(1) Seuil des Rayons-X

1. Intro.
2. Cas sans int.
3. Int.
4. Aperçu G.R.

(2) Kondo

(3)

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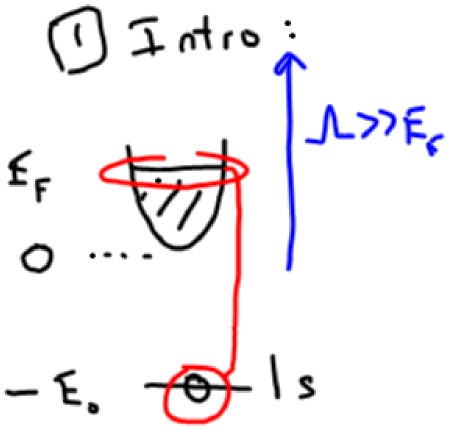
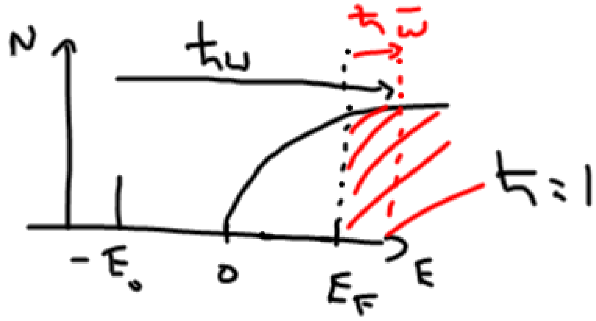
(4) Fonctionnelle B-K (approx. cons.)

(5) Principe var. dyn. + DMTF

① Intro :





$$\omega > E_F + E_0$$

$$\Gamma = 2\pi \sum_f \left| \langle d_k | \frac{A\cdot\vec{p}}{m} | \varphi_{1s} \rangle \right|^2 \delta(\omega - \epsilon_k - E_0)$$

$$= 2\pi \, N(E_F) \int d\epsilon_k \, \delta(\omega - \epsilon_k - E_0)$$

$$\vec{A}\cdot\vec{p} \propto \frac{eA}{c} \quad c^{\dagger}_{i=0} \propto \langle k | j | 1s \rangle$$

destruction du 1s

$$c^{\dagger}_{i=0} = \frac{1}{\sqrt{N}} \sum_k c^{\dagger}_k \qquad \langle 0 | c^{\dagger}_{k_1} c^{\dagger}_{k_2} \cdots$$

$$c^{+}_{i=0}\,\alpha \longrightarrow c^{+}_{i=0}\,d^{+}$$

où d^{+} crée un trou dans l'état 1s

$$\longrightarrow \quad \Theta = c^{+}_{i=0}\,d^{+} = \frac{1}{\sqrt{N}}\sum_{k} c^{+}_{k}\,d^{+}$$

$$\chi^{R} = -i\int_{0}^{\infty} dt\,\langle[\Theta(t),\Theta(0)]\rangle e^{i\omega t}$$

$$\Gamma \propto \mathrm{Im}\,\chi^{R}$$

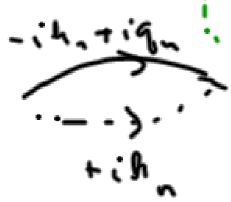
(2) Sans int.

$$H_0 = \sum_k \epsilon_k c_k^\dagger c_k + E_0 d^\dagger d$$

$$d^\dagger d \rightarrow d d^\dagger \rightarrow -d^\dagger d$$

$$\mathcal{G}(k, ik_n) = \frac{1}{ik_n - \epsilon_k} \qquad \mathcal{G} = \frac{1}{ik_n - E_0}$$

$$\chi = \frac{1}{N} \sum_k \sum_{k'} \langle T_\tau \, c_k^\dagger(\tau) \, d^\dagger(\tau) \, d(0) \, c_{k'}(0) \rangle$$



$$\chi(iq_n) = T \sum_{ik_n} \int \frac{d^d k}{(2\pi)^d} \frac{1}{-ik_n + iq_n - \epsilon_k} \frac{1}{ik_n - E_0}$$

$$\chi(iq_n) = -\int_0^{\Lambda} d\epsilon \, N(\epsilon) \, \frac{1-f(\epsilon)}{iq_n - \epsilon - E_0}$$

$$\mathrm{Im}\,\chi^R(\omega) =$$

$$\pi \int_0^{\Lambda} d\epsilon \, N(\epsilon) \, \delta(\omega - \epsilon - E_0) \left(1 - f(\epsilon)\right)$$

$$= \pi \, N(\omega - E_0) \left(1 - f(\omega - E_0)\right) \theta(\Lambda - \omega + E_0)$$

$$\boxed{\bar{\omega} = \omega - E_0 - E_F}$$

$$\boxed{\mathrm{Im}\,\chi^R = \pi \, N(E_F) \, \theta(\bar{\omega}) \, \theta(\Lambda - \bar{\omega})}$$

$$\mathrm{Re}\,\chi^R(\bar\omega) = \mathcal{P}\int \frac{d\omega'}{\pi} \, \frac{\mathrm{Im}\,\chi^R(\omega')}{\omega' - \bar\omega}$$

$$= N(E_F) \ln \left| \frac{\Lambda}{\bar\omega} \right|$$

$$\boxed{\chi^R_{6}(\bar\omega) = N(E_F) \ln\left(\left|\frac{\Lambda}{\bar\omega}\right| e^{i\pi} \right)}$$

3) Effet des interactions :

$$H = H_0 - V\, c^+_{i=0}\, d^+ d\, c_{i=0}$$

$$\chi = \chi_0 - (-V)\,\chi_0^2 + \dots\dots \to V N(E_F) \ln \frac{\Lambda}{\bar\omega}$$

$$\chi = \frac{\chi_0}{1 - V\chi_0} = \frac{\chi_0}{1 - V\left[N(E_F)\ln\left|\frac{\Lambda}{\bar\omega}\right| + i\pi N(E_F)\right]}$$

$$V N(E_F) \equiv g$$

$$1 - g\left(\ln\frac{\Lambda}{\bar\omega} + i\pi\right) = 0$$

$$\frac{1}{g} = \ln\frac{\Lambda}{\bar\omega} + i\pi \qquad \frac{1}{g} - i\pi = \ln\frac{\Lambda}{\bar\omega}$$

$$\frac{\Lambda}{\bar\omega} = e^{\frac{1}{g} - i\pi}$$

$$= -e^{\frac{1}{g}}$$

$$\boxed{\bar\omega = -\Lambda e^{-1/g}}$$

Inclure tous les ln.

Mahan 8.3.9

$$\begin{array}{c} \qquad\qquad g\chi_0 \qquad\quad (g\chi_0)^2 \qquad\quad -1(g\chi_0)^3 \cdots \\ \chi \propto \left(\chi_0 - V\chi_0^2 + \frac{2}{3}V^2\chi_0^3 - \frac{V^3}{3}\chi_0^4 + \cdots\right) \end{array}$$

?

$$\left[1 - e^{-2\chi_0 V}\right]$$

$$1 - \Big(1 - 2\chi_0 V + \frac{1}{2}(-2\chi_0 V)^2 + \cdots$$
$$\frac{1}{2\cdot 3}(-2\chi_0 V)^3 + \cdots$$

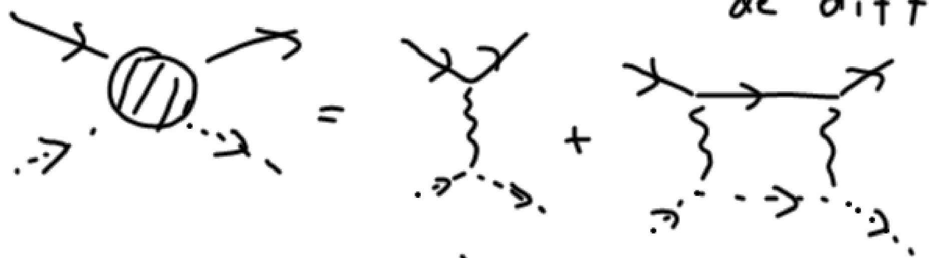
$$e^{-2\chi_0 V} = e^{-2V\left[N(E_F)\ln\left(\frac{\Lambda}{\bar\omega}e^{i\pi}\right)\right]}$$

$$g\ln\frac{\Lambda}{\bar\omega} + \cdots \quad = \left(e\right)^{-2VN(E_F)\left[\ln\frac{\Lambda}{\bar\omega} + i\pi\right]}$$

$$+\, g^2\ln\frac{\Lambda}{\bar\omega} + \cdots \quad = \left(\frac{\Lambda}{\bar\omega}\right)^{-2(g+g^2+g^3)} e^{-2gi\pi}$$

$+ g^3$.

④ Renormalisation (juste pour l'amplitude de diffusion)



$g = N(E_F) V$

$$(///) = g + g^2 \ln\left(\frac{\Lambda}{\bar{\omega}} e^{i\pi}\right)$$

$$(///) = g' + g'^2 \ln\left(\frac{\Lambda'}{\bar{\omega}} e^{i\pi}\right)$$

$$g' - g = g^2 \left[-\ln\left(\frac{\Lambda'}{\bar{\omega}} e^{i\pi}\right) + \ln\left(\frac{\Lambda}{\bar{\omega}} e^{i\pi}\right) \right]$$

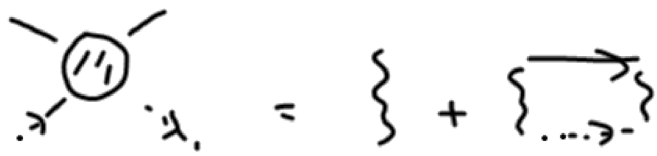
$$\Lambda' \sim \Lambda$$

$$dg = -g^2 \ln \frac{\Lambda'}{\Lambda} \qquad \Lambda' = \Lambda e^{-d\ell}$$

$$dg = -g^2 (-d\ell)$$

$$\frac{dg}{d\ell} = + g^2$$

Wilson



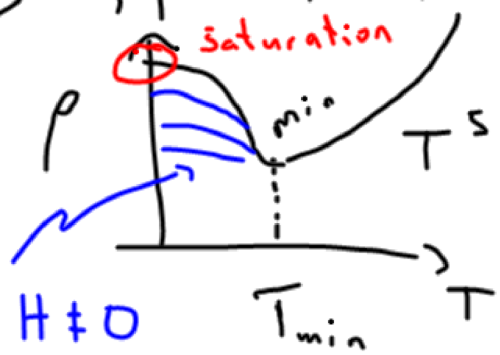
$$= g + g^2 \int_{\Lambda'}^{\Lambda} d\epsilon \, \frac{1}{\omega - \epsilon}$$

$$= g - g^2 \ln \frac{\Lambda}{\Lambda'}$$

$$g' = g - g^2 \ln \frac{1}{e^{-d\ell}}$$

$$dg = -g^2 d\ell$$

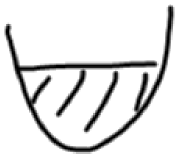
② Effet Kondo



Or impuretés magnétiques

$$R \propto T^5 + n_i\, f\left(\frac{T}{T_K}\right)$$

$$T_{min} \propto n_i^{1/5}\, T_K$$



Anderson

(inclu U)

H. de Kondo

$$\boxed{\mathcal{H} = \sum_{k\sigma} \epsilon_k c^{+}_{k\sigma} c_{k\sigma} + J\, \vec{S} \cdot \vec{s}_{i=0}}$$

spin. él.

imp. magn.

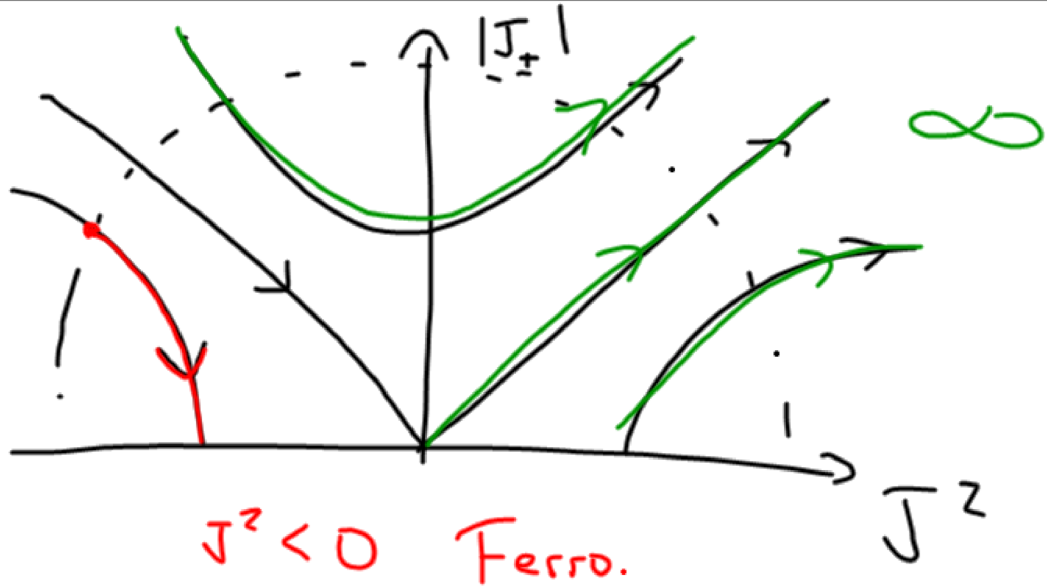
$$\mathcal{H} = \sum_{k\sigma} \epsilon_k c^{+}_{k\sigma} c_{k\sigma} + \frac{J_z}{4}\Big(c^{+}_{\uparrow} c_{\uparrow} f^{+}_{\uparrow} f_{\uparrow} + c^{+}_{\downarrow} c_{\downarrow} f^{+}_{\downarrow} f_{\downarrow} - c^{+}_{\uparrow} c_{\uparrow} f^{+}_{\downarrow} f_{\downarrow} - c^{+}_{\downarrow} c_{\downarrow} f^{+}_{\uparrow} f_{\uparrow} \Big)$$
$$+ \frac{J^{\pm}}{2}\Big(c^{+}_{\downarrow} c_{\uparrow} f^{+}_{\uparrow} f_{\downarrow} + c^{+}_{\uparrow} c_{\downarrow} f^{+}_{\downarrow} f_{\uparrow} \Big)$$

$$J^z N(E_F) \equiv \mathcal{J}^z$$
$$J^\pm N(E_F) \equiv \mathcal{J}^\pm$$

$$\frac{d\mathcal{J}^\pm}{dl} = \mathcal{J}^\pm \mathcal{J}^z \qquad \frac{d\mathcal{J}^z}{dl} = \mathcal{J}^{\pm\,2}$$

$$\mathcal{J}^\pm \frac{d\mathcal{J}^\pm}{dl} - \mathcal{J}^z \frac{d\mathcal{J}^z}{dl} = 0$$

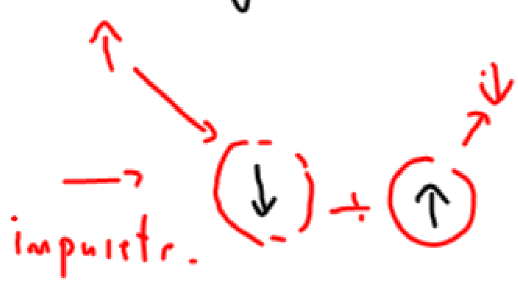
$$(\mathcal{J}^\pm)^2 - (\mathcal{J}^z)^2 = \text{Cte.}$$ Inv. de renorm.



$J^z < 0$ Ferro.

$J^z < 0$ et $J^z > |J_\pm|$

Singulet entre impureté et él. de cond.



$|\uparrow \text{ mer de Fermi } \downarrow\rangle - |\downarrow \text{ mer de Fermi } \uparrow\rangle$

$c_\downarrow = \frac{1}{\sqrt{N}} \sum_k c_{k\downarrow}$

Fonctionnelle Baym-Kadanoff + Approx. conservatives.

$$\Omega[\varphi] = -T \ln \mathrm{Tr}\Big[e^{-\beta K} T_\tau \, e^{-\int_0^\beta d\tau_1 d\tau_2 \sum_{r_1}\sum_{r_2} \varphi_\sigma(1,2)\psi^+_\sigma(1)\psi_\sigma(2)} \Big]$$

$1 \equiv (\tau_1, r_1)$

$$\boxed{\frac{\delta\Omega[\varphi]}{T\,\delta\varphi_\sigma(1,2)} = G_\sigma(2,1)}$$

$E(S) \to F(T)$

$T = \frac{\partial E}{\partial S}$

$F = E - TS$

$$\frac{\delta\varphi_\sigma(1,2)}{\delta\varphi_\sigma(3,4)} = \delta(\tau_1-\tau_3)\,\delta(\tau_2-\tau_4)\;\delta_{r_1 r_3}\,\delta_{r_2 r_4}$$

$$\boxed{\tilde{\Omega} = \Omega - \mathrm{Tr}[G\varphi]}$$

$$\mathrm{Tr}[G\varphi] = T\int d\tau_1 d\tau_2 \sum_{r_1}\sum_{r_2} G(r_1\tau_1; r_2\tau_2)\,\varphi(r_2\tau_2; r_1\tau_1)$$

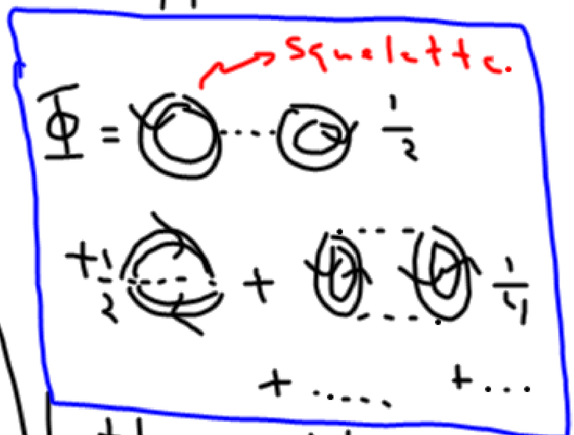
$$= T\sum_{ik_n}\sum_{k} G(k, ik_n)\,\varphi(k, ik_n)\,e^{ik_n\eta}$$

$$\boxed{\frac{\delta\tilde{\Omega}}{T\,\delta G} = \frac{1}{T}\cancel{\frac{\delta\Omega}{\delta\varphi}}\frac{\delta\varphi}{\delta G} - \varphi - G\cancel{\frac{\delta\varphi}{\delta G}} = -\varphi}$$

$$\boxed{\tilde{\Omega} = \Phi[G] - \mathrm{Tr}\left[(G_0^{-1} - G^{-1})G\right] + \mathrm{Tr}\ln G}$$

Baym-Kadanoff.

$$\rightarrow \frac{\delta \Phi}{T\delta G} = \Sigma \Rightarrow$$



Luttinger-Ward.

$$\boxed{\frac{\delta \tilde{\Omega}}{T\delta G} = \Sigma - G_0^{-1} + G^{-1} = -\varphi}$$

$$\frac{\delta}{\delta G(3,4)}\left[T\int d1\, d2\, G(1,2)\,\varphi(2,1)\right] = \varphi(4,3)$$

Dyson en présence de φ

$$\boxed{G^{-1} = G_0^{-1} - \Sigma - \varphi}$$

Principe "variationnel" dynamique.

$$\frac{\delta\tilde{\Omega}}{\delta G} = -\varphi = 0$$

$$\left.\frac{\partial\tilde{\Omega}}{\partial\mu}\right|_{T,\,\varphi=0} = \underbrace{\cancel{\left.\frac{\delta\tilde{\Omega}}{\delta G}\right|_{\varphi=0}}}_{0}\frac{\partial G}{\partial\mu} - \mathrm{Tr}\, G$$

Fonctions de réponse

$$\frac{\partial}{\partial\tau}\langle T_\tau \rho\rho\rangle = -\nabla\cdot\langle T_\tau j\rho\rangle$$

$$\frac{\partial}{\partial\tau} + \nabla\cdot$$

$$\frac{\delta(GG^{-1})}{\delta\varphi} = 0$$

$$\frac{\delta G}{\delta\varphi}G^{-1} + G\frac{\delta G^{-1}}{\delta\varphi} = 0$$

$$\frac{\delta G}{\delta\varphi} = -G\frac{\delta G^{-1}}{\delta\varphi}G \qquad G^{-1} = G_0^{-1} - \varphi - \Sigma$$

$$= GG + G\frac{\delta\Sigma}{\delta\varphi}G$$

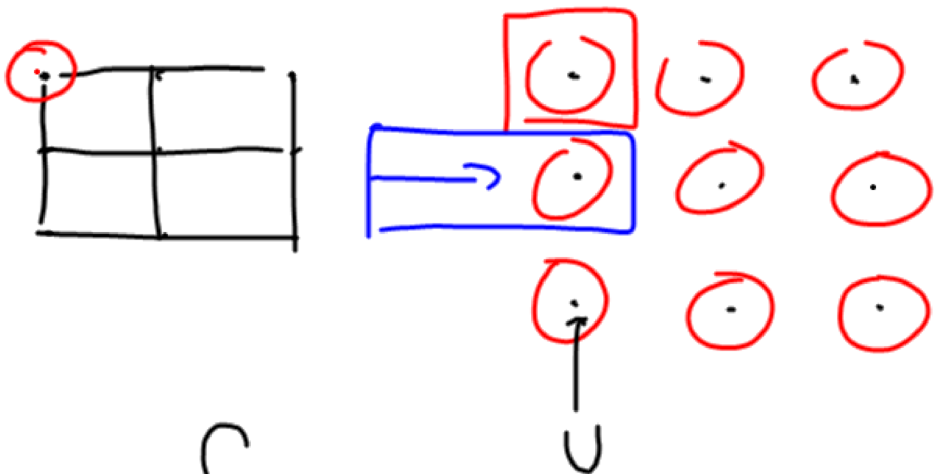
$$= GG + G\left(\frac{\delta\Sigma}{\delta G}\frac{\delta G}{\delta\varphi}\right)G$$

Vertex irréductible

$\psi^+\psi$

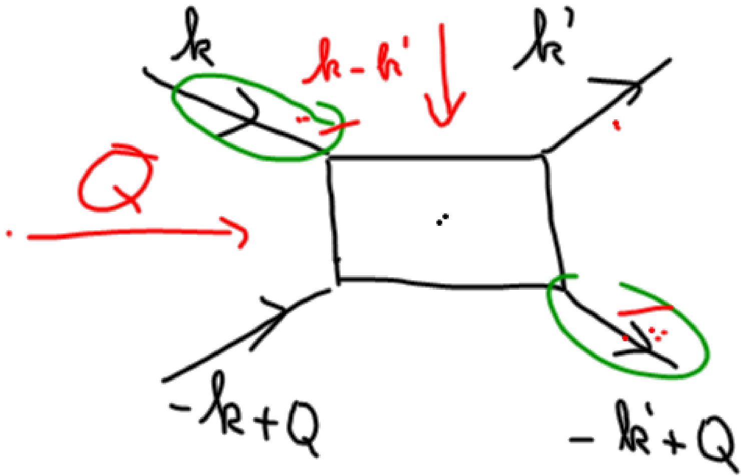
$\to e^{i\Lambda}\psi^+\psi e^{-i\Lambda}$

$$\frac{\delta\Sigma}{\delta G} = \frac{\delta\Phi}{\delta G\,\delta G}\frac{1}{T}$$



$$G_{ii}(\omega) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega + i\eta - \epsilon_k - \Sigma(\omega)}$$

$$\boxed{G_{ii}(\omega) = \frac{1}{\omega + i\eta - \Delta(\omega) - \Sigma(\omega)}}$$



$$\Gamma(k, k', Q)$$

$$\Gamma(\underbrace{k-k'}_{L\ \uparrow},\ \underbrace{k+k'-Q}_{\Delta k_r\ \uparrow},\ \underbrace{Q}_{\uparrow\ PP})$$