Résumé

$$\uparrow\ R_i = R_{i0} + u_i$$

$$H = \underbrace{K_e + V_{ee}} + \underbrace{K_i + V_{ii}} + \underbrace{V_{ie}}$$

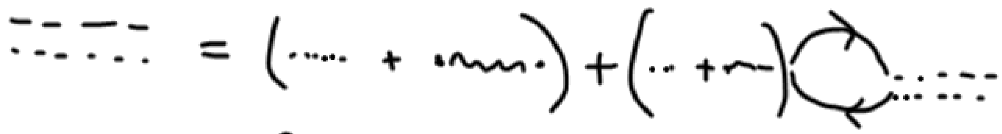
$$V_{ie} = -\frac{4\pi e^2 Z}{|r - R_i^0|} + \underbrace{\vec{u}_i \cdot \vec{\nabla}_{R_i} V_{ie}}$$

$$\vec{q}\ \frac{1}{q^2}$$

$$Z = Z_{0e} Z_{0i}\, \mathrm{Tr}\left[\underbrace{e^{-\beta(K_e+V_{ee})}\, e^{-\beta(K_i+V_{ii})}}_{Z_0^e Z_0^i}\, T_\tau e^{-\int \hat{V}_{ie} d\tau} \right]$$

$$= Z_{0e} Z_{0i} \left\langle T_\tau e^{\frac{1}{2}\int d\tau\, d\tau' \langle V_{ie}(\tau) V_{ie}(\tau') \rangle_i} \right\rangle_e$$

$$V_{eff} = \underbrace{V_c(q)\,\delta(\tau-\tau')}_{V_c} + \underbrace{|M_q|^2\,\mathcal{D}(q, \tau-\tau')}_{V_p}$$

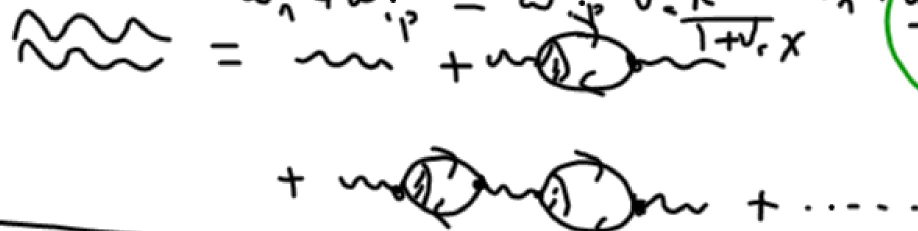


$$= V_c^e + \frac{1}{1+V_c\chi}\left[\frac{V_p}{1+V_p\dfrac{\chi}{1+V_c\chi}}\right]\frac{1}{1+V_c\chi}$$

$$\mathcal{D}^e = \frac{\mathcal{D}_0(q, iq_n)}{1 + \frac{|M_q|^2}{1+V_c\chi}\chi\,\mathcal{D}_0(q, iq_n)} \qquad \mathcal{D}_0 = \frac{-2\omega_{ip}}{\omega_n^2+\omega_{ip}^2}$$

$$= \frac{-2\omega_{ip}}{\omega_n^2+\omega_{ip}^2 - \omega_{ip}^2 V_c \frac{\chi}{1+V_c\chi}} = \frac{-2\omega_{ip}}{\omega_n^2 + \frac{\omega_{ip}^2}{1+V_c\chi}}$$

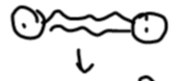
$\omega^2(q)$



$$\boxed{|M_q|^2(2\omega_{ip}) = \omega_{ip}^2 V_c}$$

$$\omega_q^2 = \frac{\omega_{ip}^2}{1+V_c\chi} = \frac{\omega_{ip}^2}{\epsilon(q)} \propto c_s^2 q^2$$

$$\epsilon(q) = 1 + \frac{q_{TF}^2}{q^2}$$

$V_{eff} = V_c^e +$ 

$$\frac{M_q}{\epsilon(q, iq_n)} \; \frac{-2\omega_{ip}}{q_n^2 + \omega_q^2} \; \frac{M_{-q}}{\epsilon(-q, iq_n)} .$$

$$\frac{-\omega_{ip}^2 V_c(q)}{\epsilon \; (q_n^2 + \omega_q^2) \; \epsilon}$$

$$\frac{-\omega_q^2 \, V_c(q)}{(q_n^2 + \omega_q^2) \, \epsilon(q, iq_n)}$$

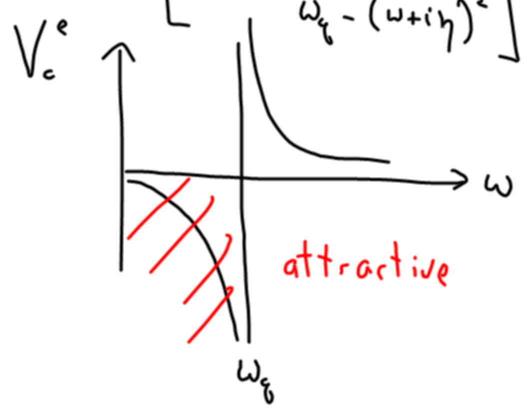
Au total.

$$V_c^e + \frac{-\omega_q^2 V_c^e}{q_n^2 + \omega_q^2}$$

$$= V_c^e \left[1 - \frac{\omega_q^2}{\omega_q^2 - (iq_n)^2} \right]$$

Prolongement:

$$= V_c^e \left[1 - \frac{\omega_q^2}{\omega_q^2 - (\omega + i\eta)^2} \right]$$



$$\mathrm{Re}\, V_{eff}^{TOT} = \frac{V(q)}{\epsilon(q)} \left[\frac{-\omega^2}{\omega_q^2 - \omega^2} \right]$$

$$= \frac{V(q)}{\epsilon(q) - \frac{\omega_q^2}{\omega^2}\epsilon(q)}$$

$$= \frac{V(q)}{\epsilon(q) - \frac{\omega_{ip}^2}{\omega^2}}$$

$$= \frac{V(q)}{1 + (\epsilon - 1)_{el} + (\epsilon - 1)_{ph}}$$

8 Supraconductivité.

1 Inst. de Cooper

1. Équation de vertex
2. Sol.

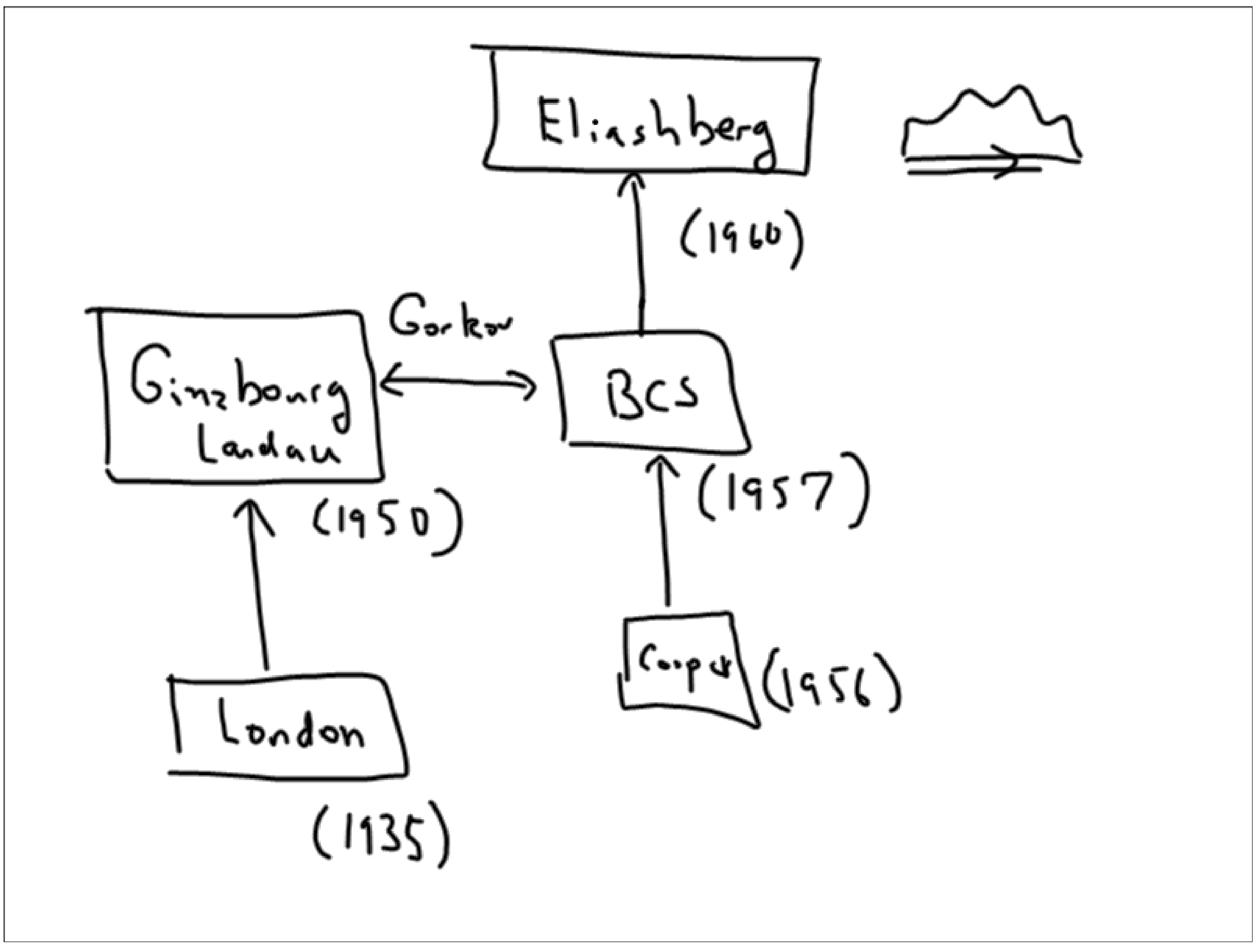
2 Phénoménologie.

3 BCS

(1) Cooper

(2) "Cohérence"

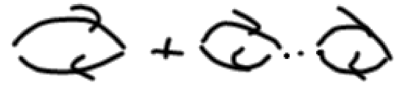
$$\prod_k \left(u_k + \underset{\substack{\uparrow \\ e^{i\theta_k}}}{v_k}\, c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)|0\rangle$$


Eliashberg
(1960)
Gorkov
Ginzbourg
Landau
BCS
(1957)
(1950)
Cooper
(1956)
London
(1935)

Instabilité de Cooper

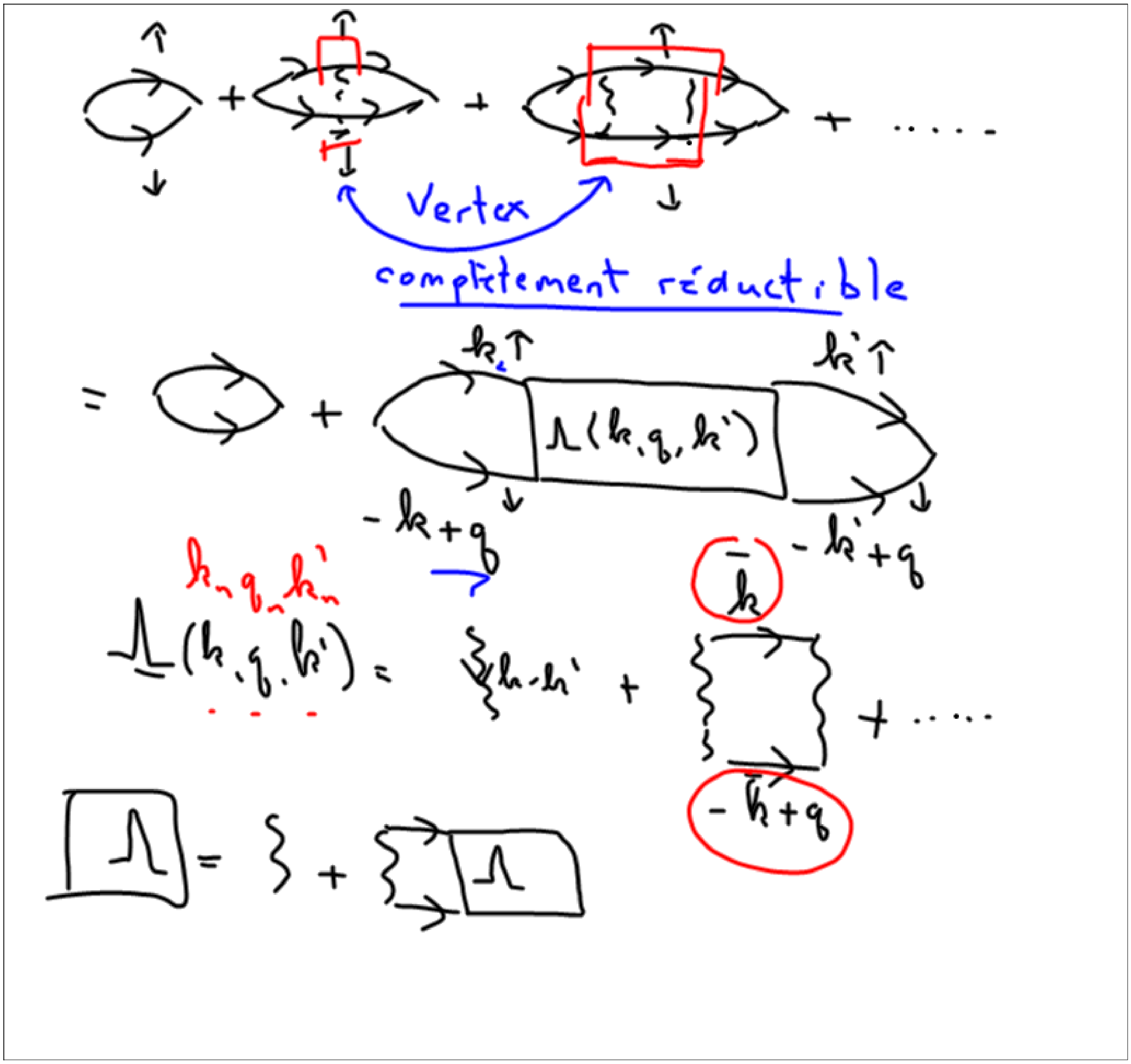
Mahan - 777 à 787

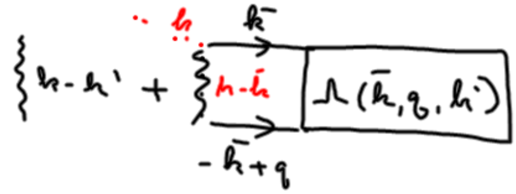
Rickayzen - 203 à 208

$\langle S^z(x,\tau)\, S^z(x',\tau') \rangle =$  $+ \dots$

$c^{+}_{\uparrow}(\tau)\, c_{\uparrow}(\tau) - c^{+}_{\downarrow}(\tau)\, c_{\downarrow}(\tau)$

$\langle T_\tau \left(c^{+}_{\uparrow}(\tau)\, c^{+}_{\downarrow}(\tau) \quad c_{\downarrow}(0)\, c_{\uparrow}(0) \right) \rangle$


Vertex
complètement réductible
k ↑
k' ↑
Λ(k,q,k')
-k+q
-k'+q
k
k_n q_n k'_n
Λ(k,q,k') =
k-k'
-k+q
Λ

$\Lambda(k, q, k') =$ 

$\Lambda_{\uparrow\downarrow}(k, q, k') = V(k-k')$

$-T \sum_{ik_n} \int \frac{d^3\bar{k}}{(2\pi)^3} \mathcal{G}_{\uparrow}(\bar{k}) \mathcal{G}_{\downarrow}(-\bar{k}+q)\, V(k-\bar{k})\, \Lambda_{\uparrow\downarrow}(\bar{k}, q, k')$

$$\Lambda_{\uparrow\downarrow}(k, q, k') = V(k-k') + \text{"}\int \frac{d^3\bar{k}}{(2\pi)^3} \frac{[1 - f(\zeta_{\bar{k}}) - f(\zeta_{-\bar{k}+q})]\text{"}}{ik_n - \zeta_{-\bar{k}+q} - \zeta_{\bar{k}}} V(k-\bar{k})\, \Omega_{\uparrow\downarrow}(\bar{k}, q, k')$$

$$1 - f_1 - f_2 = (1-f_1)(1-f_2) - f_1 f_2$$

$$\Psi = \frac{\Lambda}{\omega - \zeta_{-\bar{k}+q} - \zeta_{\bar{k}}}$$

$$(\omega - \zeta_k - \zeta_{-k+q})\Psi = V + \int V \Psi (1-f)(1-f)$$

$$\left[\omega + \frac{\hbar^2}{2m}\nabla_1^2 + \frac{\hbar^2}{2m}\nabla_2^2 - 2\mu - V(r_1 - r_2)\right]\Psi(r_1, r_2)$$

$$= V \text{ "Source"}$$

$$T \sum_{ik_n} \frac{1}{ik_n - \zeta_k} \frac{1}{-ik_n - iq_n - \zeta_{-k+q}}$$

2. Sol. pour Λ.

$$\mathcal{V}(\vec{k}-\vec{k}', ik_n - ik'_n) = \begin{cases} V(\vec{k}-\vec{k}') \text{ si } \\ \quad |k_n| \text{ et } |k'_n| < \omega_D \\ 0 \text{ autrement.} \end{cases}$$

$$V(\vec{k}-\vec{k}') = V(\vec{k}\cdot\vec{k}') \qquad |k| = |k'| \sim k_F$$

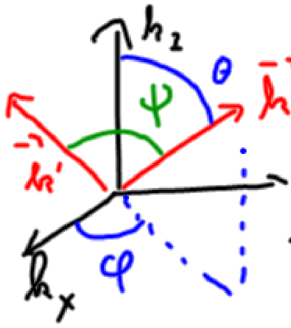
$$\Lambda(k, q, k') \xrightarrow{Y^*_{lm}} \Lambda(\hat{k}\cdot\hat{k}', ik_n, ik'_n)\, Y_{lm}$$

$$\vec{q} = 0 \quad iq_n = 0$$

En général

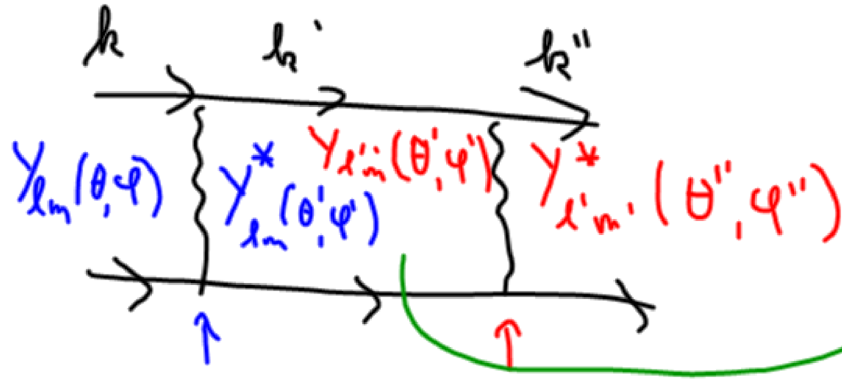
$$V(\hat{k}\cdot\hat{k}') = \sum_{l} V^{l} P_{l}(\hat{k}\cdot\hat{k}')$$

Polynômes de Legendre



$$= \sum_{l} V^{l} P_{l}(\cos\psi)$$

$$\to P_{l}(\cos\psi) = \frac{4\pi}{2l+1}\sum_{m=-l}^{l} Y_{lm}(\theta,\varphi)\, Y^{*}_{lm}(\theta',\varphi')$$



$$\int \frac{d\Omega'}{4\pi} \Rightarrow \delta_{l'l}\,\delta_{m'm}$$

$$\Lambda^{00}_{\uparrow\downarrow}(i\underline{k}_n, i\underline{k}'_n) = V^0\,\theta(\omega_D - |k_n|)\,\theta(\omega_D - |k'_n|)$$

$$-T\sum_{\bar{k}_n} N(E_F)\int d\bar{\xi}\; \frac{V^0\,\theta(\omega_D - |k_n|)\,\theta(\omega_D - |\bar{k}_n|)}{(i\bar{k}_n - \bar{\xi})(-i\bar{k}_n - \bar{\xi})}\,\Lambda^{00}_{\uparrow\downarrow}(i\bar{k}_n, i\underline{k}'_n)$$

$$\int d\bar{\xi}\,\frac{1}{\bar{\xi}^2 + \bar{k}_n^2} = \frac{1}{|\bar{k}_n|}\arctan\frac{\bar{\xi}}{|\bar{k}_n|}\Bigg|_{-\infty}^{\infty}$$

$$= \frac{\pi}{|\bar{k}_n|}$$

$$\Lambda^{00}_{\uparrow\downarrow} = V^0 - \underbrace{\boxed{T N(E_F)\pi \sum_{\bar{k}_n = -\omega_D}^{\omega_D} \frac{1}{|\bar{k}_n|}}}_{a} V^0 \Lambda^{00}_{\uparrow\downarrow}$$

$$\Lambda^{00}_{\uparrow\downarrow} = V^0 - a V^0 \Lambda^{00}_{\uparrow\downarrow}$$

$$\Lambda^{00}_{\uparrow\downarrow} = \frac{V^0}{1 + aV^0} \qquad V^0 < 0$$

$$= \infty \quad \text{si} \quad 1 + aV^0 = 0$$

$$a \equiv N(E_F)\, 2\pi T \sum_{\pi T}^{\omega_D} \frac{1}{(2n+1)\pi T}$$

$$= N(E_F)\left[\gamma + \ln \frac{2\omega_D}{\pi T}\right]$$

$\gamma \equiv 0.5772\ldots$ Cte d'Euler

$$1 + aV^\circ = 0$$

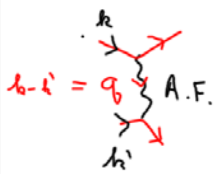
$$1 + N(E_F)\left[\gamma + \ln\frac{2\omega_D}{\pi T_c}\right] V^\circ = 0$$

$$-\frac{1}{N(E_F)V_0} = \gamma + \ln\frac{2\omega_D}{\pi T_c}$$

$$e^{-1/N(E_F)V_0} = e^{\gamma}\,\frac{2\omega_D}{\pi T_c}$$

$$\boxed{T_c = \left(\frac{2e^{\gamma}}{\pi}\omega_D\right) e^{1/N(E_F)V^\circ}}$$

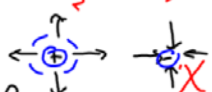
$$T_c = 1.13\,\omega_D\, e^{+1/N(E_F)V^\circ} \qquad V^\circ < 0$$



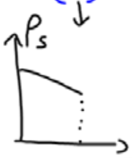
$$\vec{k} - \vec{k}' = (\pi,\pi)$$

$k_x^2 - k_y^2$

$$\Delta_k = \int \frac{d^2k'}{(2\pi)^2}\, \frac{V(k-k')\,\Delta_{k'}}{\sqrt{\epsilon_{k'}^2 + \Delta_{k'}^2}}$$



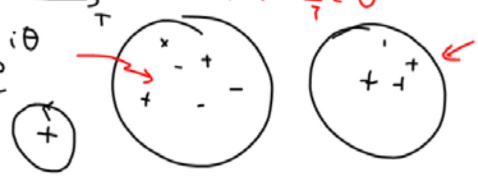
$\chi \propto \ln T$



$$1 - U\chi = 0 \qquad T_c \propto e^{-1/UN(E_F)}$$

$$1 - UN(E_F)\ln\frac{E_F}{T} = 0$$

$e^{i\theta}$



$$\langle j\;\; j\rangle \rightarrow \langle j\rangle\langle j\rangle + e^{-r/\xi}$$

$$\langle \Delta(r)\, \Delta^{\dagger}(0)\rangle = 1/r^{2-\eta}$$

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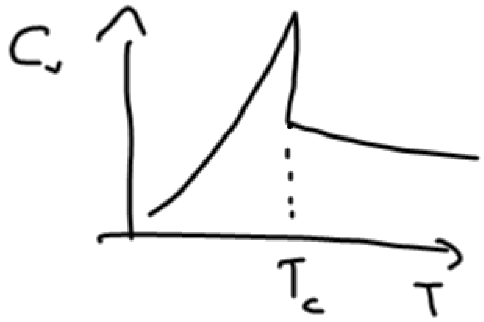
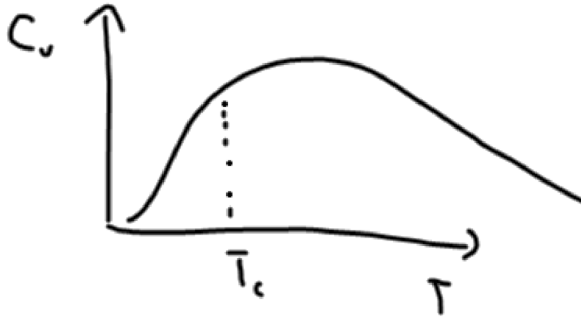


$$\xi \propto (T-T_c)^{-1/2}$$



$$\xi \propto e^{1/(T-T_c)^{1/2}}$$