1. Seconde Quant.
    1. corps
    2. corps
    Heisenberg.
2. Motivation pour la déf. G^R
    · ex. Hamiltonien quadratique.
3. Repr. d'interaction - Produit chronologique.
4. Contour Kadanoff · Baym
    Keldysh-Schwinger
5. Matsubara

$$\psi(r) = \sum_{\alpha} \langle r | \alpha \rangle a_{\alpha}$$

$$\{\psi(r), \psi^{+}(r')\} = \delta^{3}(r-r')$$

$$\mathcal{H}_{1} = \int d^{3}r\, d^{3}r'\; \psi^{+}(r) \langle r | H_{1} | r' \rangle \psi(r')$$

$$\mathcal{H}_{2} = \int dr\, dr'\; \psi^{+}(r) \psi^{+}(r') \psi(r') \psi(r) \frac{1}{2} V(r-r')$$

2. Fonction de Green, N particules

1 particule.

$$G^R(r,t;r't') = -i\langle r|e^{-iH(t-t')}|r'\rangle\,\Theta(t-t')$$

N-corps $= -i\langle GS|\psi(r)\,e^{-iH(t-t')}\,\psi^+(r')|GS\rangle\,\Theta(t-t')$

e^{iHt} $\qquad$ $e^{-iHt'}$

$$\Longrightarrow \quad = -i\langle GS|\{\psi(r,t),\psi^+(r',t')\}|GS\rangle\,\Theta(t-t')$$

$$|GS\rangle = \int d^3r_1\dots d^3r_N\;|r_1\dots r_N\rangle\langle r_1\dots r_N|\psi\rangle$$

$$\chi^R_{\rho\rho} = 2i\langle[\rho,\rho]\rangle\,\Theta(t-t')$$

1 corps $\langle 0|\psi^+(r)\,\psi(r')|0\rangle = 0$

N-corps $\langle GS|\psi^+(r)\,\psi(r')|GS\rangle \neq 0$

Trou $\psi^+(r,t)\,\psi(r',0)\quad \theta(t)$

Particule. $\psi(r',0)\,\psi^+(r,t)\quad \theta(t)$

$e^{-i|E|t}$

$e^{i|E|t}$

Exemple H quadratique.

$$\langle r|H|r_1\rangle = -\frac{\nabla^2}{2m}\langle r|r_1\rangle = -\frac{\nabla^2}{2m}\delta(r-r_1)$$

$$\to \hat{H} = \int dr\, dr_1\, \Psi^+(r)\langle r|H|r_1\rangle \Psi(r_1)$$

$$= -\frac{1}{2m}\int dr\, \Psi^+(r)\frac{\nabla^2}{2m}\Psi(r)$$

$\boxed{\text{Où } |\alpha\rangle = \text{base propre de } H}$

$$\Psi(r,t) = \sum_\alpha \varphi_\alpha(r)\, a_\alpha(t) = \sum \varphi_\alpha(r)\, a_\alpha e^{-i\epsilon_\alpha t}$$

$$\{\Psi(r,t), \Psi^+(r',t')\} = \sum_{\alpha\alpha'} \varphi_\alpha(r) e^{-i\epsilon_\alpha t}\{a_\alpha, a^+_{\alpha'}\} e^{i\epsilon_{\alpha'} t'}\varphi^*_{\alpha'}(r')$$

$$= \sum_\alpha \varphi_\alpha(r)\varphi^*_\alpha(r')\, e^{-i\epsilon_\alpha(t-t')}$$

$$G^R = -i\sum_\alpha \varphi_\alpha(r)\varphi^*_\alpha(r')\, e^{-i\epsilon_\alpha(t-t')}\langle GS|GS\rangle\, \theta(t-t')$$

$$\int_0^\infty dt\, e^{i(\omega+i\eta-\epsilon_\alpha)t} = \left.\frac{e^{i(\omega+i\eta-\epsilon_\alpha)t}}{i(\omega+i\eta-\epsilon_\alpha)}\right|_0^\infty$$

$$= \sum_\alpha \frac{\varphi_\alpha(r)\varphi^*_\alpha(r')}{\omega+i\eta-\epsilon_\alpha}$$

$$\mathrm{Im}\, G^R = -\sum_\alpha \varphi_\alpha(r)\varphi^*_\alpha(r')\,\delta(\omega-\epsilon_\alpha)\pi$$

$$\int dr\left(-\frac{1}{\pi}\mathrm{Im}\, G^R(r,r;\omega)\right) = \sum_\alpha \delta(\omega-\epsilon_\alpha)$$

$$\int dr\, \varphi_\alpha(r)\varphi^*_\alpha(r) = 1$$

Eqs du mvt

$$i\frac{\partial}{\partial t}G^R(r,t;r't') = \delta(t-t')\delta(r-r')$$

$$-i\int dr_1 \langle r|H|r_1\rangle \langle GS|\left\{\psi(r_1,t), \psi^\dagger(r',t')\right\}|GS\rangle\,\theta(t-t')$$

$$\boxed{i\frac{\partial}{\partial t}G^R(r,t;r't') = \delta(t-t')\delta(r-r') + \int dr_1 \langle r|H|r_1\rangle G^R(r_1,t;r',t')}$$

$$= -\int dr_1 \langle r|H|r_1\rangle \dot{\psi}(r_1)$$

4.3 Représentation d'interaction et T_t

$H = H_0 + V \qquad [H_0, V] \neq 0$

$$\sum_i \langle i | e^{-\beta H} \psi_H(t) \psi_H(t') | i \rangle$$

$$\psi_H(t) = e^{iHt} \psi_S(t_{\text{explicite}}) e^{-iHt}$$

Si $H \neq H(t)$

$$\psi_H(t) = U^+(t,0) \psi_S \, U(t,0)$$

$$U(t,t') = U(t,0) U(0,t')$$

$$U(t,0) U(0,t) = 1$$

$$U(t,0) U^+(t,0) = 1$$

$$U^+(t,0) = U(0,t)$$

$$\psi_H(t) = U(0,t) \psi_S U(t,0)$$

Repr. int.

$$\hat{\Psi}(t) = e^{iH_0 t}\, \Psi_s\, e^{-iH_0 t}$$

$$\langle i| e^{-\beta H}\, U(0,t)\, e^{-iH_0 t}\, e^{iH_0 t}\, \Psi_s\, e^{-iH_0 t}\, e^{iH_0 t}\, U(t,0)\, U(0,t')\, \psi^{+}\, U(t',0)|i\rangle$$

$$\langle i| e^{-\beta H}\, \hat{U}(0,t)\, \hat{\Psi}(t)\, \hat{U}(t,0)\, \hat{U}(0,t')\, \hat{\Psi}(t')\, \hat{U}(t',0)|i\rangle$$

$$\boxed{\hat{U}(t,0) = e^{iH_0 t}\, U(t,0)}$$

Eqs. du mvt.:

$$\hat{U}(t,0) = e^{iH_0 t} U(t,0)$$

$$i \frac{\partial}{\partial t} \hat{U}(t,0) = e^{iH_0 t} \left[-H_0 + H \right] U(t,0)$$

$$= e^{iH_0 t} V e^{-iH_0 t} e^{iH_0 t} U(t,0)$$

$$\boxed{i \frac{\partial}{\partial t} \hat{U}(t,0) = \hat{V}(t) \hat{U}(t,0)}$$

$$\rightarrow \hat{U}(t,0) = e^{-i \int \hat{V}(t) dt} \quad ?$$

$$\frac{\partial}{\partial t} \ln U = \frac{1}{U} \frac{\partial U}{\partial t}$$

$$= \frac{1}{U(t)} \left[\frac{U(t+\Delta t) - U(t)}{\Delta t} \right]$$

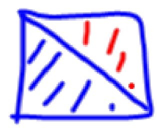
$$\overset{?}{=} \left[\frac{U(t+\Delta t) - U(t)}{\Delta t} \right] \frac{1}{U(t)}$$

$$\int_{t_0}^{t} i \frac{\partial}{\partial t} \hat{U}(t, t_0) =$$

sept. 30 - 10:18

$$\hat{U}(t,t_0) = 1 - i \int_{t_0}^{t} dt' \, \hat{V}(t') \, \underline{\hat{U}(t',t_0)}$$

$$\hat{U}(t,t_0) = 1 - i \int_{t_0}^{t} dt' \, \hat{V}(t') + (-i)^2 \int_{t_0}^{t} dt' \, \hat{V}(t') \int_{t_0}^{t'} dt'' \, \hat{V}(t'') + (-i)^3 \int_{t_0}^{t} dt' \, \hat{V}(t') \int_{t_0}^{t'} dt'' \, \hat{V}(t'') \int_{t_0}^{t''} dt''' \, \hat{V}(t''') + \dots$$



$$(-i)^3 \frac{1}{3!} T_t \int_{t_0}^{t} dt' \, \hat{V}(t') \int_{t_0}^{t} \hat{V}(t'') \, dt'' \int_{t_0}^{t} dt''' \, \hat{V}(t''')$$

$$\hat{U}(t,t_0) = T_t\left[\sum_{n=0}^{\infty} \frac{(-i)^n}{n!}\left(\int_{t_0}^{t} dt'\,\hat{V}(t')\right)^n\right]$$

$$= T_t\left[e^{-i\int_{t_0}^{t}\hat{V}(t')dt'}\right]$$

↑ Produit chronologique

Feynman + Dyson

$$U^{\dagger}(t,0) = U(0,t)$$

$t>0$

Définir T_t sur un contour

4.4 K.B. — K.S.

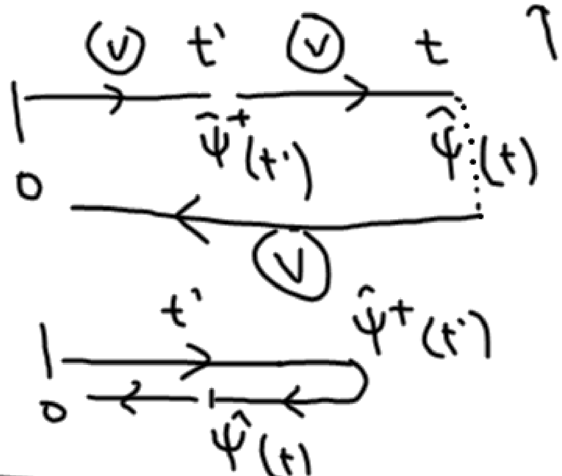
$-i\int \bar{V} u r$

$(-i)^2 \int V \int q$

$$G^> \propto \{i| e^{-\beta H} \hat{U}(0,t)\hat{\Psi}(t)\hat{U}(t,t')\hat{\Psi}^+(t')\hat{U}(t',0)|i\}$$

$\longrightarrow \hat{\Psi}(t)$

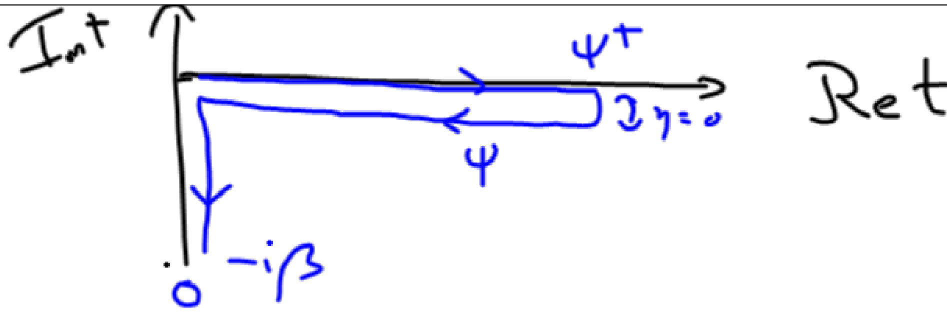
$G_c \quad G_A$



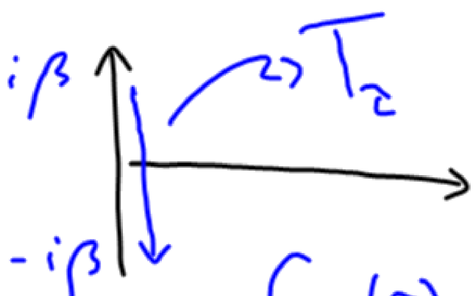
$$\boxed{G^>(r,t;r',t') \equiv -i\langle \Psi_H(r,t)\Psi_H^+(r',t')\rangle}$$

$$G^R(t-t') = -i\langle \{\Psi_H(t),\Psi^+(t')\}\rangle \Theta(t-t')$$

$$= \left[G^>(t-t') - G^<(t-t')\right]\Theta(t-t')$$



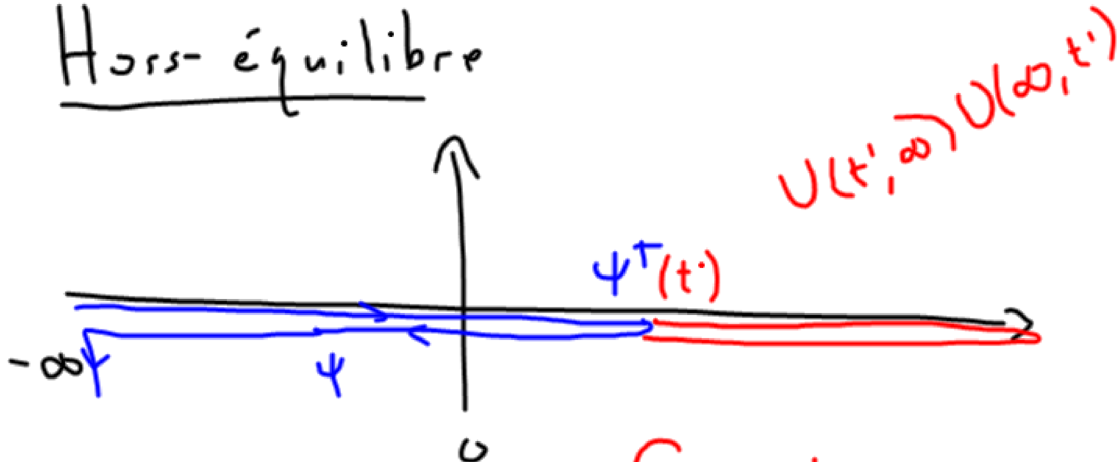
Matsubara



$$G_M(\tau) = -\langle T_\tau \psi_H(\tau)\psi_H^+(0)\rangle$$

$$\longrightarrow G^R$$

Hors-équilibre



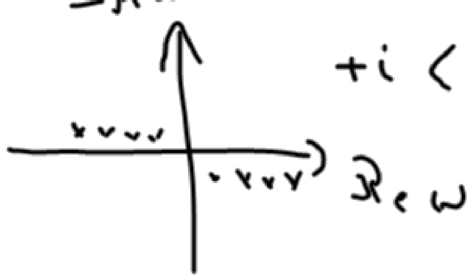
$G^>$

G Matrice. $\longrightarrow$ Keldysh

$$G_c(t-t') = -i\,\langle T_c\, \psi(t)\,\psi^+(t')\rangle$$

$$= -i\,\langle \psi(t)\,\psi^+(t')\rangle\,\theta(t-t')$$

$$+ i\,\langle \psi^+(t)\,\psi(t')\rangle\,\theta(t'-t)$$



sept. 30 - 11:00