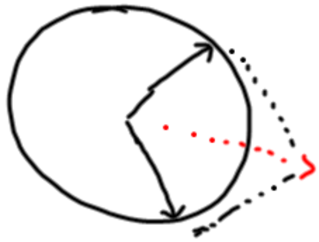
4.6.6. Liquides de Fermi



$\mathrm{Im}\,\Sigma^R(k,\omega) = 0$ à $\omega = 0$

Dépendance en ω

Analytique:

$$\mathrm{Im}\,\Sigma^R(k,\omega) = a\omega - \gamma\omega^2 + \dots$$

$a=0$ par causalité.

$$\rightarrow \boxed{\mathrm{Im}\,\Sigma^R(k,\omega) = -\gamma\omega^2}$$

$$G^R = G_0^R + G_0^R\,\Sigma^R\,G^R$$

$$\Sigma^R(k,\omega) - \Sigma^R(k,\infty) = \int \frac{d\omega'}{\pi}\,\frac{\mathrm{Im}\,\Sigma^R(k,\omega')}{\omega' - \omega - i\eta}$$

$$\mathrm{Re}\,\Sigma^R(k,\omega) - \Sigma^R(k,\infty) = \int \frac{d\omega'}{\pi}\,\frac{\mathrm{Im}\,\Sigma^R}{\omega'} + \omega \int \frac{d\omega'}{\pi}\,\frac{\mathrm{Im}\,\Sigma^R}{\omega'^2} + \dots$$

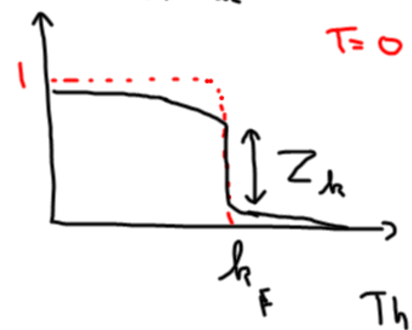
$$Z_k^{-1} = \left(1 - \frac{\partial}{\partial\omega}\mathrm{Re}\,\Sigma^R\right)$$

$$\mathrm{Re}\,\Sigma^R = \mathrm{cte} + \omega(-\gamma)$$

$$Z_k^{-1} = (1+\gamma) > 1 \qquad \boxed{Z_k < 1}$$

4.6.7 Distribution de k

$$n_k = \langle c_k^\dagger c_k \rangle$$



Théorème de Luttinger

$$\langle c_k^\dagger c_k \rangle = \lim_{\tau \to 0^-} G(k,\tau)$$

$$G(k,\tau) = -\langle T_\tau c_k(\tau) c_k^\dagger \rangle$$

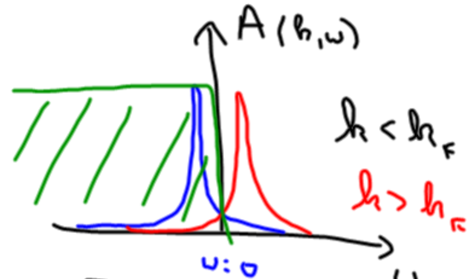
$$G(k,\tau) = T \sum_n e^{ik_n\tau} \int \frac{d\omega}{2\pi} \frac{A(k,\omega)}{ik_n - \omega}$$

$$= \int \frac{d\omega}{2\pi} \frac{A(k,\omega)\, e^{-\omega\tau}}{e^{\beta\omega}+1}$$

$$\lim_{\tau \to 0^-} G(k,\tau) = \int \frac{d\omega}{2\pi} f(\omega) A(k,\omega) = n_k$$

T=0

$$n_k = \int_{-\infty}^{0} \frac{d\omega}{2\pi} A(k,\omega)$$



$$\langle c_{k^-}^\dagger c_{k^-} \rangle - \langle c_{k^+}^\dagger c_{k^+} \rangle = Z_k$$

Comportement asymptotique.

$$G(k, ik_n) = \int \frac{d\omega}{2\pi} \frac{A(k,\omega)}{ik_n - \omega}$$

$$\lim_{ik_n \to \infty} G(k, ik_n) = \frac{\int \frac{d\omega}{2\pi} A(k,\omega)}{ik_n} = \frac{1}{ik_n}$$

$$G(k, ik_n) = \frac{1}{ik_n - \xi_k - \Sigma(k, ik_n)}$$

$$\lim_{ik_n \to \infty} \Sigma(k, ik_n) = \text{cte.} \qquad \text{Hartree-Fock}$$

4.7 3 théorèmes.

4.7.1 Théorème de Wick

$$\langle T_\tau \psi(\tau)\psi^+(0)\rangle = \frac{\mathrm{Tr}\left[e^{-\beta H_0} T_\tau \hat{U}(\beta,0)\hat{U}(0,\tau)\hat{\psi}\,\hat{U}(\tau,0)\right]}{\mathrm{Tr}\left[e^{-\beta H_0} T_\tau \hat{U}(\beta,0)\right]}$$

$$\hat{U}(\tau,0) = T_\tau e^{-\int_0^\beta \hat{V}(\tau)d\tau}$$

$$\langle T_\tau\, \psi(\tau)\psi^+ \int_0^\beta d\tau'\, \psi^+(\tau')\psi^+(\tau')\frac{1}{|r-r'|}\psi(\tau')\psi(\tau') \rangle$$

<u>Simple</u>: 1 electron:

$$\langle a_1 a_1^+ \rangle = \frac{\mathrm{Tr}\, e^{-\beta K_0} a_1 a_1^+}{\mathrm{Tr}\, e^{-\beta K_0}}$$

$$= \frac{\langle 0|\, a\, a_1^+\, |0\rangle + \left(\langle 0| a_1\right) e^{-\beta \xi_1} a_1 a_1^+ \left(a_1^+ |0\rangle\right)}{\langle 0|0\rangle + \left(\langle 0| a_1\right) e^{-\beta \xi_1} \left(a_1^+ |0\rangle\right)}$$

$$= \frac{1}{1 + e^{-\beta \xi_1}}$$

2 électrons:

$$|0\rangle \qquad a_1^+|0\rangle \qquad a_2^+|0\rangle \qquad a_1^+a_2^+|0\rangle$$

$$\rightsquigarrow \langle 0|(1+a_1)(1+a_2)\,\mathcal{O}\,(1+a_1^+)(1+a_2^+)|0\rangle$$

$$\langle 0|(1+a_1)\,\mathcal{O}\,(1+a_1^+)|0\rangle$$

$$\langle 0|\mathcal{O}|0\rangle$$

$$\langle 0|a_1\,\mathcal{O}\,a_1^+|0\rangle$$

$$\langle a_1 a_1^+ a_2 a_2^+\rangle = \frac{1}{1+e^{-\beta\xi_1}}\,\frac{1}{1+e^{-\beta\xi_2}}\,\frac{1+e^{-\beta\xi_3}}{1+e^{-\beta\xi_3}}$$

$$= \langle a_1 a_1^+\rangle\langle a_2 a_2^+\rangle$$

$$\langle a_i(\tau_i)\, a_j(\tau_j)\, a_k^+(\tau_k)\, a_l^+(\tau_l)\rangle$$

$$a_\alpha(\tau) = e^{-\tau \xi_\alpha} a_\alpha \qquad \text{si } H = \sum_\alpha \xi_\alpha a_\alpha^+ a_\alpha$$

$$a_i(\tau) = \langle i|\alpha\rangle\, a_\alpha(\tau) \quad \text{avec } \sum_\alpha \text{ implicite.}$$

$$= e^{-\tau\xi_\alpha} \langle i|\alpha\rangle\, a_\alpha$$

$$\to \langle i|\alpha\rangle\langle j|\beta\rangle \langle a_\alpha a_\beta a_\gamma^+ a_\delta^+\rangle \langle\gamma|k\rangle\langle\delta|l\rangle$$

$$e^{-\xi_\alpha\tau_i - \xi_\beta\tau_j + \xi_\gamma\tau_k + \xi_\delta\tau_l}$$

$$\langle a_\alpha a_\beta a_\gamma^+ a_\delta^+\rangle = \langle a_\alpha a_\alpha^+\rangle\langle a_\beta a_\beta^+\rangle \delta_{\alpha\delta}\delta_{\beta\gamma}$$

$\beta = \gamma$
$\alpha = \delta$
$\alpha \neq \beta$

$$- \langle a_\alpha a_\alpha^+\rangle\langle a_\beta a_\beta^+\rangle \delta_{\alpha\gamma}\delta_{\beta\delta}$$