Résumé:

$$\underline{\Psi}_k = \begin{pmatrix} c_{k\uparrow} \\ c^+_{-k\downarrow} \end{pmatrix} \qquad \underline{\Psi}^+_k = \left(c^+_{k\uparrow} \quad c_{-k\downarrow} \right)$$

$$\tilde{H}_0 = \sum_k \left(\underline{\Psi}^+_k \left[\epsilon_k \tau_3 - \mathrm{Re}\Delta\tau_1 + \mathrm{Im}\Delta\tau_2 \right] \underline{\Psi}_k \right)$$

$$\underline{\mathcal{G}}_{\alpha\beta}(k,\tau) = -\langle T_\tau \, \underline{\Psi}^\alpha_k(\tau) \, \underline{\Psi}^{\beta+}_k(0) \rangle$$

$$\epsilon_k c^+_k c_k \longrightarrow \frac{1}{ik_n - \epsilon_k}$$

$$\mathcal{G} = \left[ik_n \underline{I} - \epsilon_k \tau_3 + \mathrm{Re}\Delta\tau_1 - \mathrm{Im}\Delta\tau_2 \right]^{-1}$$

$$(I + a\cdot\tau)(I + b\cdot\tau) = I + a\cdot b\, I + i(\vec{a}\times\vec{b})\cdot\vec{\tau}$$

$$\mathcal{G}(k, i\omega_n) = \frac{i\omega_n + \xi_k - \mathrm{Re}\Delta\,\tau_1 + \mathrm{Im}\Delta\,\tau_2}{(i\omega_n)^2 - (\underbrace{\xi_k^2 + \Delta^2}_{E_k^2})}$$

$$\Delta = V \sum_k \langle c_{-k\downarrow} c_{k\uparrow} \rangle$$

$$= V \sum_k T \sum_{i\omega_n} \mathcal{G}^{12}(k, i\omega_n)\, e^{-i\omega_n 0^-}$$

$$\Delta = V\,\Delta \sum_k \frac{(1 - 2f(E_k))}{2E_k} \qquad E_k = \sqrt{\xi_k^2 + \Delta^2}$$

$$\frac{2\Delta(T=0)}{k_B T_c} = 3.53$$

Densité d'états de Q.P.

$$\boxed{\rho_\uparrow(\omega) = -\frac{1}{\pi}\int \frac{d^3k}{(2\pi)^3}\,\mathrm{Im}\left[G^R_\uparrow(k,\omega)\right]}$$

$$\rightarrow A_\uparrow(k,\omega) = -2\,\mathrm{Im}\,G^R_\uparrow(k,\omega)$$

$$= -2\,\mathrm{Im}\left[\frac{\omega+\xi_k}{(\omega+i\eta)^2 - E_k^2}\right]$$

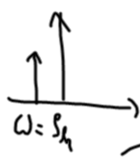
$$= -2\,\mathrm{Im}\left[\frac{\omega+\xi_k}{2E_k}\left(\frac{1}{\omega+i\eta-E_k} - \frac{1}{\omega+i\eta+E_k}\right)\right]$$

$$= 2\pi\left[\frac{\omega+\xi_k}{2E_k}\,\delta(\omega-E_k) - \frac{\omega+\xi_k}{2E_k}\,\delta(\omega+E_k)\right]$$

$$= 2\pi\left[\frac{1}{2}\left(1+\frac{\xi_k}{E_k}\right)\delta(\omega-E_k) + \frac{1}{2}\left(1-\frac{\xi_k}{E_k}\right)\delta(\omega+E_k)\right]$$

$$= 2\pi\left[u_k^2\,\delta(\omega-E_k) + v_k^2\,\delta(\omega+E_k)\right]$$

u_k et v_k "facteurs de cohérence"



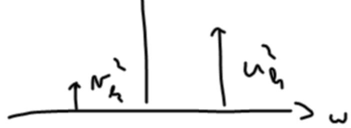
$$\rightsquigarrow \boxed{\gamma^+_{k\uparrow} = u_k c^+_{k\uparrow} + v_k c_{-k\downarrow}}$$

$$\gamma_{k\uparrow}|BCS\rangle = 0$$

$$H_{BCS}\left(\gamma^+_{k\uparrow}|BCS\rangle\right) = E_k\left(\gamma^+_{k\uparrow}|BCS\rangle\right)$$

$$T\,\mathcal{G}\,T^+ \longrightarrow G^{\text{diag.}}$$

$$\Rightarrow A(k,\omega)$$



$u_k v_k$

$$\langle BCS|\,c^+ c\,|BCS\rangle$$

$$u\gamma^+ - v\gamma$$

$$\rho_\uparrow(\omega) = \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2\pi}\,A(k,\omega)$$

$\omega > 0$

$$= N(E_F)\int d\xi \left(\frac{1}{2} + \cancelto{0}{\frac{\xi}{2E}}\right)\delta(\omega - E)$$

$$= \frac{1}{2}N(E_F)\int dE\,\frac{E}{\xi}\,\delta(\omega - E)$$

$\omega < \Delta$

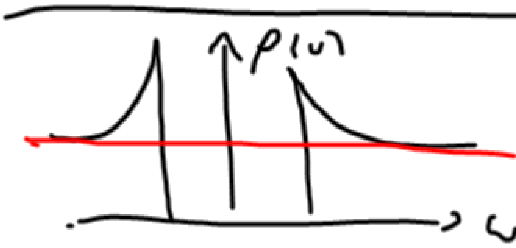
$$E^2 = \xi^2 + \Delta^2$$

$$E\,dE = \xi\,d\xi$$

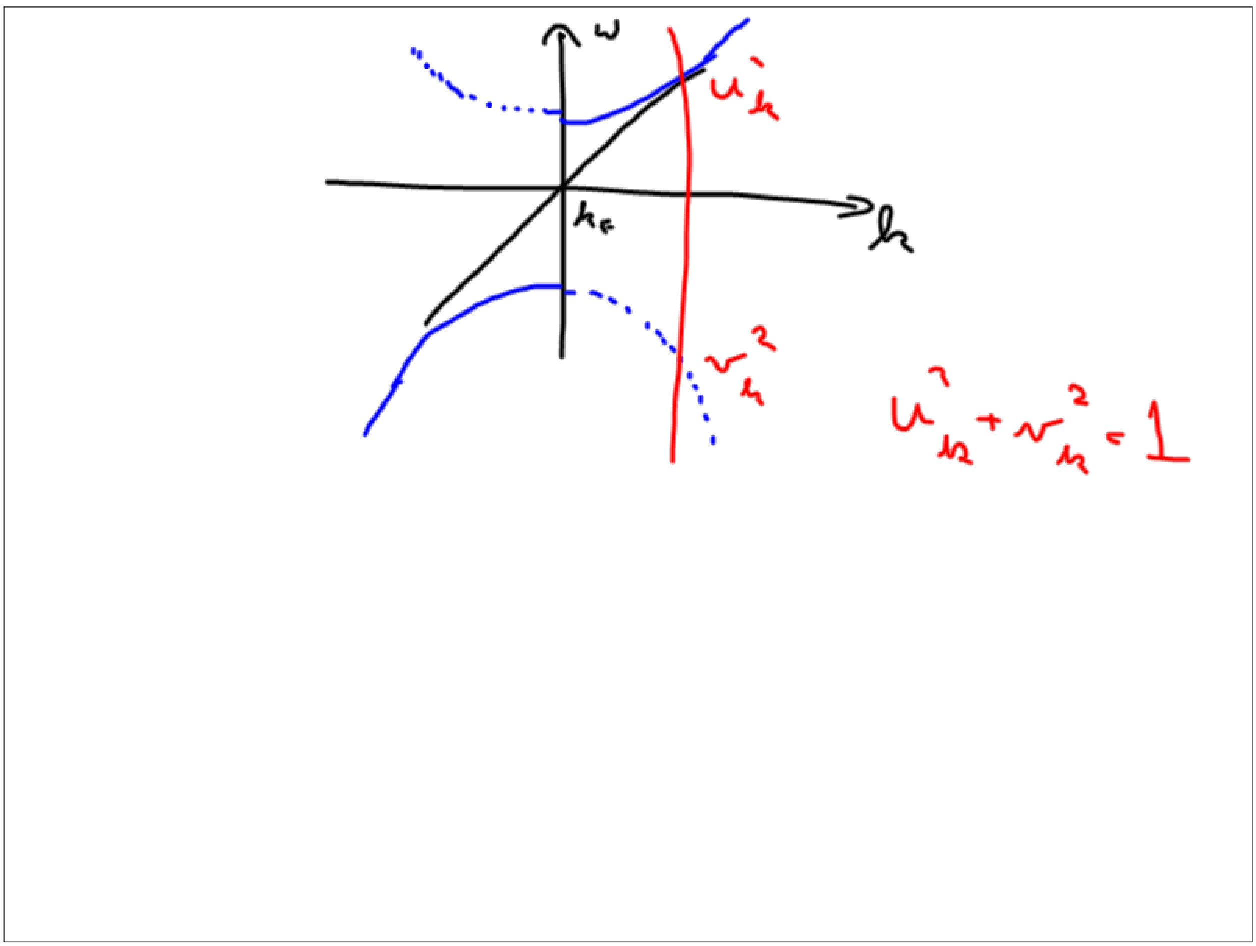
$$= \frac{1}{2}N(E_F)\int dE\,\frac{E}{\sqrt{E^2-\Delta^2}}\,\delta(\omega - E)$$

$\rho_\uparrow + \rho_\downarrow$

$$\boxed{\rho_\uparrow(\omega) = \frac{1}{2}N(E_F)\,\frac{\omega}{\sqrt{\omega^2-\Delta^2}}}$$



Giaver



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Phase quelconque

$$\cancel{\Delta} = \cancel{\Delta}\, V \int dE \, \frac{1-2f(E)}{2E} \, N(E_F)$$

$$\Delta = |\Delta| e^{i\varphi}$$

$$j \propto \Psi^+ \nabla \Psi$$

Désordre c^+_μ états propres

$$T c^+_\mu = \tilde{c}^+_\mu$$

$\uparrow$

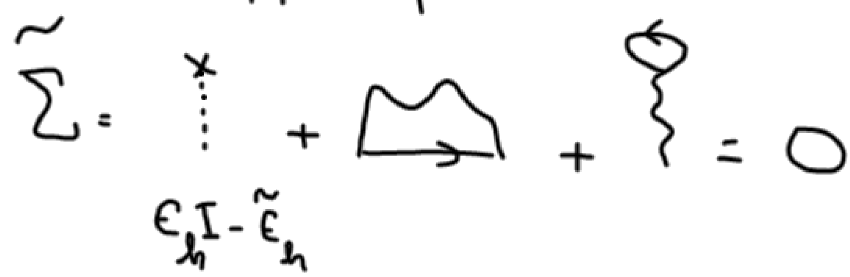
symétrie sous inv. du temps

Paires $\langle c^+_\mu \tilde{c}^+_\mu \rangle$

Autre façon d'obtenir BCS.

$$\tilde{H}_{int} = H_0 - \underbrace{\tilde{H}_0}_{BCS} + H_{int}$$

Milieu effectif.



Eliashberg.

$Z(\omega)$ $\quad$ $\chi(\omega)$ $\quad$ $\Delta(\omega)$

$$\Sigma(k) = \left[1 - Z(k)\right] ik_n I + \chi(k)\tau_3 + \Delta(k)\tau_1$$

$\vec{k}, ik_n$

Coulomb écranté $\qquad$ Phonon

$$\Sigma(k) = $$  $$+$$ 

contient Σ

$$\cancel{\frac{\partial \Sigma^R(\vec{k},\omega)}{\partial \xi_k}} \sim \frac{\Sigma^R}{E_F} \rightarrow 0$$

$$\frac{\partial \Sigma^R(\vec{k},\omega)}{\partial \omega} \sim \frac{\Sigma^R}{\omega_D}$$

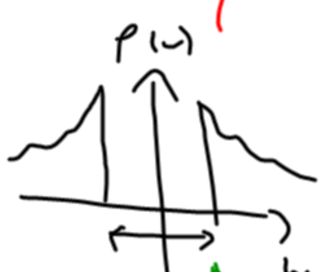
Migdal, i.e. pas de correction de vertex

$$\sqrt{\frac{m}{M}} \ll 1$$

$$\mathcal{G} = \frac{1}{ik_n - \xi_k\tau_3 - \Sigma}$$

$$\boxed{G^R = \frac{\omega Z + \xi_k\tau_3 - \tau_1\Delta(\omega)}{\left(\omega Z(\omega)\right)^2 - \left(\xi_k^2 + \Delta^2(\omega)\right)}}$$

$$\rho(\omega) \propto \frac{\omega}{\sqrt{\omega^2 - \frac{\Delta^2(\omega)}{Z(\omega)}}}$$



$$\omega = \frac{\Delta(\omega)}{Z(\omega)}$$

"Survol du cours"

Fct de corr. et exp.

$$\Gamma_{i\to f} = \frac{2\pi}{\hbar} \underset{\uparrow \atop \rho}{N_{fi}} \,|\, \underline{\delta(E_f - E_i - \hbar\omega)}$$

$$\frac{d\hat{\sigma}}{dE_f\, d\Omega_f} \propto \langle \rho_k(\omega)\, \rho_{-k}(\omega) \rangle$$

Rép. linéaire.

$$\delta H(t) = -\int d^3r\; \underset{\uparrow\ \text{Opér.}}{A_i(\vec{r})}\; \underset{\uparrow\ \text{Class.}}{a_i(\vec{r}, t)}$$

$$\delta \langle B(\vec{r},t)\rangle = \int_{-\infty}^{\infty} dt'\, d^3r'\, \chi^R_{BA_i}(r,t;r',t')\, a_i(r',t')$$

$$\chi^R_{BA} = \frac{i}{\hbar}\langle [B,A]\rangle\, \theta(t-t')$$

$$\rightarrow \quad \chi^R_{BA} = 2i\,\chi''_{BA_i}\,\theta(t-t')$$

$$\chi''_{A_iA_j}(\omega) = -\chi''_{A_iA_j}(-\omega)$$ in. du t hermitien

$$\chi' = P\int \frac{d\omega'}{\pi}\frac{\chi''(\omega')}{\omega'-\omega} \qquad \chi''(\omega) = -P\int \frac{\chi'(\omega')\,d\omega'}{\omega'-\omega}\,\frac{1}{\pi}$$

Dissipation positive: $\omega \chi'' \geq 0$

Fluc. Diss.

$$S_{A_i A_j}(\omega) = \frac{2}{1 - e^{-\beta\omega}} \chi''_{A_i A_j}(\omega)$$

$\langle |V(\omega)|^2 \rangle = 2 k_B T R$

$$S_{A_i A_j}(-\omega) = e^{-\beta\omega} S_{A_j A_i}(\omega)$$

Règles de somme:

T.D. $\frac{\partial n}{\partial \mu} = \int \frac{d\omega}{\pi} \frac{\chi''(\omega)}{\omega} = \beta \langle n n \rangle$

$$\int \frac{d\omega}{\pi} \omega^n \chi''_{A_i A_j}(\omega) = \left\langle \left[\left[\left[A_i, \frac{H}{\hbar} \right], \frac{H}{\hbar} \dots \right] A_j \right\rangle$$

f: $\int \frac{d\omega}{\pi} \omega \, \chi''_{nn} = \frac{n \cancel{e^2}}{m}$

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathrm{Re}\left[\sigma_{xx}(q_x, \omega)\right] = \frac{ne^2}{2m} = \frac{\omega_p^2}{8\pi}$$

$$\sigma \propto \frac{1}{\omega} jj \rightarrow \omega \frac{nn}{\cancel{q^2}}$$

$$qj = \omega n$$

$$\sigma_{yy}(q_x, \omega) = \frac{1}{i(\omega + i\eta)}\left[\chi^R_{j_y j_y}(q_x, \omega) - \frac{ne^2}{m}\right]$$

$$\frac{1}{\epsilon^L} = 1 - \frac{4\pi}{q^2} \chi^R_{\rho\rho}$$

$$\epsilon^T = \left(1 - \frac{\omega_p^2}{(\omega + i\eta)^2}\right) I + \frac{4\pi}{(\omega + i\eta)^2} \chi^R_{jj}$$

$$D = \pi \lim_{\omega \to 0} \left[\frac{ne^2}{m} - \mathrm{Re}\,\chi^R_{jj}\right]$$

	D	D_s
M	D	0
I	0	0
S	D	D_s

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Fct. de Green à 1 particule

$$\psi(r,t) = \langle r | e^{-iHt} | \psi_H \rangle$$

$$\psi(r,t)\,\theta(t-t') = i\int d^3r'\, G^R(r,t;r',t')\,\psi_0(r',t')$$

$$G(r,t;r',t') = -i \langle r | e^{-iH(t-t')} | r' \rangle\, \theta(t-t')$$

Règles de somme. $\hbar = 1$

Perturbations:

$$\frac{1}{x+y} = \frac{1}{x} - \frac{1}{x}\, y\, \frac{1}{x+y}$$

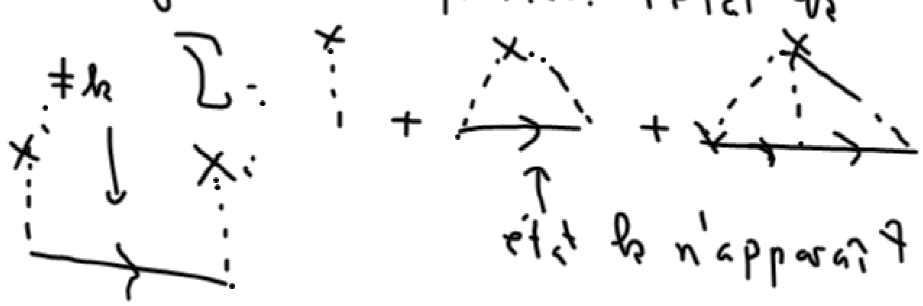
$$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 \dots$$

$$= G_0 + G_0 V G \quad \text{Lippmann-Schwinger}$$

$$= G_0 + G_0 \Sigma G \leftarrow$$

ne revient pas dans l'état k

Moyenne sur impuretés.



état k n'apparaît pas.

À T fini et N-corps

$$G^R(r,t;r',t') = -i\left\langle \left\{ \psi_H(r,t), \psi^+_H(r',t') \right\} \right\rangle \theta(t-t')$$

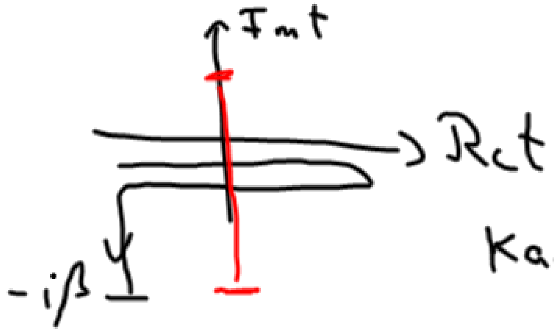
Repr. d'int:

$$\hat{U}(t,0) = e^{iH_0 t}\, U(t,0)$$

$$\uparrow$$

$$\psi_H(t)\,\psi^+_H(t') \rightarrow \hat{U}(0,t)\,\hat{\psi}(t)\,\hat{U}(t,t')\,\hat{\psi}^+(t')\,\hat{U}(t',0)$$

$$\hat{U}(t,t_0) = T_\tau\, e^{-i\int dt\, \hat{V}(t)}$$



Kadanoff-Baym

$$\mathcal{G}(r,r';\tau-\tau') = -\langle T_\tau \, \psi(r,\tau)\,\psi^\dagger(r';\tau')\rangle$$

$$-\beta \leq \tau \leq \beta$$

$$\psi(r,\tau) = e^{\tau K}\,\psi\, e^{-K\tau}$$

$$K = H - \mu N$$

$$\mathcal{G}(r,r';\tau) = \frac{1}{\beta}\sum_{n=-\infty}^{\infty} e^{-i\omega_n \tau}\,\mathcal{G}(r,r';i\omega_n)$$

$$\mathcal{G}(r,r';i\omega_n) = \int_0^\beta d\tau\, e^{i\omega_n \tau}\,\mathcal{G}(r,r';\tau)$$

$$\mathcal{G}(r,r';i\omega_n) = \int \frac{d\omega'}{2\pi}\,\frac{A(r,r';\omega')}{i\omega_n - \omega'}$$

Rep. de Lehmann

Si pas d'int :

$$\mathcal{G}^0(k,i\omega_n) = \frac{1}{i\omega_n - (\epsilon_k - \mu)} \leftarrow$$

Fct. retardé cas général.

$$G^R(p,\omega) = \frac{1}{\omega + i\eta - \xi_p - \Sigma^R(p,\omega)}$$

Quasiparticule :

$$(E_p - \mu) = \xi_p + \mathrm{Re}\,\Sigma^R(p, E_p - \mu)$$

$$Z(p) = \frac{1}{1 - \frac{\partial \mathrm{Re}\Sigma}{\partial \omega}} \longrightarrow G^R \sim \frac{Z}{\omega - (E_p - \mu)}$$

$$\frac{m}{m^*} = \lim_{F.S.} \frac{1 + \frac{\partial}{\partial \xi_p} \mathrm{Re}\Sigma}{1 - \frac{\partial}{\partial \omega} \mathrm{Re}\Sigma}$$

$+$ inc.

Théorie des perturbations

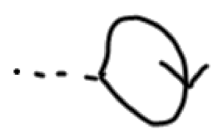
Wick

Graphs. connexes.

$$\frac{\langle e^{-f(x)} A(x) \rangle}{\langle e^{-f(x)} \rangle} = \langle e^{-f(x)} A \rangle_c$$

$$\ln \langle e^{-f(x)} \rangle = \langle e^{-f(x)} \rangle_c - 1$$

Règles de Feynmann

- Diag. top. diff.
- $\longrightarrow \mathcal{G}$ 
- Conserver k, ω aux vertex
- $V(k, \omega_n) = V(k)$ coulomb.
- $\displaystyle\int \frac{d^3k}{(2\pi)^3} \, T \sum_n$ $\forall k$ non fixés par cons.
- $\sum$ indices de spin
- $(-)^n (-)^F$ n = ordre, F = # boucles
-  $e^{i\omega_n \eta}$

Principe var. Feynmann

$$-T\ln Z \leq -T\ln Z_0 + \langle H - \tilde{H}_0 \rangle_0$$

Gaz d'électrons :

H.F. $$\tilde{\epsilon}_k = \epsilon_k - \int \frac{d^3k}{(2\pi)^3} \frac{4\pi e^2}{|k-k'|} f(\tilde{\epsilon}_k)$$

$\chi_{\rho\rho}$

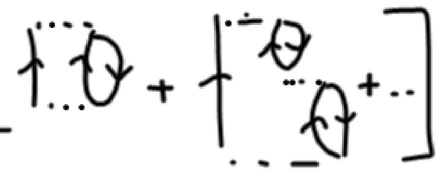
RPA

$$\boxed{\lim_{\omega \to 0} \frac{4\pi e^2}{q^2 + q_{TF}^2}}$$

V^{eff}

$$V^{eff} = \frac{V(q)}{1 - V(q)\,\Pi^{(1)}(q, iq_n)}$$

$$\int d\bar{1}\,\Sigma(1,\bar{1})\,\mathcal{G}(\bar{1},1^+) = 2\beta\langle V\rangle$$

$\mathcal{G}\Big[$  $\Big]$ $= \displaystyle\int \frac{d^3q}{(2\pi)^3} V_q \Big[T \sum_{iq_n} \chi_{nn}(q, iq_n) - n_0 \Big]$

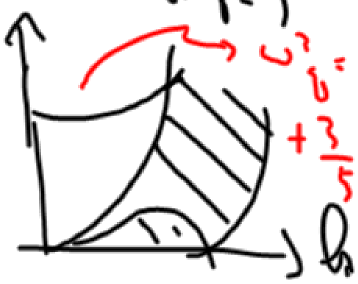
Ecrantage linéaire.

$$\epsilon_L^{RPA}(q, iq_n) = 1 - V(q)\,\Pi^{(1)}(q, iq_n)$$

a) Continuum p-t

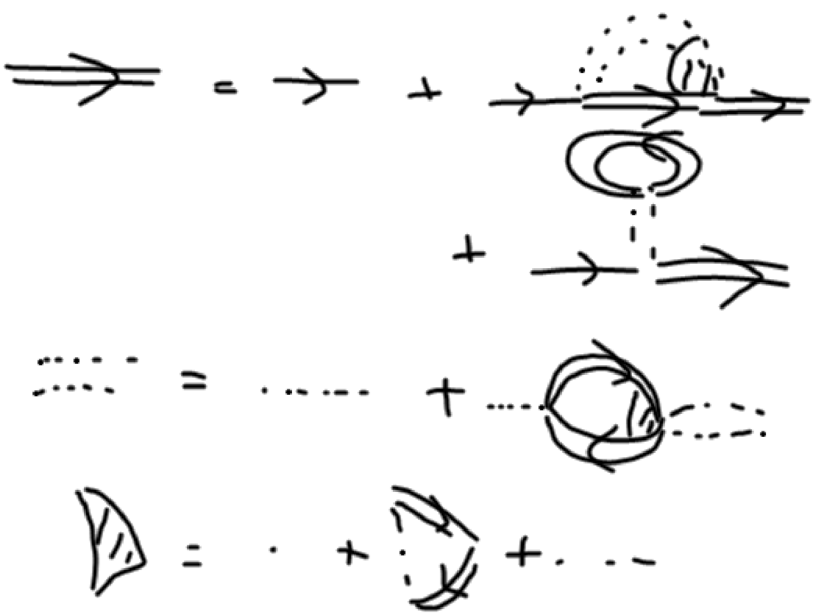
b) Ecrantage

c) Plasmon



$$\omega_q^2 = \omega_p^2 + \frac{3}{5} v_F^2 q^2$$

$$\Omega = -T \ln Z_0 + \frac{1}{2}\int \frac{d\lambda}{\lambda}\int d1\, d1' \, \Sigma_\lambda(1,1')\, G_\lambda(1',1^+)$$

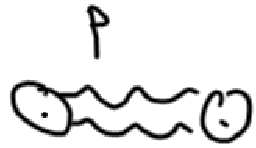


Phonons

$$\mathcal{D}(k,\tau) = -2\omega_k \langle T_\tau Q_k(\tau) Q_{-k}(0) \rangle$$

$$= \frac{-2\omega_k}{\omega_n^2 + \omega_k^2}$$

$$H = H_e + H_{ee} + \underbrace{H_i + H_{ii}} + \underbrace{H_{ei}}$$

Int. eff =  + 

vertex écranté

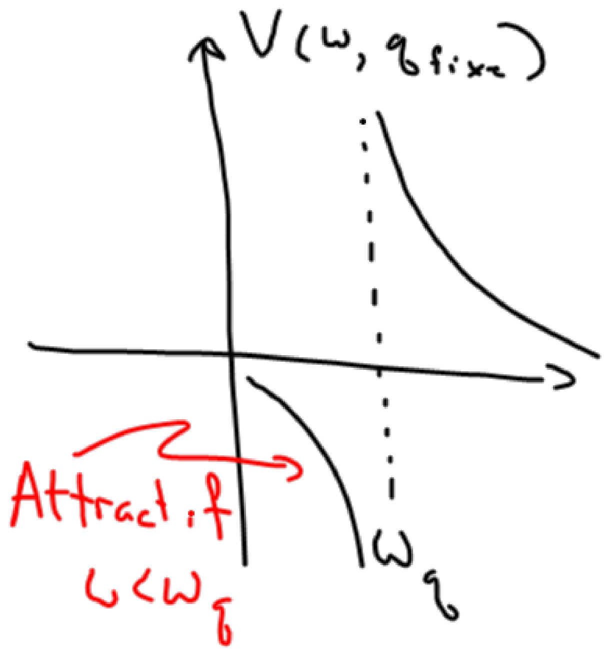
Coulomb écranté.

Phonon écranté

$$\omega_q^2 = \frac{\omega_{ip}^2}{\epsilon(q,0)} \longrightarrow \omega_q = c_s q$$

$$c_s = \frac{\omega_{ip}}{q_{TF}} = \left[\frac{Zm}{3M}\right]^{1/2} v_F$$

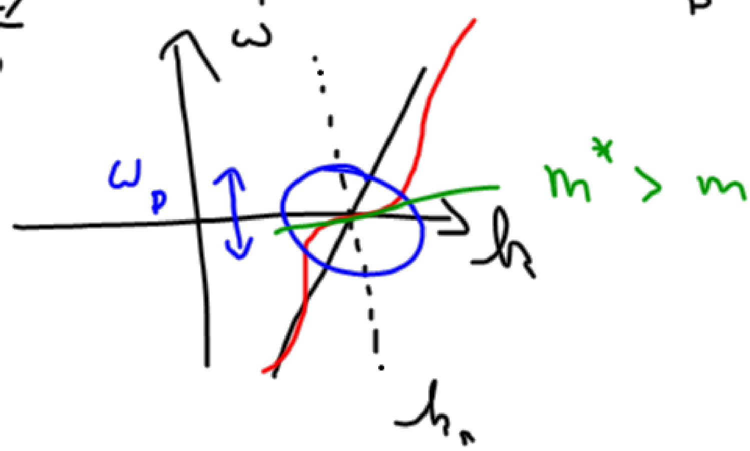
Bohm Staver.



$$\frac{m^*}{m} = 1 + \lambda = 1 - \frac{\partial \mathrm{Re}\,\Sigma}{\partial \omega}$$

$$\cancel{\left.\frac{\partial \Sigma}{\partial k}\right|_{k}} \sim \frac{\Sigma}{E_F} \qquad \frac{\partial \Sigma}{\partial \omega} \sim \frac{\Sigma}{\omega_D}$$

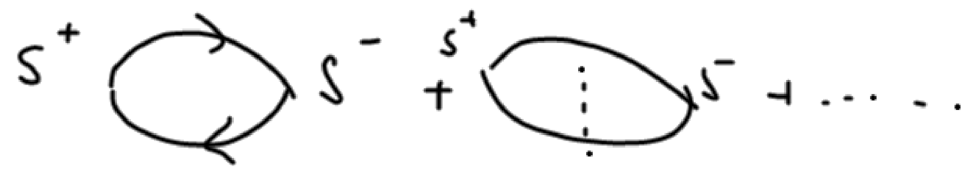
0



Magnétisme et trans. de phase

$$H = -t\sum_{\substack{\langle ij\rangle \\ \sigma}} \left(c^\dagger_{i\sigma} c_{j\sigma} + c.h.\right) + U\sum_i n_{i\uparrow} n_{i\downarrow}$$

Inst. de phase normale.

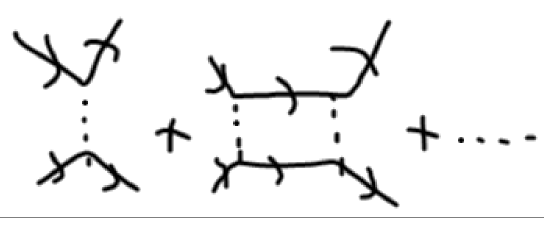


$$\chi_{sp} = \frac{\chi_0}{1-\frac{U}{2}\chi_0} = \infty$$

à $T = T_c$, U fixe

à $U = U_c$, $T = 0$

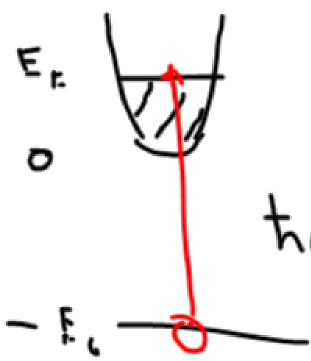
U renormalisé



Supra

à $T = T_c$

Critère de Thouless



$\hbar\omega = E_F + E_0$