M  M. $\quad \omega_q = 0 \quad \infty$

$Q = 0$ Ferro.

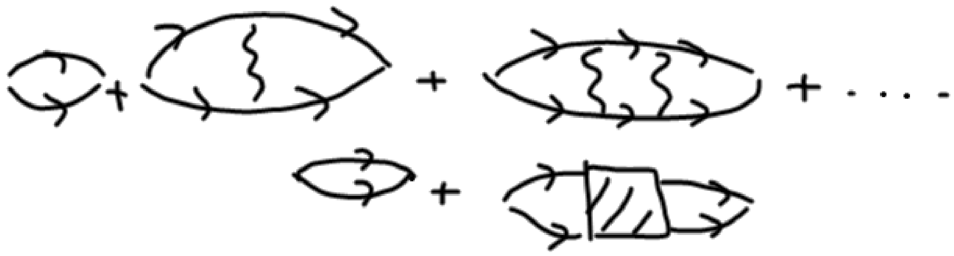
SDW $\quad Q = (\pi, \pi, \pi)$ Antiferro.

ODS

ρ  $\rho \Rightarrow \infty \quad \omega_q = 0$

ODC $\quad d = 1 \quad 2k_F = Q$

CDW



$\downarrow \infty \qquad T = T_c$

$$\chi_{pp}(q=0,\ iq_n=0)$$

$$= \int_0^\beta d\tau \left\langle T_\tau \underbrace{c^\dagger_{k\uparrow}(\tau)\, c^\dagger_{-k\downarrow}(\tau)}_{\Delta^\dagger_k} \underbrace{c_{k'\downarrow}(0)\, c_{-k'\uparrow}(0)}_{\Delta_{k'}} \right\rangle$$

$$\langle \Delta^\dagger \Delta \rangle \to \infty$$

$l = 0$

$$\langle c^{+}_{k\uparrow} \, c^{+}_{-k\downarrow} \rangle = -\langle c^{+}_{k\downarrow} \, c^{+}_{-k\uparrow} \rangle$$

$$= -\langle c^{+}_{-k\downarrow} \, c^{+}_{k\uparrow} \rangle$$

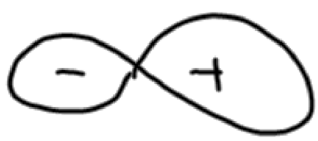
$$\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) Y_{00}$$

$l = 1$ ←

$$\langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \rangle = \langle c^\dagger_{k\downarrow} c^\dagger_{-k\uparrow} \rangle$$

^3He

$$= -\langle c^\dagger_{-k\downarrow} c^\dagger_{k\uparrow} \rangle$$



$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

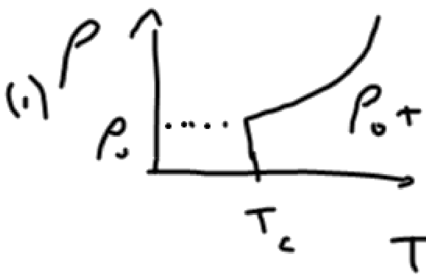
Sr_2RuO_4

$S^2 = 0$

Phénoménologie:

Bloch

(1)

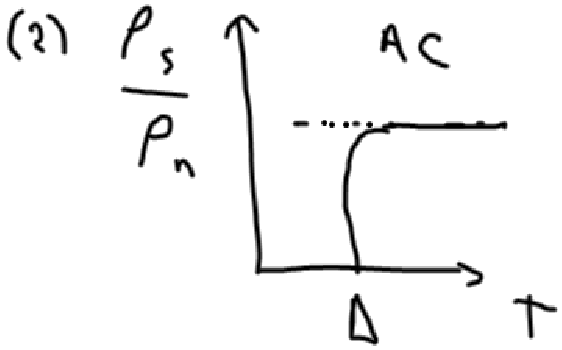


$\rho_0 + \beta T^5$

$\cos\theta \to \theta^2$

T^2

(2)



$\Delta = \hbar\omega_g = 3.5\, k_B T$

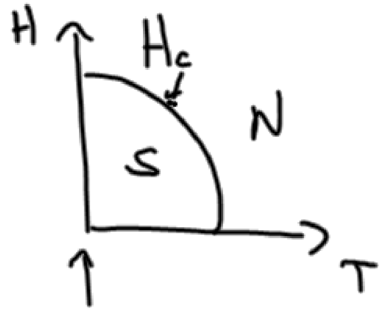
(3) Courant persistant

 $\tau > 10^5$ ans

(4) Flux quantifié

$$\Phi_0 = \frac{hc}{2e} = 2 \times 10^{-7} \text{ gauss/cm}^2$$

(5)



$$H_c(T) = H_c(0)\left(1-\left(\frac{T}{T_c}\right)^2\right)$$

(6) Meissner :

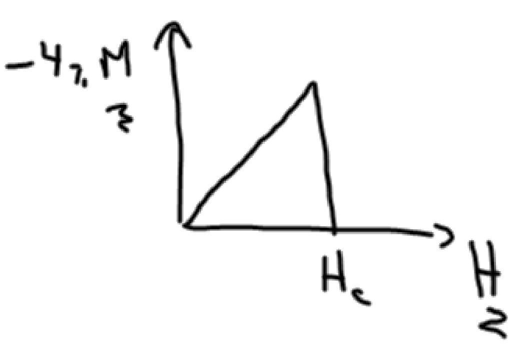
Type I

$$M = \chi H \qquad \chi = -\frac{1}{4\pi}$$

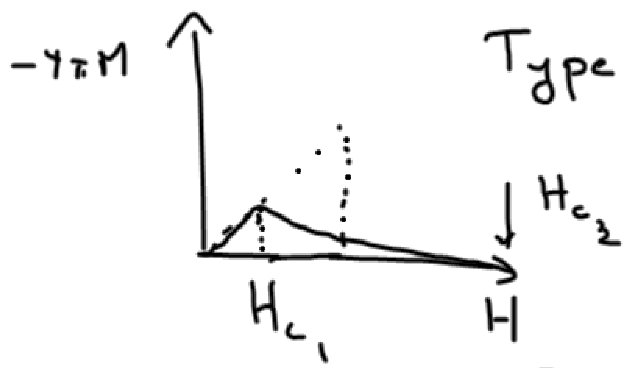
$$B = H + 4\pi M$$

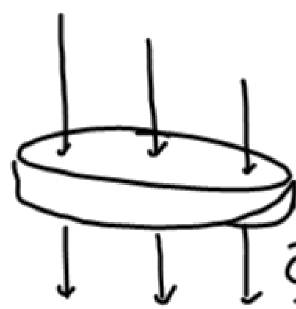
$B = 0$ à l'intérieur

$$F_N(T) = F_S(T) + \frac{H_c^2(T)}{8\pi}$$



Type 2





$\Phi_0 = \frac{hc}{2e}$

$\frac{e}{\hbar c}\int \vec{A}\cdot d\vec{l}$ $2\pi n$

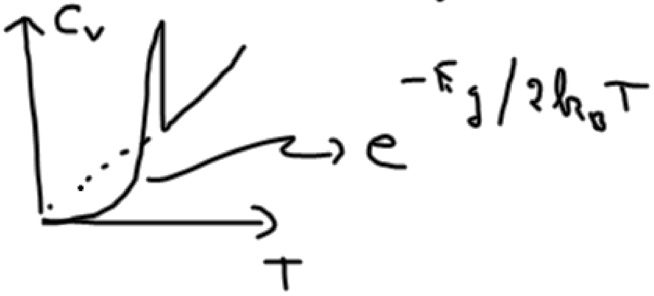
$\frac{hc}{e} = \int A\cdot dl$

(7) Effet isotopique

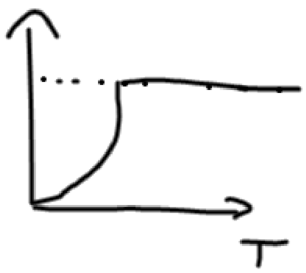
$$\underline{T_c \propto M^{-1/2}} \quad \text{Frölich (50)}$$

$$T \propto \omega_D \propto \sqrt{\frac{K}{M}}$$

(8)

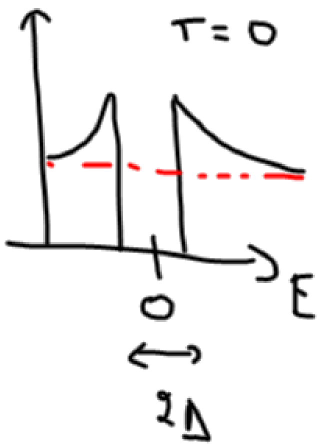


(9) $\frac{\alpha_s}{\alpha_N}$



(10) N(E)

Tunnel



③ $T < T_c$ BCS (1957)

$\langle M \rangle$

$$\underbrace{\langle c^\dagger_{k\uparrow}(\tau)\, c^\dagger_{-k\downarrow}(\tau)\rangle}_{\neq 0} \langle c_{k'\downarrow}(0)\, c_{-k'\uparrow}(0)\rangle$$

"Paramètre d'ordre"

$$\Delta^+ = V \sum_k \langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\rangle = V \sum_i \langle c^\dagger_{i\uparrow} c^\dagger_{i\downarrow}\rangle$$

Analogies Ferro.

$\langle \underline{M_Q(t) M_{-Q}(0)} \rangle \to \infty$	$\langle \underline{c^+_\uparrow c^+_\downarrow}\; \underline{c_\downarrow c_\uparrow} \rangle \to \infty$
$\langle M^z_{Q=0} \rangle \neq 0 \quad T<T_c$ $\sum_k \langle c^+_{k\uparrow} c_{k\uparrow} - c^+_{k\downarrow} c_{k\downarrow} \rangle \neq 0$	$\langle \Delta^+_{Q=0} \rangle \neq 0 \quad T<T_c$ $\sum_k \langle c^+_{k\uparrow} c^+_{-k\downarrow} - c^+_{k\downarrow} c^+_{-k\uparrow} \rangle \neq 0$
Symétrie brisée: <u>rot.</u>	Sym. Brisée: <u>jauge</u> $\partial\rho/\partial t + \nabla\cdot j = 0$
Ergodicité $\mathrm{Tr}\, e^{-\beta H} \to \mathrm{Tr}\, e^{-\beta(H - hM^z)}$ $\lim_{h\to 0}$ à la fin	$\mathrm{Tr}\, e^{-\beta(H - \mu N - \delta \sum_k c^+_{k\uparrow} c^+_{-k\downarrow} + c.h.)}$ $\lim_{\delta\to 0}$ à la fin.

$$\longrightarrow \frac{A}{q^2 + \xi^{-2}} = \chi(q, \omega = 0) \longrightarrow e^{-r/\xi}$$

$$\longrightarrow \frac{A}{\xi^{-2}} = \chi(0, \omega = 0) \longrightarrow Ctc$$

$$\xi = \infty$$

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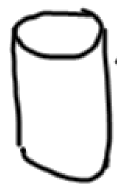
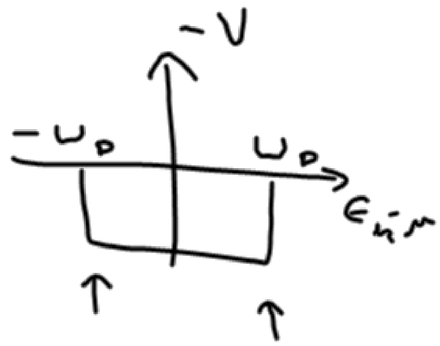
Mode de Goldstone	Mode de Goldstone
Ondes de spin	Onde de "charge"
$M^z \propto \mathrm{Re}\, e^{i\theta}$	$\langle c^{+}_{k\uparrow} c^{+}_{-k\downarrow} \rangle \propto e^{i\theta}$
$\nabla \theta$	$\nabla \theta$
$\omega = 0$	$\omega = \omega_p$
	Mécanisme de Higgs-Anderson



$\rightsquigarrow \lambda$ de pénétration relié à ω_p

$\boxed{18.3}$ Théorie BCS:



Hamiltonien BCS:

$$H = \sum_k \epsilon_k \, c^\dagger_{k\sigma} c_{k\sigma} - \frac{1}{2} \sum_{\substack{k k' \cancel{q} \\ \sigma\sigma'}} V \overbrace{\langle c^\dagger_{k\sigma} c^\dagger_{-k+q\,\sigma'} \rangle}^{b^*_q \delta_{q,0}} \overbrace{\langle c_{-k'+q\,\sigma'} c_{k'\sigma} \rangle}^{b_q \delta_{q,0}}$$

$\sigma = \uparrow$, $\sigma' = \downarrow$

$(V > 0)$

Coupure (cutoff) à $|(\epsilon_k - \mu)| < \omega_D$

$$\langle c^\dagger_{k\uparrow} c^\dagger_{q-k\downarrow} \rangle = b^*_q \, \delta_{q0}$$

Hamiltonien d'essai :

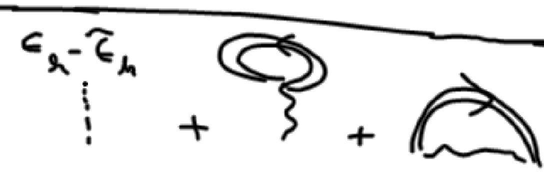
$$\tilde{H}_0 = \sum_k \epsilon_k c^+_{k\sigma} c_{k\sigma}$$

$$- \frac{1}{2} V \sum_k 2\, c^+_{k\uparrow} c^+_{-k\downarrow} \sum_{k'} b_{k'}$$

$$- \frac{1}{2} V \sum_k 2\, b^*_k \sum_{k'} c_{k\downarrow} c_{-k\uparrow}$$

$$\rightarrow \Delta = V \sum_k b_k = V \sum_k \langle c_{-k\downarrow} c_{k\uparrow} \rangle \leftarrow$$

$$\boxed{\tilde{H}_0 = \sum_{k\sigma} \epsilon_k c^+_{k\sigma} c_{k\sigma} - \sum_k c^+_{k\uparrow} c^+_{-k\downarrow} \Delta - \sum_k c_{k\downarrow} c_{-k\uparrow} \Delta^*}$$

$\tilde{\Sigma} =$  $= 0$

$$\alpha_{k\sigma} = u_k c_{k\sigma} + v_k c^+_{-k-\sigma} \quad \text{Bogoliubov Valentin}$$

↑ facteurs de cohérence

$T=0 \quad |\Psi_{BCS}\rangle = \prod_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) |0\rangle$

$$\propto \prod_k u_k\, e^{\frac{v_k}{u_k} c^+_{k\sigma} c^+_{-k\sigma}} |0\rangle$$

Formalisme de Nambu

$$\Psi_k = \begin{pmatrix} c_{k\uparrow} \\ c^{+}_{-k\downarrow} \end{pmatrix} \qquad \Psi^{+}_k = \left(c^{+}_{k\uparrow} \quad c_{-k\downarrow} \right)$$

$$\left\{ \Psi^{\alpha}_k , \Psi^{+\beta}_k \right\} = \delta_{\alpha\beta}$$

$$\mathcal{G}^{\alpha\beta}(k,\tau) = -\langle T_\tau \Psi^{\alpha}_{k}(\tau) \Psi^{+\beta}_{k}(0) \rangle$$

$$\boxed{\tilde{H}_0 = \sum_k \left(\Psi^{+}_k \left[\epsilon_k \tau_3 - \Delta \tau_+ - \Delta^{*} \tau_- \right] \Psi_k \right)}$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \tau_{\pm} = \frac{\tau_1 \pm i\tau_2}{2} = \frac{1}{2}\left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \pm i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \right]$$

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\Delta\tau_+ + \Delta^*\tau_- = \frac{\Delta}{2}(\tau_1 + i\tau_2) + \frac{\Delta^*}{2}(\tau_1 - i\tau_2)$$

$$= (\mathrm{Re}\,\Delta)\tau_1 - (\mathrm{Im}\,\Delta)\tau_2$$

$$\tilde{H}_0 = \sum_k \Psi^+_k \left[\epsilon_k\tau_3 - \mathrm{Re}\,\Delta\,\tau_1 + \mathrm{Im}\,\Delta\,\tau_2\right]\Psi_k$$

$$\mathcal{G}(k, ik_n) = \left(ik_n I - \underline{\xi_k\tau_3 + \mathrm{Re}\,\Delta\,\tau_1 - \mathrm{Im}\,\Delta\,\tau_2} \right)^{-1}$$

$$(aI + \vec{b}\cdot\vec{\tau})(aI - \vec{b}\cdot\vec{\tau}) = a^2 - (\vec{b}\cdot\vec{\tau})(\vec{b}\cdot\vec{\tau})$$

$$(\vec{a}\cdot\vec{\tau})(\vec{b}\cdot\vec{\tau}) = \vec{a}\cdot\vec{b}\,I + i\cancel{(\vec{a}\times\vec{b})}\cdot\vec{\tau}$$

$$\tau_x\tau_y = i\tau_z \qquad\qquad = (a^2 - b^2)\,I$$

$$\tau_x^2 = I$$

$$\boxed{\mathcal{G}(k, ik_n) = \frac{ik_n I + \xi_k\tau_3 - \mathrm{Re}\,\Delta\,\tau_1 + \mathrm{Im}\,\Delta\,\tau_2}{(ik_n)^2 - (\xi_k^2 + |\Delta|^2)}}$$

$$E_k^2 = \xi_k^2 + |\Delta|^2 \qquad \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\Delta = V \sum_k \langle c_{-k\downarrow} c_{k\uparrow} \rangle$$

$$= V \sum_k \langle \Psi^{2+}_k \Psi^1_k \rangle = -V \sum_k \langle T_\tau \Psi^1_k(0^-) \Psi^{2*}_k \rangle$$

$$= V \sum_k T \sum_{ik_n} \mathcal{G}^{12}(k, ik_n)\, e^{-ik_n 0^-}$$

$$\Delta = V \sum_k T \sum_{ik_n} \frac{-\Delta\, e^{ik_n\eta}}{(ik_n)^2 - E_k^2} \qquad \eta > 0$$

$$\left[\frac{1}{ik_n - E_k} - \frac{1}{ik_n + E_k}\right]\frac{1}{2E_k}$$

$$\boxed{1 = V \sum_{k} \frac{(1 - 2f(E_k))}{2E_k}}$$

$\rightarrow E_k^2 = \xi_k^2 + |\Delta|^2$ Équation du gap de BCS

a) $T = 0$

$$1 = \frac{V}{2} N(E_F) \int_{-\omega_D}^{\omega_D} \frac{d\xi}{\sqrt{\xi^2 + |\Delta|^2}}$$

$$= V N(E_F) \operatorname{arcsinh} \frac{\omega_D}{|\Delta|}$$

$$\boxed{|\Delta| = \frac{\omega_D}{\sinh\left(\frac{1}{N(E_F)V}\right)} \simeq 2\omega_D e^{-1/N(E_F)V}}$$

$T = 0$

b) $T = T_c$, $\Delta = 0$

$$1 = N(E_F)V \int_0^{\omega_D} \frac{1-2f(\xi)}{\xi} d\xi$$

$$= N(E_F)V \int_0^{\omega_D/T_c} \frac{\tanh \xi/2T_c}{\xi/T_c} d\xi/T_c$$

$$1 = N(E_F)V \Bigg[\ln\frac{\omega_D}{T_c} \tanh\frac{\omega_D}{2T_c} \leftarrow 1$$

$$- \underbrace{\int_0^{\infty} \ln\frac{\xi}{T_c} \frac{\partial}{\partial\xi} \tanh\left(\frac{\xi}{2T_c}\right) d\xi}_{\gamma} \Bigg]$$

$$\boxed{T_c = \frac{2e^{\gamma}}{\pi} \omega_D e^{-1/N(E_F)V}}$$

$$\frac{2\Delta(T=0)}{k_B T_c} = 3.53$$

Al = 3.53

Cd = 3.44

Hg = 3.95

Pb = 3.95

"Couplage fort"

(Eliashberg)

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Densité d'états :

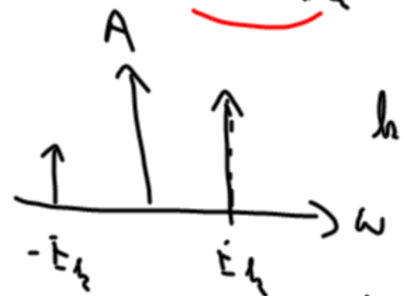
$$\rho_\uparrow(\omega) = -\frac{1}{\pi}\int \frac{d^3k}{(2\pi)^3}\, \mathrm{Im}\, G^R_\uparrow(k,\omega)$$

$$A_\uparrow(k,\omega) = -2\,\mathrm{Im}\, G^R(k,\omega)$$

$$= -2\,\mathrm{Im}\left[\frac{\omega+\xi_k}{(\omega+i\eta)^2 - E_k^2}\right]$$

$$= -2\,\mathrm{Im}\left[\frac{\omega+\xi_k}{2E_k}\left(\frac{1}{\omega-E_k+i\eta} - \frac{1}{\omega+E_k+i\eta}\right)\right]$$

$$= 2\pi\left[\frac{1}{2}\left(1+\frac{\xi_k}{E_k}\right)\delta(\omega-E_k) + \frac{1}{2}\left(1-\frac{\xi_k}{E_k}\right)\delta(\omega+E_k)\right]$$



k fixe.