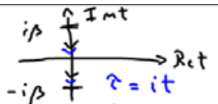
4.4 K.B.+K.S.

4.5 $\mathcal{G}$ Matsubara

1. Def.
2. Antipériodicité → Fréquences de Matsubara.
3. Repr. spectrale.
4. Règles de prolongement.
5. $\mathcal{G}(k, ik_n)$ + sans interaction.
6. Sommes sur ik_n
7. Comp. asymtotique $(ik_n \to \infty)$

4.6 Signif. physique du poids spectral.

1. Sans interaction.
2. Repr. de Lehman $G^R = \sum_\alpha \frac{\varphi_\alpha(-)\varphi_\alpha^*(1)}{\omega + i\eta - E_\alpha}$
3. Interp. prob. de A.
4. ARPES photoémission.
5. Quasiparticules
6. Inter. de ARPES avec liquides de Fermi
7. Distr. de n_k dans syst. en interaction.



$\tau = it$

$-\beta < 0 < \beta$

$$\mathcal{G}(r,r';\tau) = -\langle T_\tau \psi(\tau)\psi^+(0)\rangle$$

$$\boxed{\begin{aligned}\psi(\tau) &= e^{K\tau}\psi_s e^{-K\tau}\\ K &= H-\mu N\end{aligned}}$$

$$\psi^+(\tau) = e^{K\tau}\psi_s^+ e^{-K\tau}$$

$$\mathcal{G}(r,r';\tau) = -\theta(\tau)\langle\psi(r,\tau)\psi^+(r',0)\rangle + \theta(-\tau)\langle\psi^+(r',0)\psi(r,0)\rangle$$

$\tau < 0$

$$\mathcal{G}(r,r';\tau) = -\mathcal{G}(r,r';\tau+\beta)$$

$$\boxed{\mathcal{G}(r,r';\tau) = T\sum_{ik_n} e^{-ik_n\tau}\mathcal{G}(r,r';ik_n)}$$

$k_n = (2n+1)\pi T \qquad T = 1/\beta$

$$\boxed{\mathcal{G}(r,r';ik_n) = \int_0^\beta d\tau\, e^{ik_n\tau}\mathcal{G}(r,r';\tau)}$$

$$\boxed{\mathcal{G}(r,r';ik_n) = \int\frac{d\omega'}{2\pi}\frac{A(r,r';\omega')}{ik_n-\omega'}}$$

Repr. spectrale

$$G(z) \equiv \int\frac{d\omega'}{2\pi}\frac{A(\omega')}{z-\omega'}$$

$z = ik_n \quad G(z) = \mathcal{G}(ik_n)$

$z = \omega + i\eta \quad G(z) = G^R(\omega)$

Sans interaction

$$\mathcal{G}_0(k,\tau) = -e^{-\xi_k\tau}\left[(1-f_k)\theta(\tau) - f_k\theta(-\tau)\right]$$

$$= -\langle c_k(\tau)c_k^+(0)\rangle\theta(\tau) + \langle c_k^+(0)c_k(\tau)\rangle\theta(-\tau)$$

$$c_k(\tau) = e^{-\xi_k\tau}c_k$$

$$H-\mu N = \sum_k \xi_k c_k^+ c_k$$

$$\boxed{\mathcal{G}(k,ik_n) = \frac{1}{ik_n-\xi_k}}$$

$$\Big(\underset{ik_n}{\frac{\partial}{\partial\tau}} - \xi_k\Big)\mathcal{G}(k,\tau) = \delta(\tau)$$

4.5.6. Sommes sur ik_n

$$G(k,\tau) = -\langle T_\tau c_k(\tau) c_k^+(0)\rangle$$

$$G(k,0^-) = \langle c_k^+ c_k\rangle = f_k$$

$$G(k,0^+) = -\langle c_k c_k^+\rangle = -1 + f_k$$

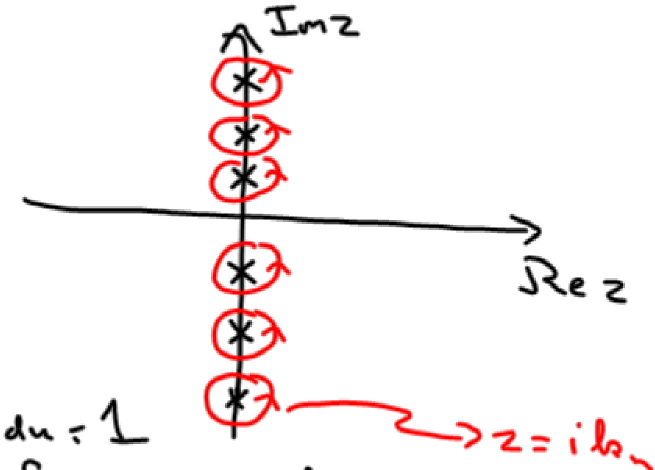
$$\boxed{G(k,\tau) = T\sum_n \frac{e^{-ik_n\tau}}{ik_n - \xi_k}}$$

$$G(k,0^\pm) = T\sum_n \frac{e^{-ik_n 0^\pm}}{ik_n - \xi_k}$$

$$\sum_n \frac{1}{ik_n - \xi_k} = \sum_{n>0}^{\infty} \frac{1}{ik_n - \xi_k} + \frac{1}{-ik_n - \xi_k}$$

$$= \sum_{n=0}^{\infty} \frac{-2\xi_k}{k_n^2 + \xi_k^2} \overset{?}{=} \frac{1}{2}$$

$$\sum_n \frac{e^{-i\omega_n \tau}}{i\omega_n - \xi_k}$$



Note :

$$-\beta \frac{1}{e^{\beta z}+1}$$ a un résidu = 1 aux fréquences $i\omega_n$.

Preuve : $z = i\omega_n + \delta z$

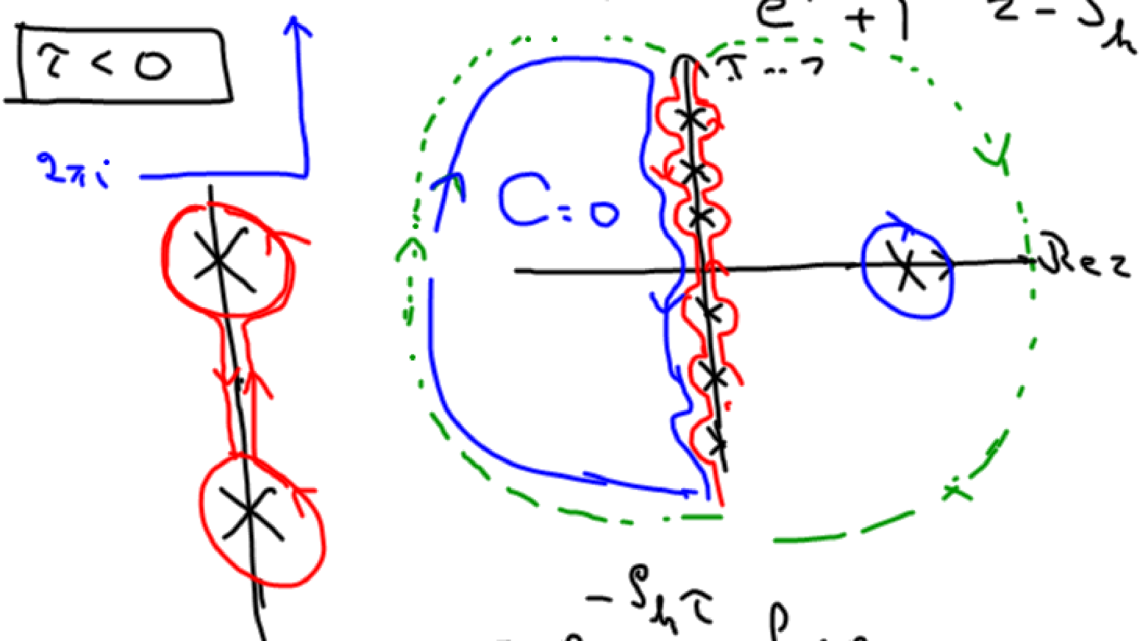
$$-\beta \frac{1}{e^{\beta i\omega_n + \beta \delta z}+1} = -\beta \frac{1}{-e^{\beta\delta z}+1}$$

$$= -\beta \frac{1}{-(1+\beta\delta z)+1}$$

$$= \frac{1}{\delta z} = \frac{1}{z - i\omega_n}$$

$$T\sum_n \frac{e^{-ik_n\tau}}{ik_n-\xi_k} = -\oint \frac{dz}{2\pi i}\,\frac{e^{-z\tau}}{e^{\beta z}+1}\,\frac{1}{z-\xi_k}$$

$\boxed{\tau<0}$



$$= e^{-\xi_k\tau} f(\xi_k)$$

$\boxed{\tau > 0}$

$$\beta \frac{1}{e^{-\beta z} + 1}$$

Résidus à $z = i k_n$

$$\beta \frac{1}{e^{-\beta(i k_n + \delta z)} + 1} \sim \frac{\beta}{-1(1 - \beta \delta z) + 1}$$

$$T \sum_n \frac{e^{-i k_n \tau}}{i k_n - \xi_k} = \oint \frac{dz}{2\pi i} \frac{e^{-z\tau}}{e^{-\beta z} + 1} \frac{1}{z - \xi_k} \qquad \sim \frac{1}{\delta z}$$

$$\left(1 - \frac{1}{e^{\beta \xi_k} + 1}\right) = \frac{e^{\beta \xi_k}}{e^{\beta \xi_k} + 1} = - \frac{e^{-\xi_k \tau}}{1 + e^{\beta \xi_k}} e^{\beta \xi_k} = -e^{-\xi_k \tau}(1 - f_k)$$

4.6 Expérience ARPES: Motivation

$$\vec{A}\cdot\vec{J}$$

$$\langle 0_{photon} \langle k' | \langle n | \left((a^+ + a)\, c^+_{k'} c_k \right) | m \rangle | 0_{libre} \rangle | 1 photon \rangle$$

$$\frac{\partial^2 \sigma}{\partial\Omega\, \partial\omega} \propto \sum_m e^{-\beta K_m} \left| \langle n | c_k | m \rangle \right|^2 \delta(\omega - (K_m - K_n))$$

4.1.1 Sans interaction

$$\mathcal{G}(k, ik_n) = \frac{1}{ik_n - \xi_k}$$

$$G^R(k,\omega) = \frac{1}{\omega + i\eta - \xi_k} \longrightarrow -2\,\mathrm{Im}\,G^R = +2\pi\,\delta(\omega - \xi_k)$$

$$= \int \frac{d\omega'}{2\pi} \frac{A(k,\omega')}{\omega + i\eta - \omega'}$$

$$A(k,\omega') = 2\pi\,\delta(\omega' - \xi_k)$$

$$A(k,\omega') = -2\,\mathrm{Im}\,G^R(k,\omega')$$

$$A(r,r';t) = \langle \{ \Psi(r,t), \Psi^+(r',0) \} \rangle$$

$$= e^{\beta\Omega} \sum_{mn} \langle n| e^{-\beta K_n} e^{iK_n t} \Psi_s(r) e^{-iK_m t} |m\rangle$$

$$\langle m| \Psi_s^+(r') |n\rangle + \text{autre}$$

$$K|n\rangle = (E_n - \mu N_n) |n\rangle$$

$$A(r,r';\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} A(r,r';t)$$

$$= e^{\beta\Omega} \sum_{nm} e^{-\beta K_n} 2\pi \delta(\omega - (K_m - K_n)) \langle n|\Psi_s(r)|m\rangle$$

$$\langle m|\Psi_s^+(r')|n\rangle$$

$$+ \text{autre}$$

$$A(k,\omega) = e^{\beta\Omega}$$

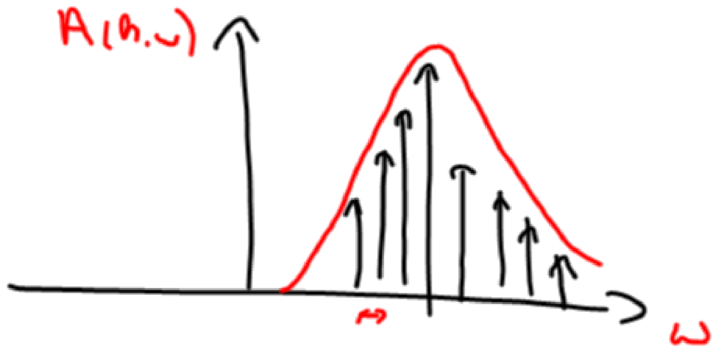
$$\sum_{m,n} e^{-\beta K_n} \langle n|c_k|m\rangle \langle m|c_k^+|n\rangle 2\pi\delta(\omega - (E_m - E_n - \mu))$$

$$+ e^{-\beta K_n} \langle n|c_k^+|m\rangle \langle m|c_k|n\rangle 2\pi\delta(\omega - (E_n - E_m - \mu))$$

$$\omega = (E_n - E_m) - \mu$$

$$= \sum_{m,n} \left(e^{-\beta K_n} + e^{-\beta K_m} \right) |\langle m|c_k^+|n\rangle|^2 \, 2\pi\delta(\omega - (E_m - E_n - \mu))$$

$A(k,\omega)$



$$\int \frac{d\omega'}{2\pi} A(k,\omega') = A(k,t=0)$$

$$A(k,t=0) = \langle \{ c_k, c_k^\dagger \} \rangle = 1$$

$$\frac{\partial^2\sigma}{\partial\Omega\,\partial\omega} \propto \sum_{mn} e^{-K_n\beta} |\langle m|c_k|n\rangle|^2$$

$$\delta(\omega + \mu - (E_n - E_m))$$
$$= \delta(\omega - (K_n - K_m))$$

$$\propto \int dt\, e^{it(\omega - K_n + K_m)}$$

e^{ikt} $\quad$ e^{-ikt}

$$e^{-\beta K_n} \langle n|c_k^+|m\rangle\langle m|c_k|n\rangle$$

$$\propto \int dt\, e^{i\omega t} \langle c_k^+ c_k(t)\rangle \leftarrow$$

$$\propto f(\omega)\, A(k,\omega)$$

$$\left.\frac{\partial^2\sigma}{\partial\Omega\,\partial\omega}\right|_{\text{electrons}} \propto S_{\rho\rho}(\omega) \sim \frac{2}{e^{-\beta\omega}-1}\chi'' \qquad 1-n_B$$

$$\langle c_k(t)\, c_k^\dagger \rangle = \langle c_k^\dagger\, c_k(t+i\beta) \rangle$$

à prouver

$$= \frac{1}{Z}\, \mathrm{Tr}\left[e^{-\beta K} \left(e^{iKt}\, c_k\, e^{-iKt} \right) c_k^\dagger \right]$$

$$= \frac{1}{Z}\, \mathrm{Tr}\left[e^{-\beta K}\, e^{\beta K - iKt}\, c_k^\dagger\, e^{-\beta K}\, e^{iKt}\, c_k \right]$$

$t + i\beta$

$$A(k,\omega) = \int dt\, e^{i\omega t} \langle c_k^\dagger c_k(t) + c_k(t) c_k^\dagger \rangle$$

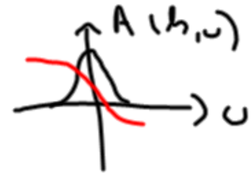
$$= \int dt\, e^{i\omega t} \langle c_k^\dagger c_k(t) \rangle$$

$$+ \int dt\, e^{i\omega(t + i\beta - i\beta)} \langle c_k^\dagger c_k(t+i\beta) \rangle$$

$$= \left(1 + e^{\beta\omega}\right) \int dt\, e^{i\omega t} \langle c_k^\dagger c_k(t) \rangle$$

$$A \propto \frac{1}{f(\omega)} \frac{\partial^2 \sigma}{\partial\Omega\, \partial\omega}$$

$$\boxed{\frac{\partial^2 \sigma}{\partial\Omega\, \partial\omega} \propto f(\omega)\, A(k,\omega)}$$



$$A(k,t) = \langle c_k(t)\, c_k^{\dagger} \rangle + \langle c_k^{\dagger}\, c_k(t) \rangle$$

$$+ i\, G^{>} - i\, G^{<}$$

$$\big(1 - f(\omega)\big) A(k,\omega) + f(\omega)\, A(k,\omega)$$

oct. 7 - 10:56