

MECHANISM

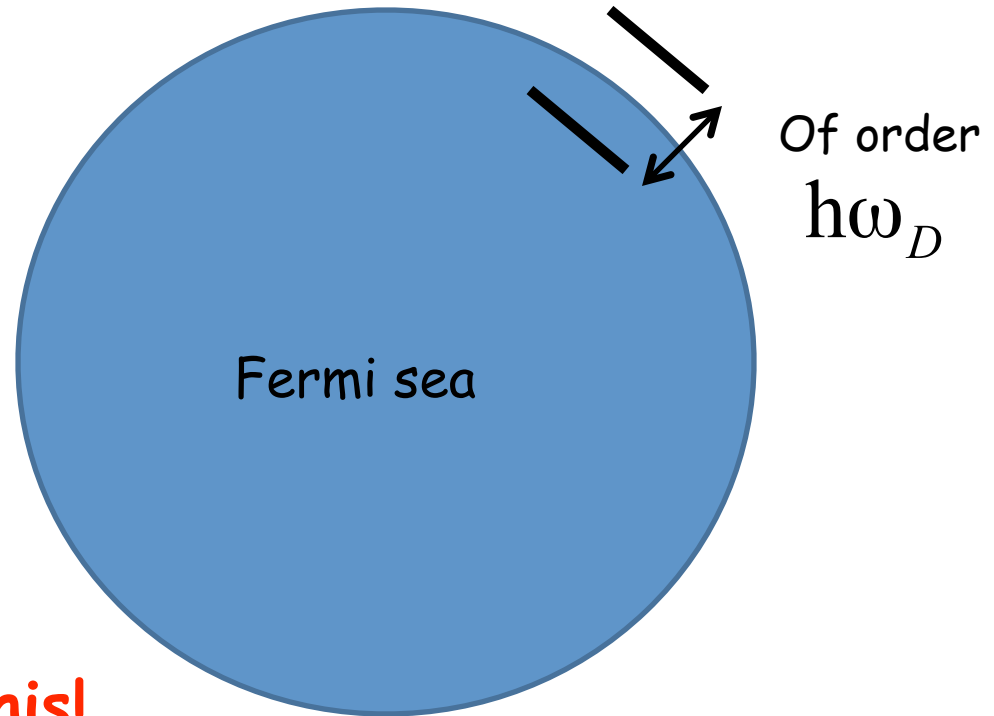
requires going beyond BCS theory to include inelastic scattering

In conventional superconductors we use Eliashberg theory to include the electron-

A serious limitation of BCS theory is that it is not good for relating the T_c value to microscopic parameters characterizing the normal state complexities of a metal.

$$T_c = 1.13\hbar\omega_D e^{-1/N(0)V}$$

**Just qualitative model
Not quantitative in its
conceptualization**



Need to go beyond this!

**Interaction beyond band structure
are introduced through a self energy**

**Think of electron as propagating in a
medium**

**consisting of all other electrons and
phonon**

**i.e. lattice vibrations AND in cuprates
spin fluctuations [magnetism is
around the corner]**

Electron acquires a lifetime

[imaginary part of self energy] and

Electron spectral density

$$A(\mathbf{k}, \omega) = \frac{1}{\pi} \frac{\Sigma''(\mathbf{k}, \omega)}{[\omega - \varepsilon_{\mathbf{k}} - \Sigma'(\mathbf{k}, \omega)]^2 + [\Sigma''(\mathbf{k}, \omega)]^2}$$

Real part self energy

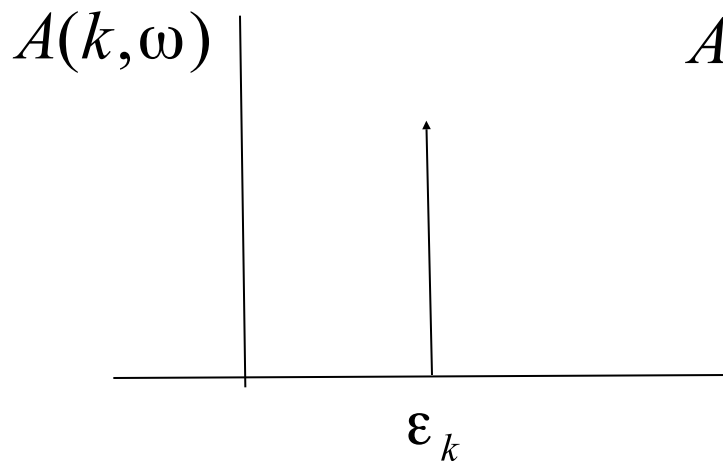
Imaginary part self energy

Self energy $\Sigma(\mathbf{k}, \omega)$ gives renormalizations

Electron Spectral Function $A(\mathbf{k}, \omega)$

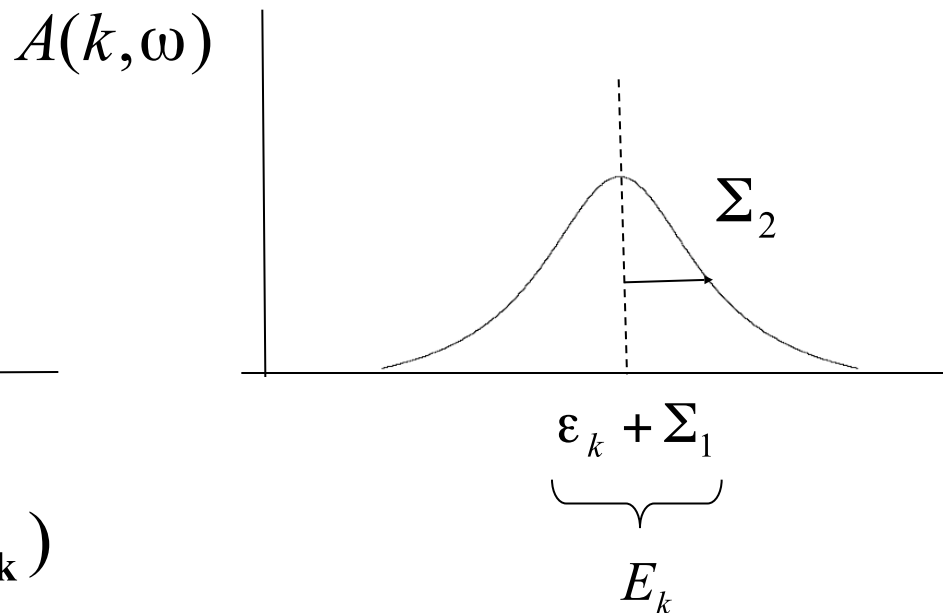
$A(\mathbf{k}, \omega)$ is measured in Angle-Resolved Photoemission Spectroscopy (ARPES)

$$A(\mathbf{k}, \omega) = \frac{1}{\pi} \frac{|\Sigma_2|}{(\omega - \epsilon_{\mathbf{k}} - \Sigma_1)^2 + \Sigma_2^2}$$



$$A(\mathbf{k}, \omega) = \delta(\omega - \epsilon_{\mathbf{k}})$$

Non-interacting case



Interacting case

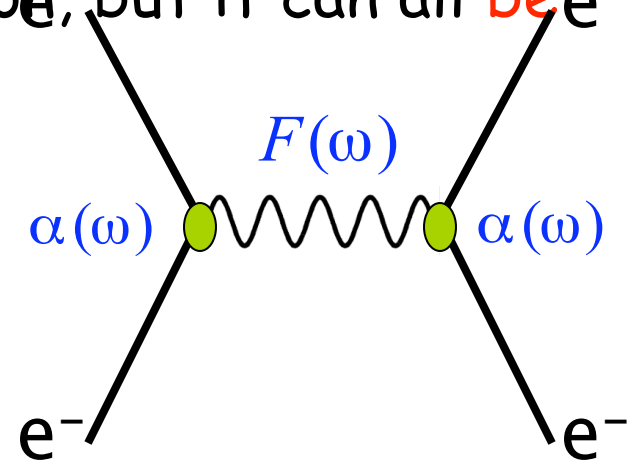
To describe the electron-phonon interaction as it enters Eliashberg theory, in principle we need:

- 1] phonons - frequencies and polarization vectors
- 2] electronic band structure and wavefunctions
- 3] coupling between electron and an ion displaced from equilibrium

This is a lot of detailed information, but it can all be condensed into a single function:

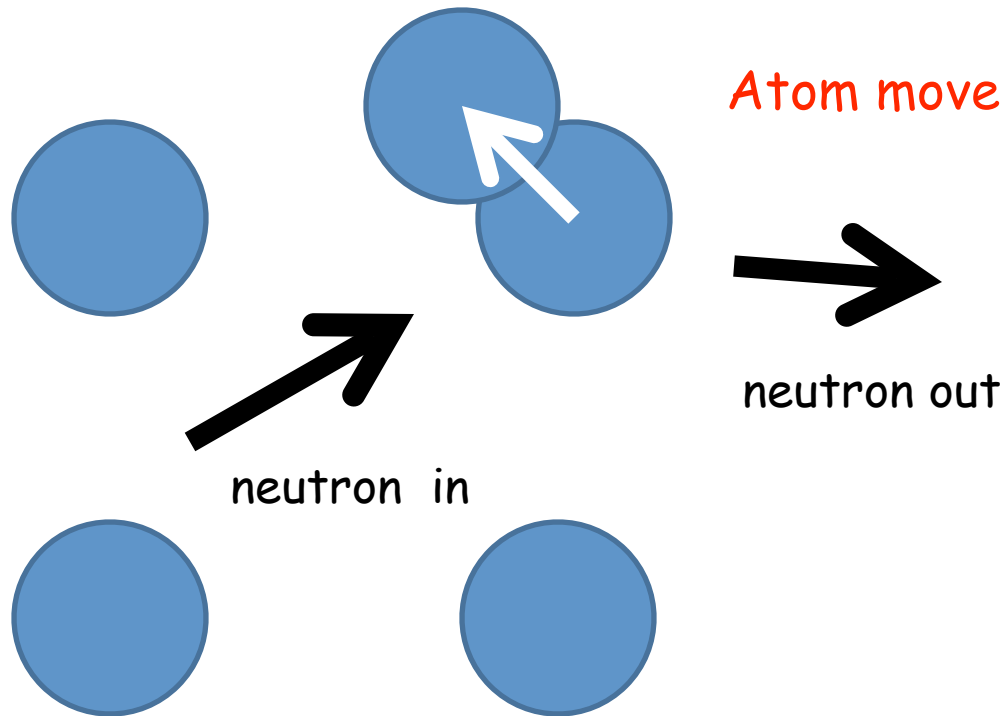
$$\alpha^2(\omega) F(\omega) \equiv \alpha^2 F(\omega)$$

Analogous to the phonon frequency distribution $F(\omega)$ or the electronic density of states $N(\epsilon)$.



Phonons: dispersion $\omega(k)$, frequency distribution $F(\omega)$

Lattice vibrations or phonons are measured by inelastic neutron scattering



Phonon is created:
Energy and momentum
lost by neutron.
Goes into the phonon

Neutron groups

definite momentum transfer to phonon analyzed as a function of energy transfer to phonon

Gives $\omega(q)$ relationship

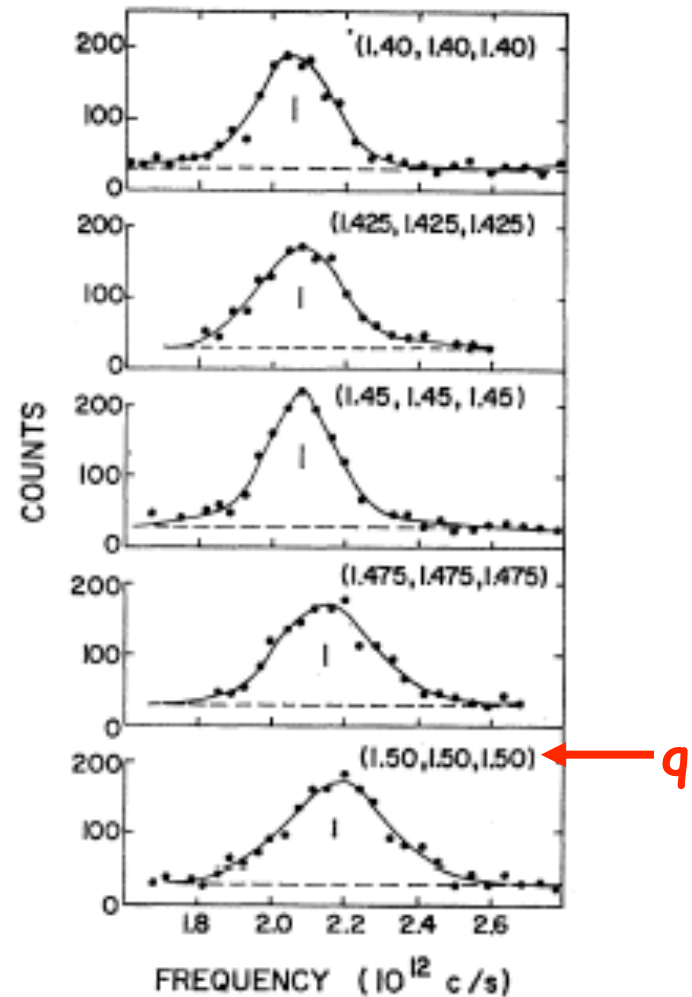
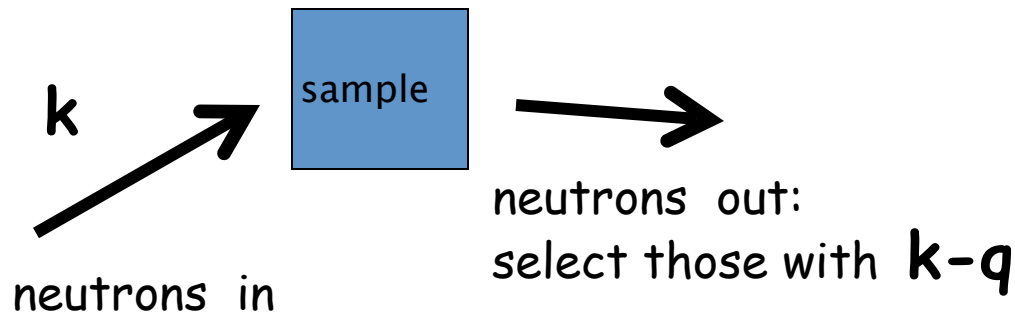


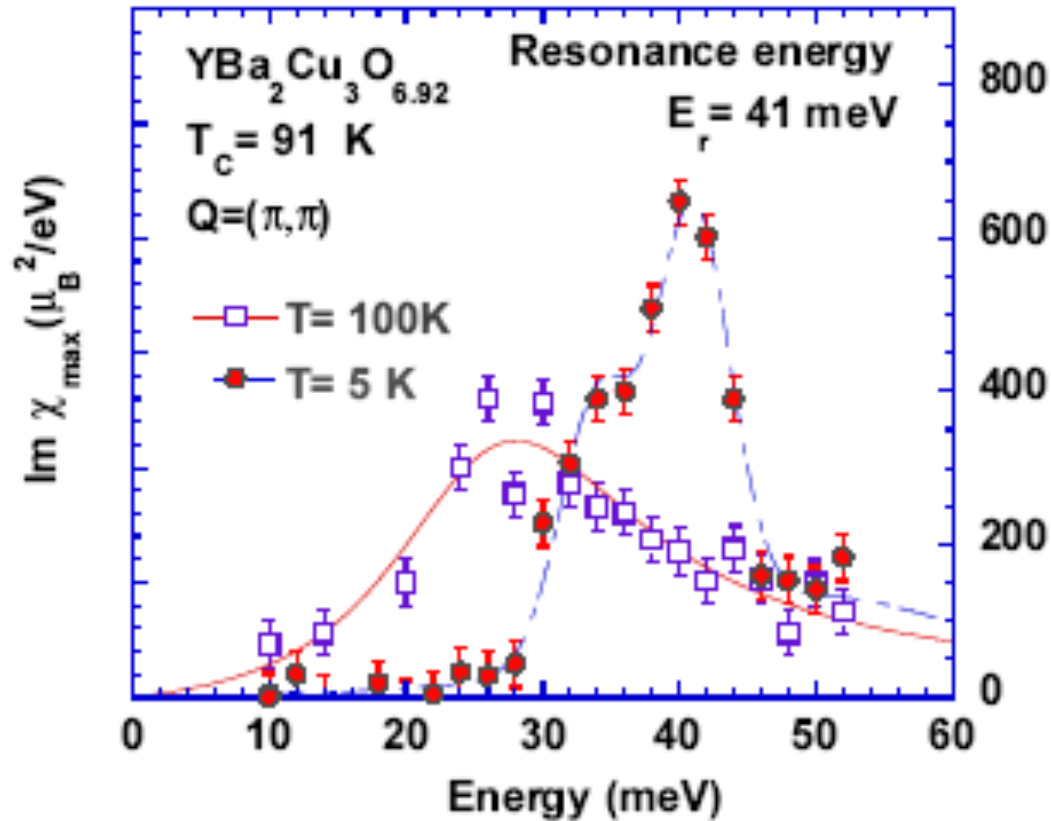
Fig. 4.10. A set of 'constant q ' scans in Pb taken at various points along the diagonal in the Brillouin zone. Reproduced from Ref. [89]

B. Brockhouse et al., Phys. Rev. 128, 1099 (1962).

For the high Tc oxides **spin polarized** inelastic neutron scattering [monitor spin flip] can be used to get the magnetic excitations
spin waves or spin fluctuations[damping]

See a very famous 41mev resonance
In YBCO ,different energies in other materials

Famous 41 meV spin one resonance



YBCO

Bourges et. al. see later for reference

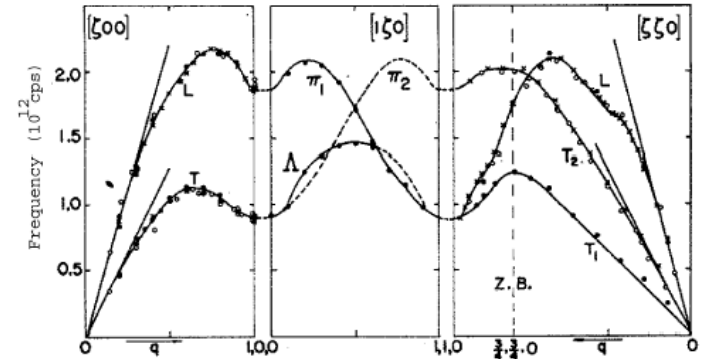
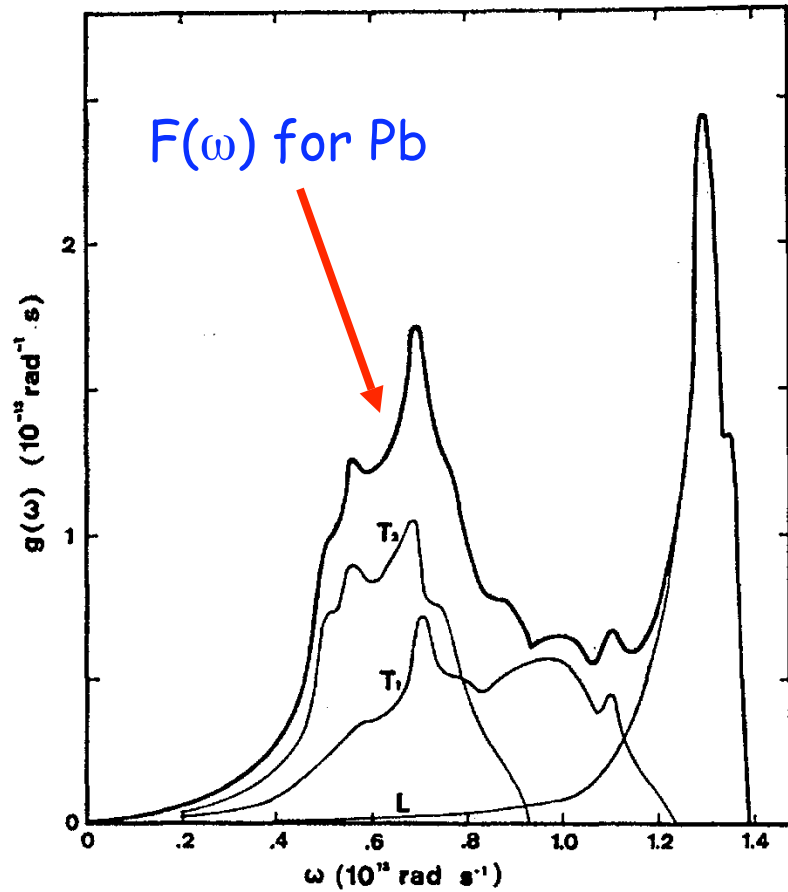
Phonon frequency distribution

$$F(\omega) = \frac{1}{N} \sum_{\mathbf{k}j} \delta(\omega - \omega_j(\mathbf{k}))$$

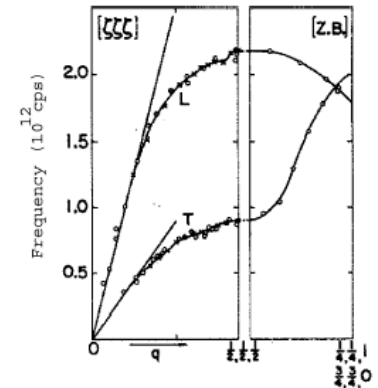
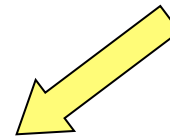
N is number of points in FBZ; j is phonon branch index

Note that frequency distribution is a single function while dispersion curves are many frames, one for each direction in FBZ, yet for the lattice specific heat knowing $F(\omega)$ is enough!! - much, much simpler

For simple metal, one atom per unit cell $F(\omega)$ is normalized to 3



$\omega(\mathbf{k})$



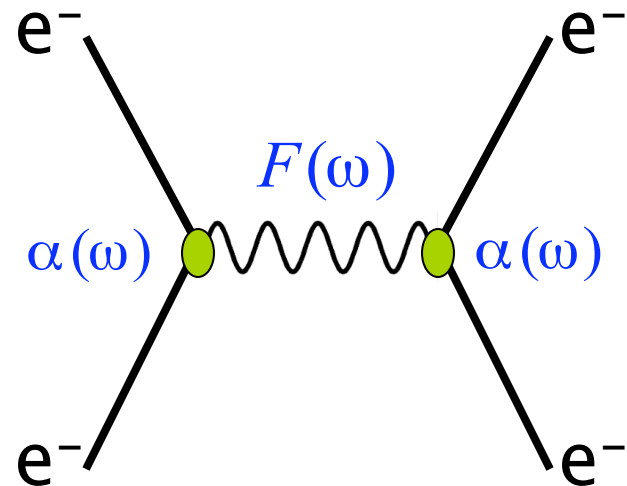
$$F(\omega) = \frac{1}{N} \sum_{\mathbf{k}_j} \delta(\omega - \omega_j(\mathbf{k}))$$

Stedman et al., Phys. Rev. **162**, 549 (1967).

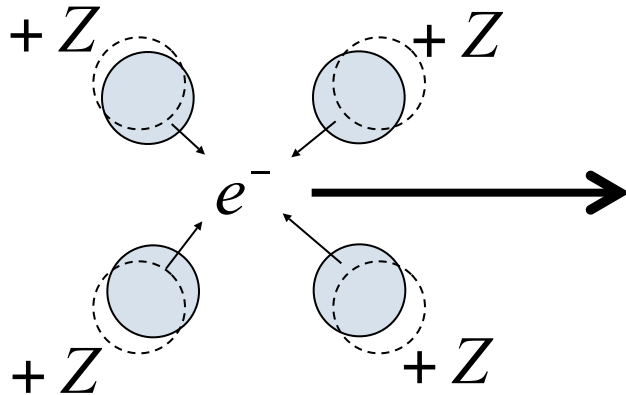
If you image these through the electrons
use
weighted distribution with electron-
phonon
weighting

$$\alpha^2(\omega)F(\omega) \equiv \alpha^2 F(\omega)$$

Analogous to the phonon frequency
distribution $F(\omega)$ or the electronic
density of states $N(\epsilon)$.



Furthermore,



Renormalization due to electron-phonon interaction can change dispersion curve (ie. energy-momentum relationship ϵ_k).

Electron Self-Energy

$$\Sigma(\vec{k}, \omega) = \text{[Diagram of a self-energy loop: a horizontal line with a jagged loop above it.]}$$

$$\text{[Diagram of a self-energy loop]} = \text{[Diagram of a self-energy loop]} + \text{[Diagram of a self-energy loop with a smaller self-energy loop inside]} + \text{[Diagram of a self-energy loop with two smaller self-energy loops inside]} + \text{[Diagram of a self-energy loop with three smaller self-energy loops inside]} + \dots$$

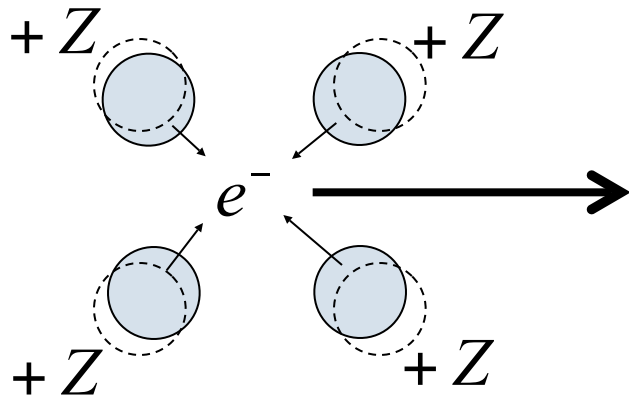
Medium changes electron energy dispersion curve and gives a finite lifetime to be in a particular k state, ie, $\epsilon \rightarrow \epsilon + i\Gamma$

$$\Sigma(\omega) = \int_0^{\infty} d\Omega \alpha^2 F(\Omega) \ln \left| \frac{\Omega - \omega}{\Omega + \omega} \right| - i\pi \int_0^{|\omega|} d\Omega \alpha^2 F(\Omega)$$

Example of how $\alpha^2 F(\omega)$ determines properties

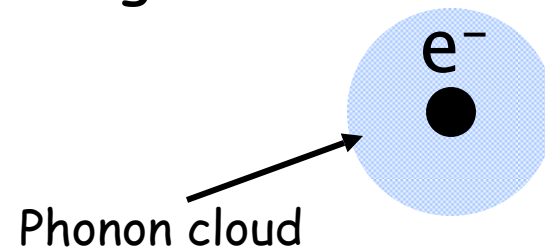
λ the electron-phonon mass renormalization

Electron pulls ions off equilibrium positions



Heavier by $[1+\lambda]$
i.e. $m^* = m[1+\lambda]$

Electron moves with polarization cloud of lattice charge



Electron spectral density for noninteracting case

$$A(\mathbf{k}, \omega) = \delta(\omega - \varepsilon_{\mathbf{k}})$$

Want to include lifetimes, change to Lorentzian with width Γ , also change energy to $E_{\mathbf{k}}$

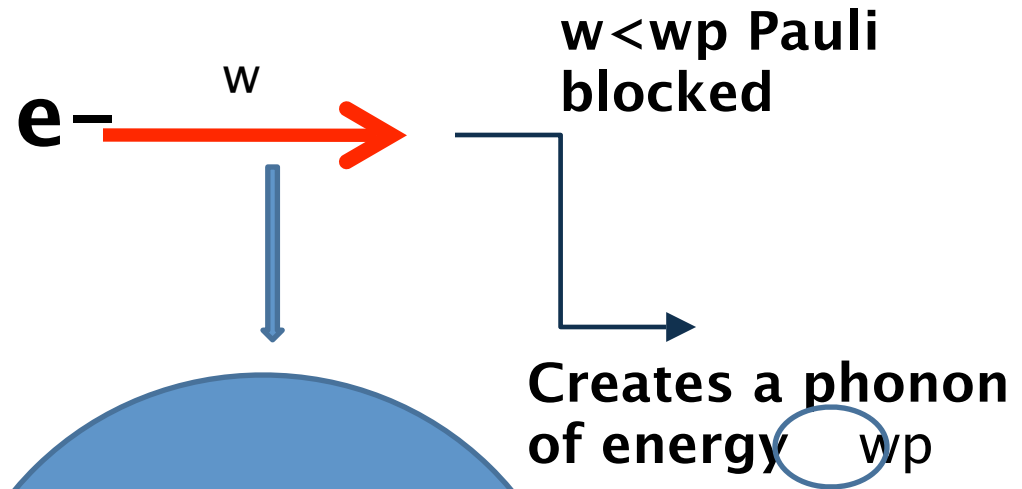
$$A(\mathbf{k}, \omega) = \frac{1}{\pi} \frac{\Gamma}{(\omega - E_{\mathbf{k}})^2 + \Gamma^2}$$

New dispersion

Broadening

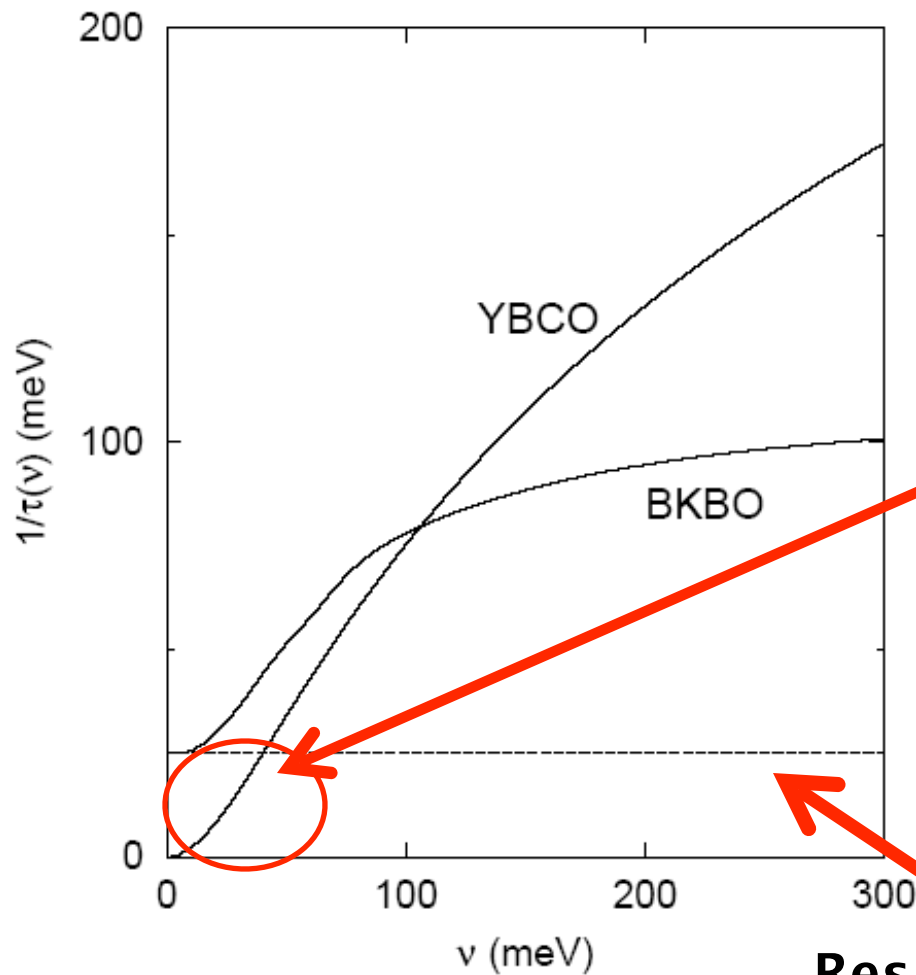
$$E_{\mathbf{k}} = \varepsilon_{\mathbf{k}} + \text{Re}[\Sigma(E_{\mathbf{k}})]$$

Quasiparticle scattering rate due to phonon creation, asks how long an electron stays in state $|k\rangle$

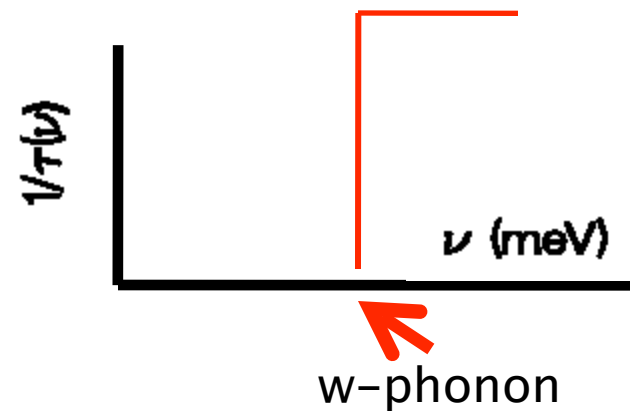


Can only scatter creating a phonon if its energy above the Fermi energy is larger than the phonon energy ω_p

Scattering rate due to phonon is zero till energy ' ω_p ' at which point it increases out of zero



Profile here reflects phonon frequency distribution



Residual elastic scatt

Scattering rate is zero till excitation energy of electron is large enough to create a phonon at which point it rises out of zero.

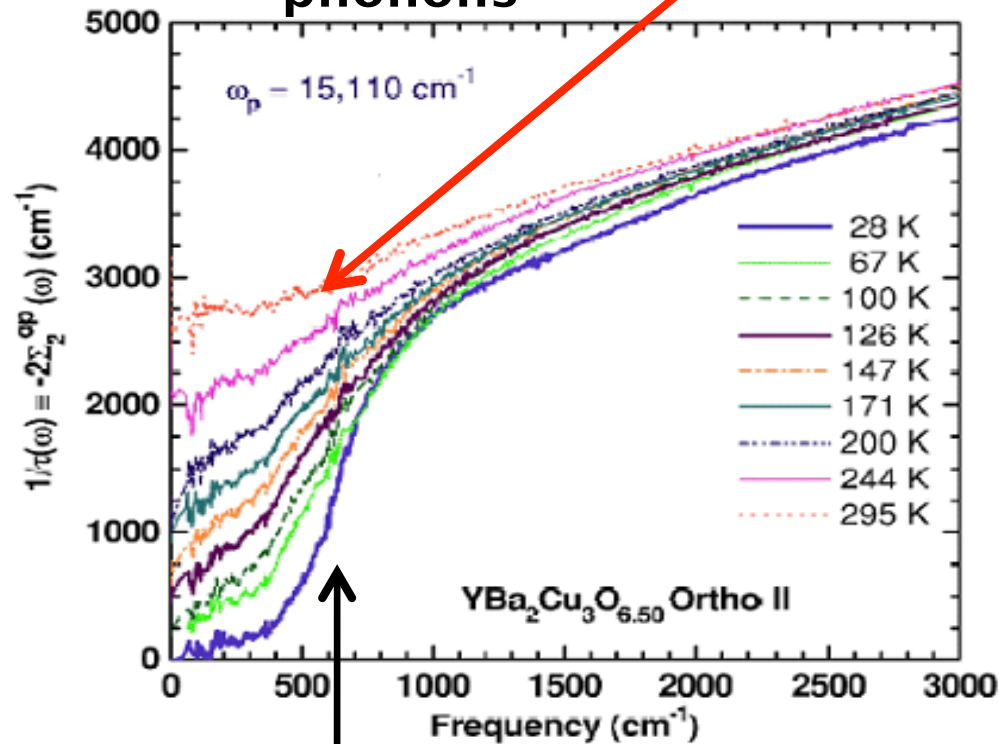
Rise reflects the available phonon frequency distribution $F(\omega)$

The real thing

a-axis

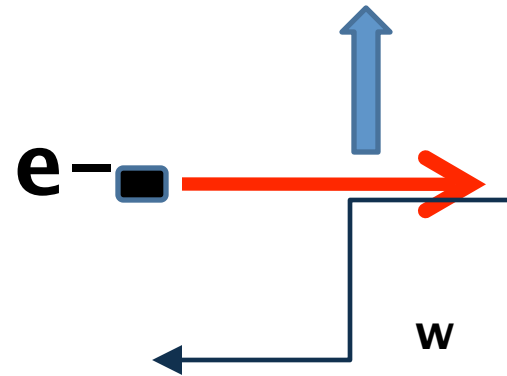
$T_c = 59\text{K}$

Blurred with increasing temp . New scattering processes become possible because of thermal population of real phonons



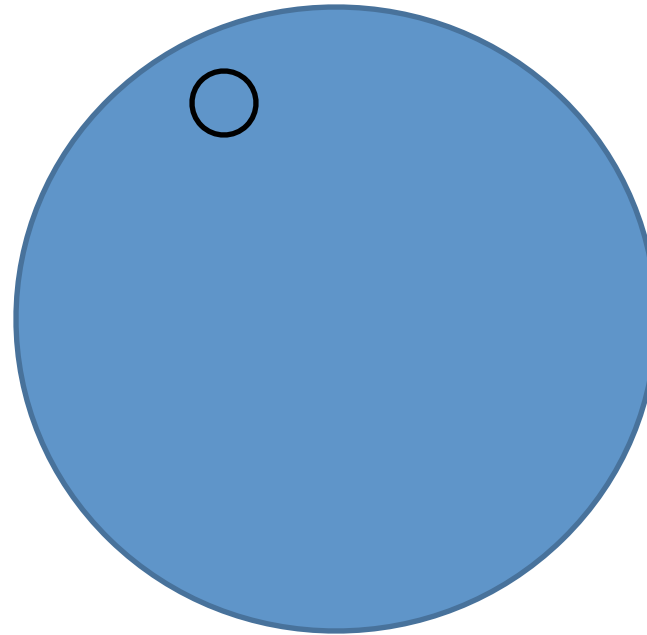
The mode in Y123 Ortho-II

Quasiparticle scattering rate due to absorption of a real thermally created phonon

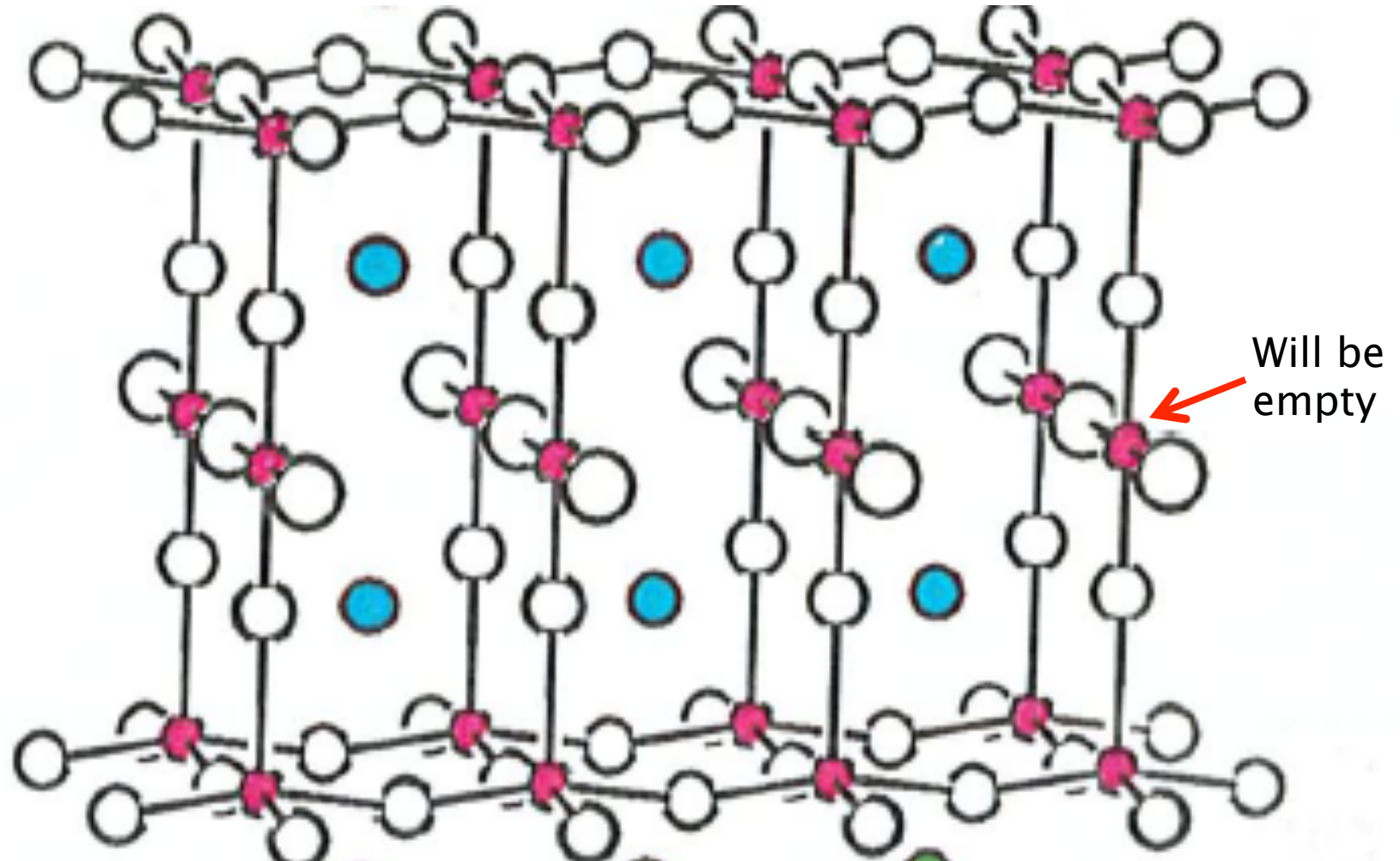


Thermally excited Phonon w is absorbed by electron. Empty states are always available for such scattering

No threshold for such scattering with real phonons



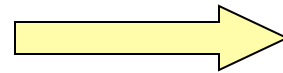
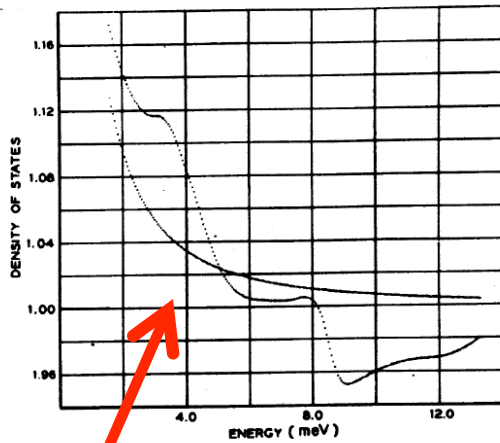
Ortho II, alternating full and empty Cu-O chain



YBaCuO

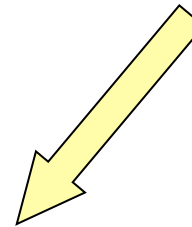
123

Can invert I-V characteristics to get electron-phonon spectral density through Eliashberg equations



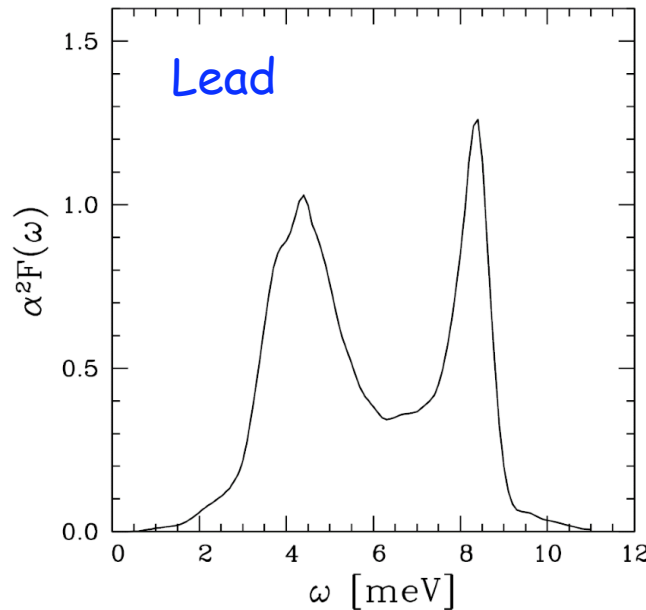
$$dI/dV = N_S(\omega)$$

$$N_S(\omega) = G [\alpha^2 F(\omega); \mu^*]$$



McMillan and Rowell, Phys. Rev. Lett. 14, 108 (1965)

Details matter!
Image of phonon spectra



Electron-phonon spectral density in Pb

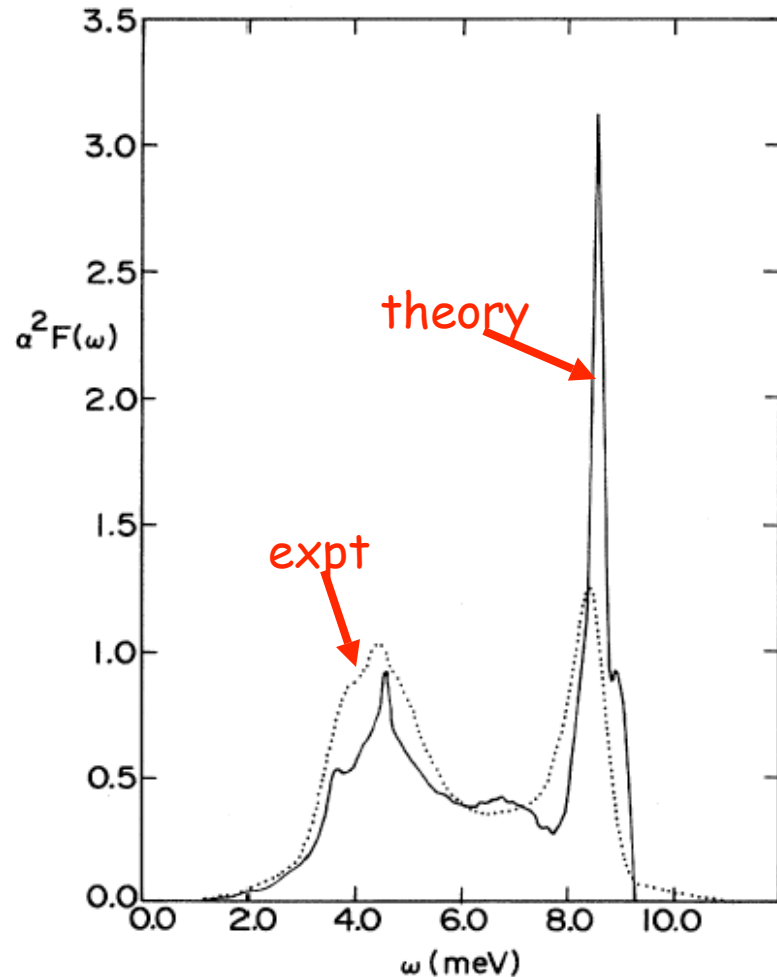
Tunneling:

McMillan and Rowell, Phys. Rev. Lett. **14**, 108 (1965).

Calculations:

Tomlinson and Carbotte, Phys. Rev. B **13**, 4738 (1976).

This function contains all of the complicated information on electron and phonon dynamics and the coupling between them that enters superconductivity.



Remarkable condensation of required information!

Inversion of optical data

How to get information on mechanism from optics

Can get distribution of bosons that scatter electrons from an analyses of scattering rates

What is recovered is a weighted boson distribution i.e. weighted by appropriate electron -boson coupling

In conventional superconductors, tunneling has been method of choice to get information on gap and phonons

Optics has been hard, good metals reflectance near 1

In poor metals such as oxides, optics has been great!

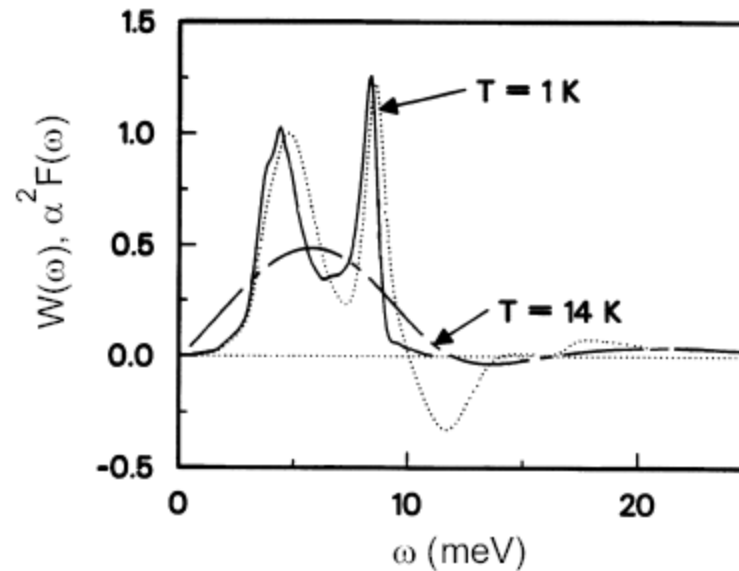
INVERSION

$$\alpha^2 F(\nu) \sim W(\omega) = \frac{1}{2\pi} \frac{\omega_p^2}{4\pi} \frac{d^2}{d\nu^2} \left\{ \nu \operatorname{Re} \frac{1}{\sigma(\nu)} \right\}$$

- Calculation of conductivity from Pb
- Electron-phonon spectral density and differentiation to get it back

Normal state only

Solid curve is input and others are output at two temperatures

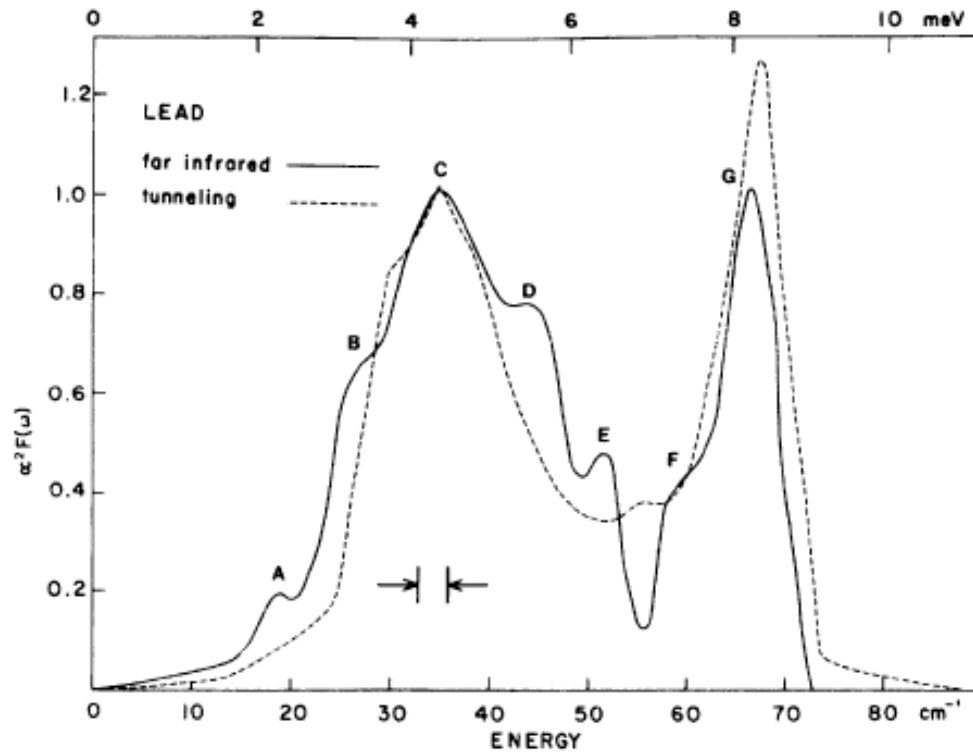


In theory!

F. Marsiglio, T. Startseva and J.P. Carbotte, Phys. Lett. A 245, 172 (1998)

Electron-phonon spectral density from optics

Old data for Pb
good agreement
with tunneling



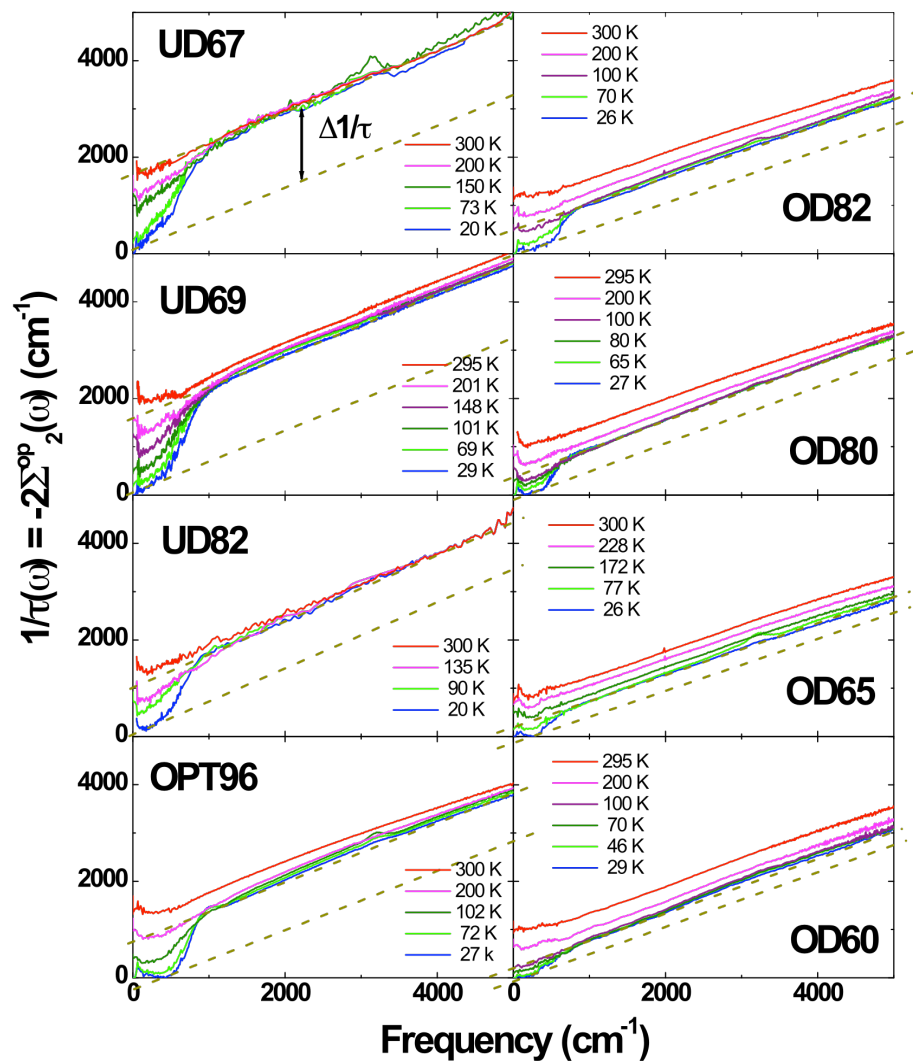
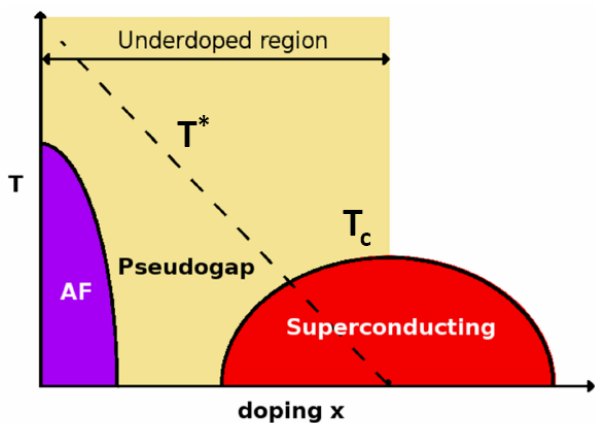
It works beautifully!

B. Farnworth and T. Timusk, *Phys. Rev. B* **10**, 2799 (1974);
ibid **14**, 5119 (1976).

Hwang et. al. [Timusk-group] Nature,427,714 [2004]

Optical scattering rate in Bi2212 across the doping range

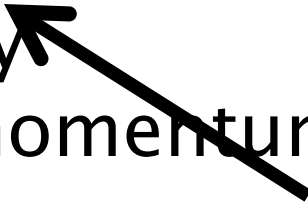
from underdoped to



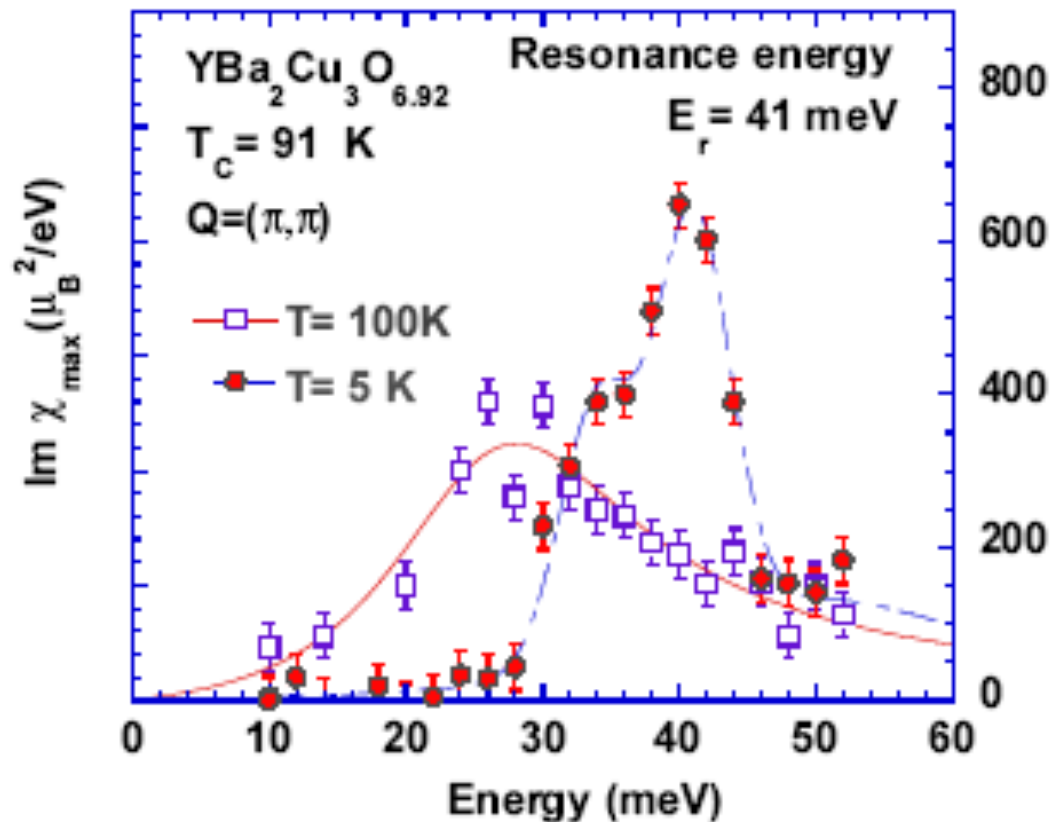
What should we see if its spin fluctuation exchange ?

Expect to see a picture of local spin susceptibility i e BZ average

In cuprates see a broad background and sometimes a prominent spin one resonance in magnetic susceptibility at specific momentum $[\pi, \pi]$ **great ,easier to see**

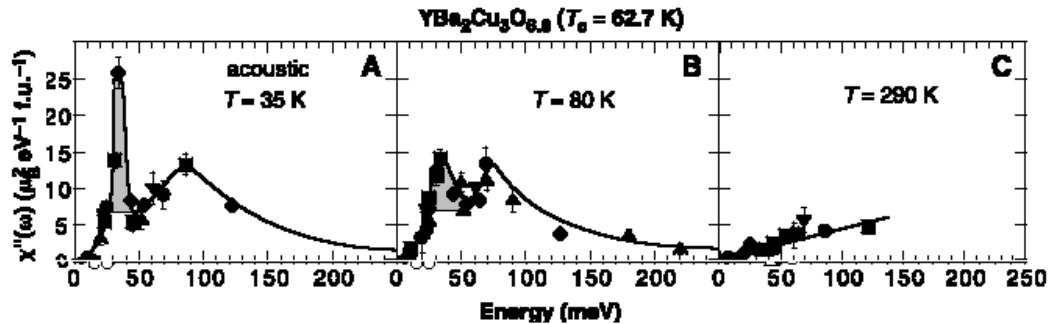


Famous 41 meV spin one resonance



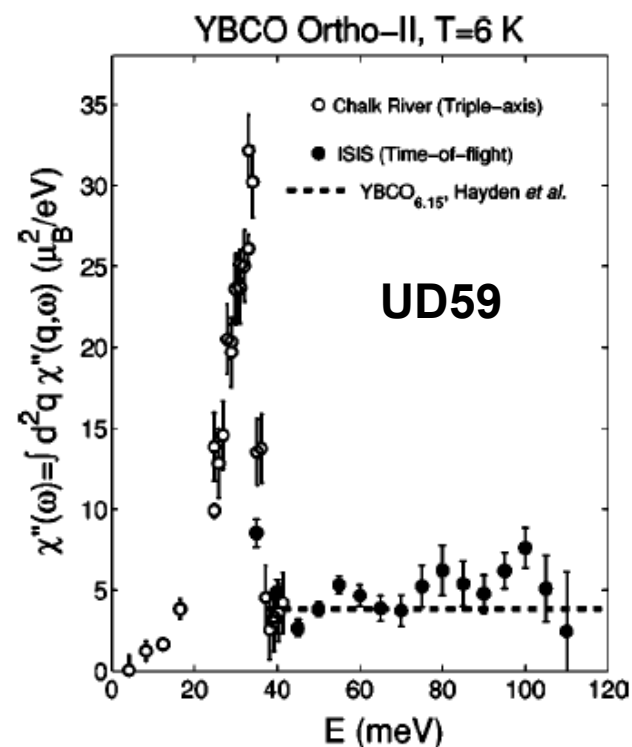
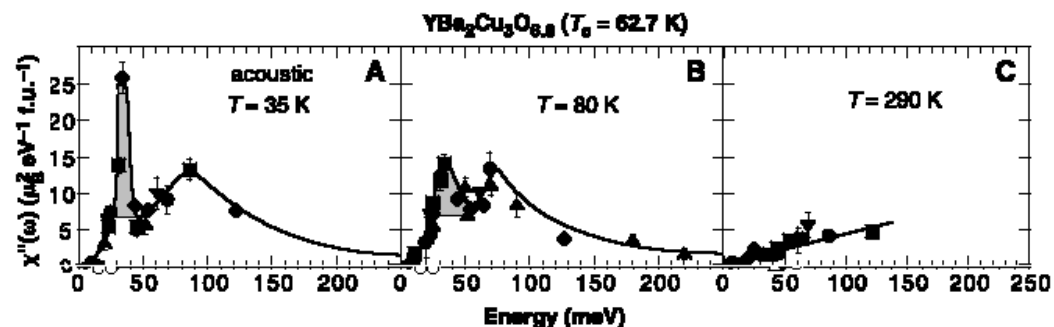
Ph. Bourges, Y. Sidis, H.F. Fong, B. Keimer, L.P. Regnault, J. Bossy, A.S. Ivanov, D.L. Lilius, and I.A. Aksay, in High Temperature Superconductivity, eds. S.E. Barnes, *et al.*, CP483 (American Institute of Physics, Amsterdam, 1999), p. 207.

The neutron resonance mode



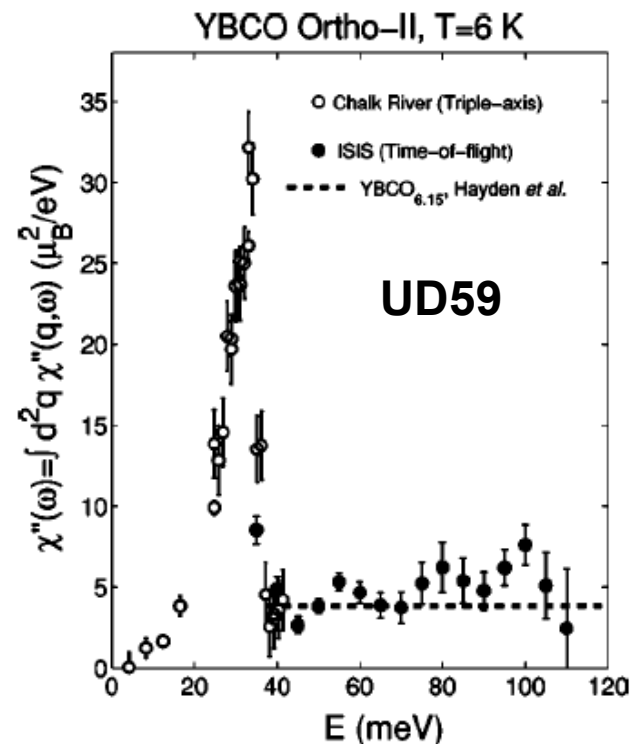
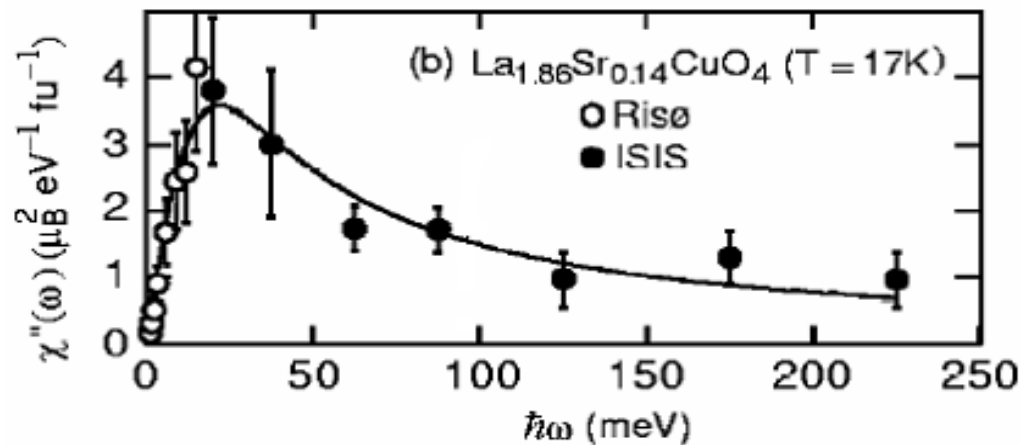
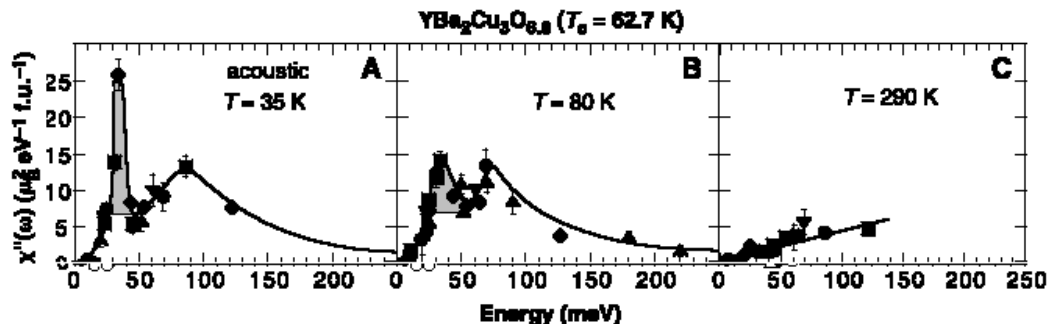
Hayden et. Al. PRL 76,1344 [1996]
Stock et.al. PRB 71,024522 [2005]
Bourges et.al. PRB 53,876 [1996]
and other Bourges references

The neutron resonance mode



Hayden *et al.* PRL 76,1344 [1996]
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and other Bourges references

The neutron resonance mode

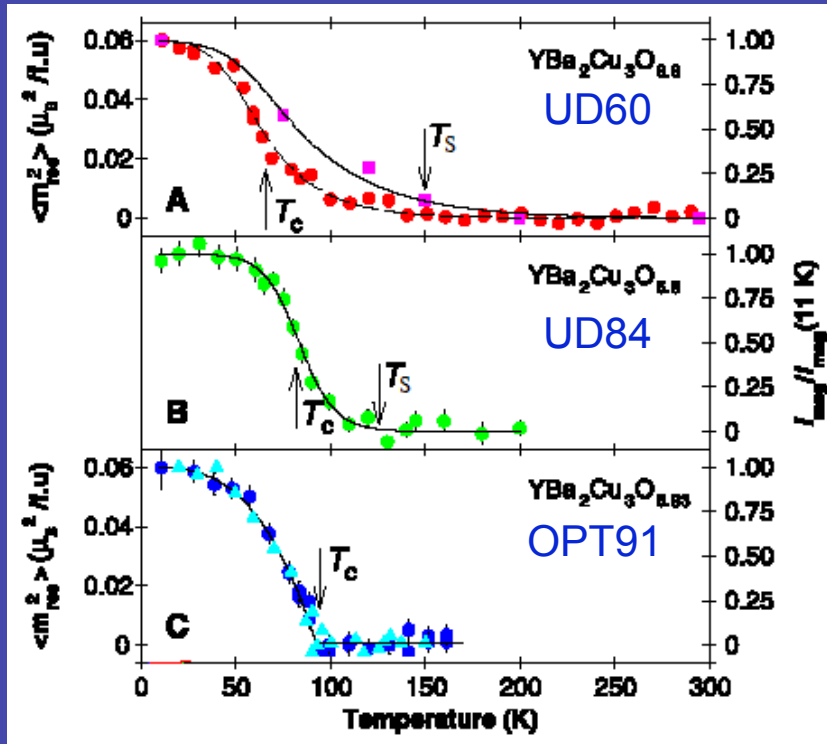


Hayden et. Al. PRL 76,1344 [1996]
 Stock et.al. PRB 71,024522 [2005]
 Bourges et.al. PRB 53,876 [1996]
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Temperature and doping dependence ...

Temperature

Y123

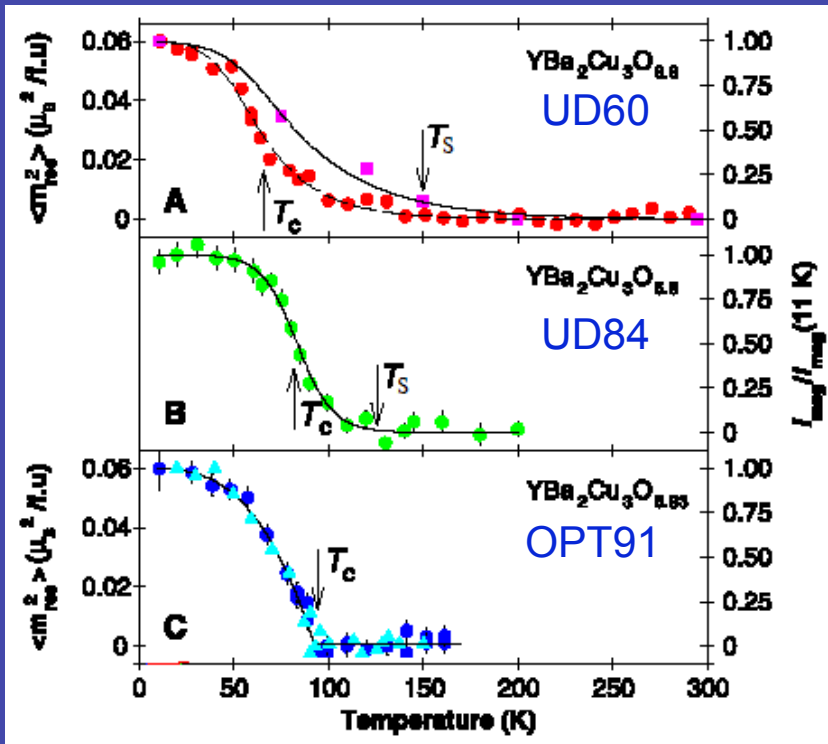


Dai *et al.*, Science 284, 1344 (1999)

Temperature and doping dependence ...

Temperature

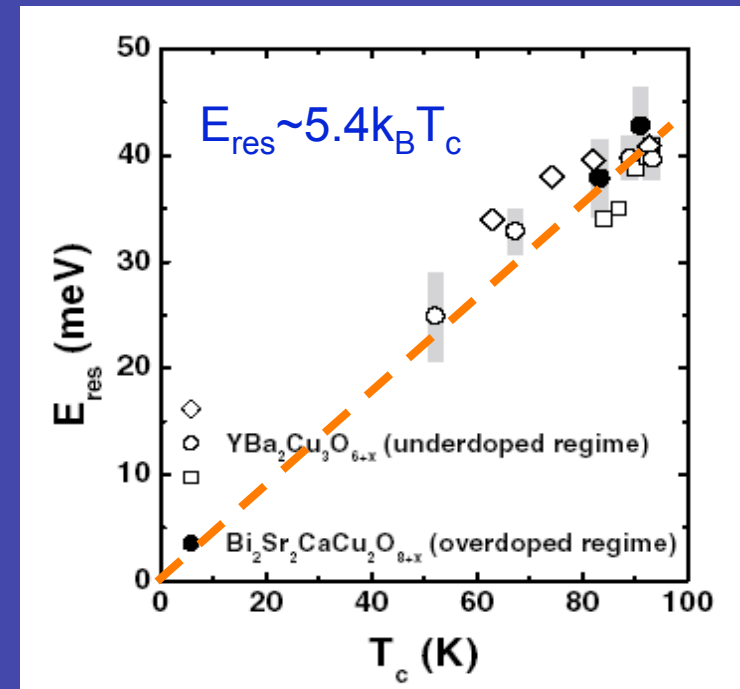
Y123



Dai *et al.*, Science 284, 1344 (1999)

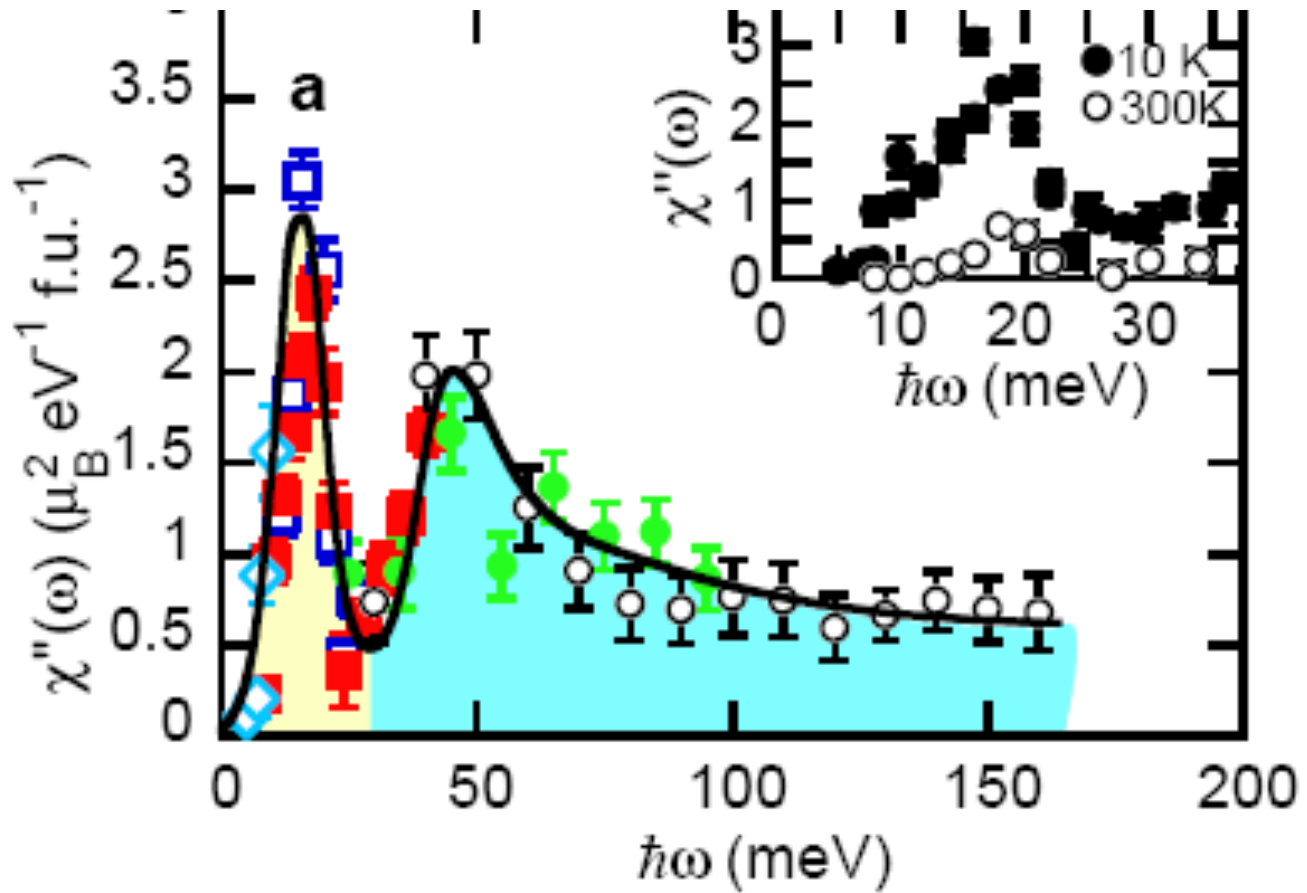
Doping

Y123 & Bi-2212



He *et al.*, PRL 86, 1610 (2001)

Two energy scales in spin susceptibility of La_{1.64}Sr_{0.16}CuO₄



What does optics see?

Phonons

Spin fluctuations

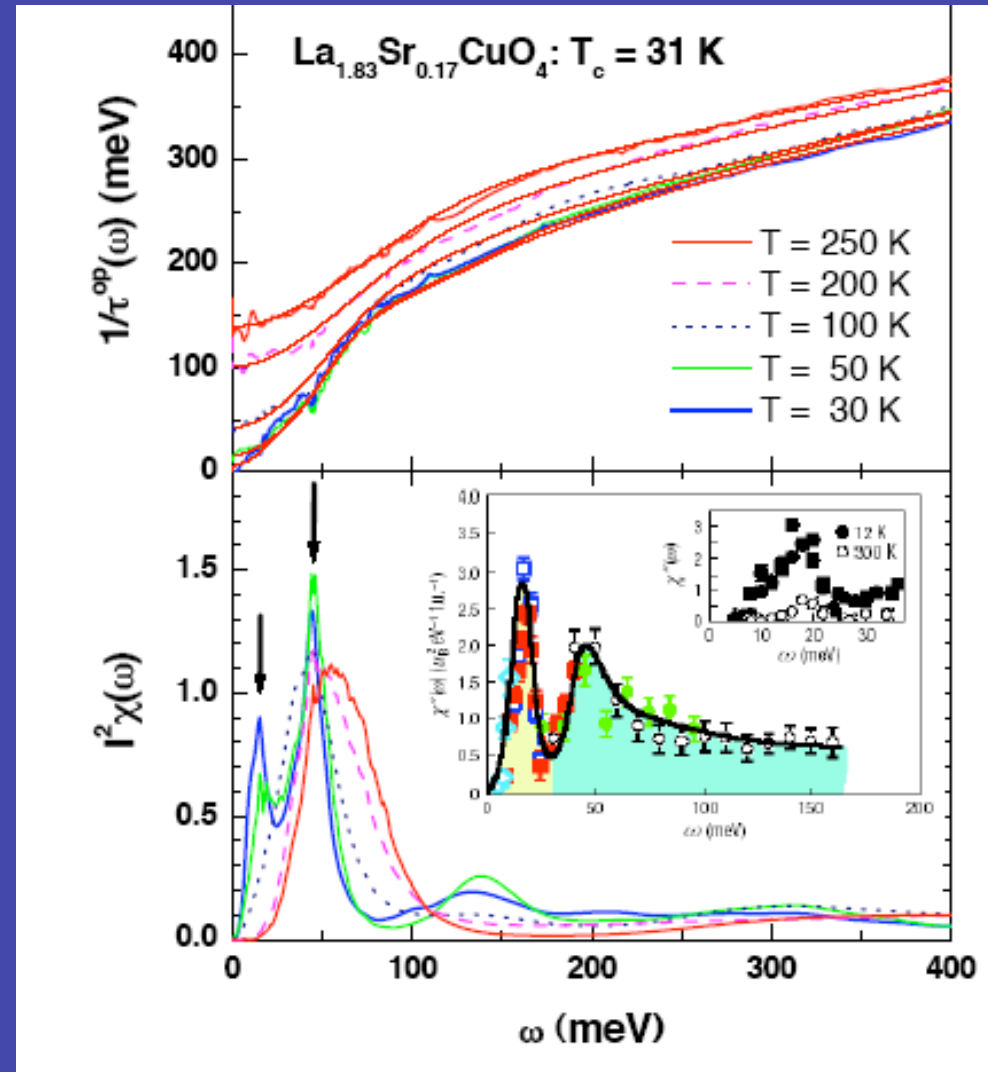
Both? [most likely case]?

Listen carefully and tell me
what you think

Inversion in LSCO

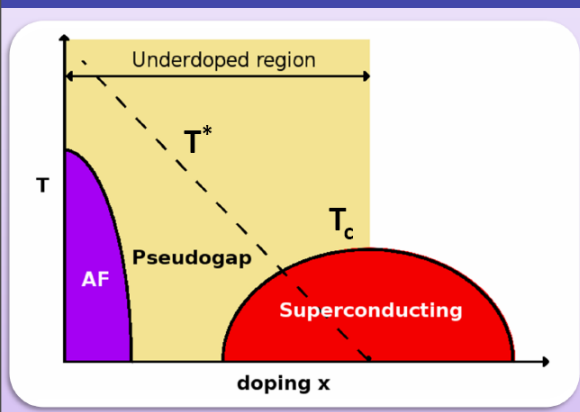
Get two peak structure at low T
And evolution to single peak at room temp
Good agreement with neutrons

Looks real good !

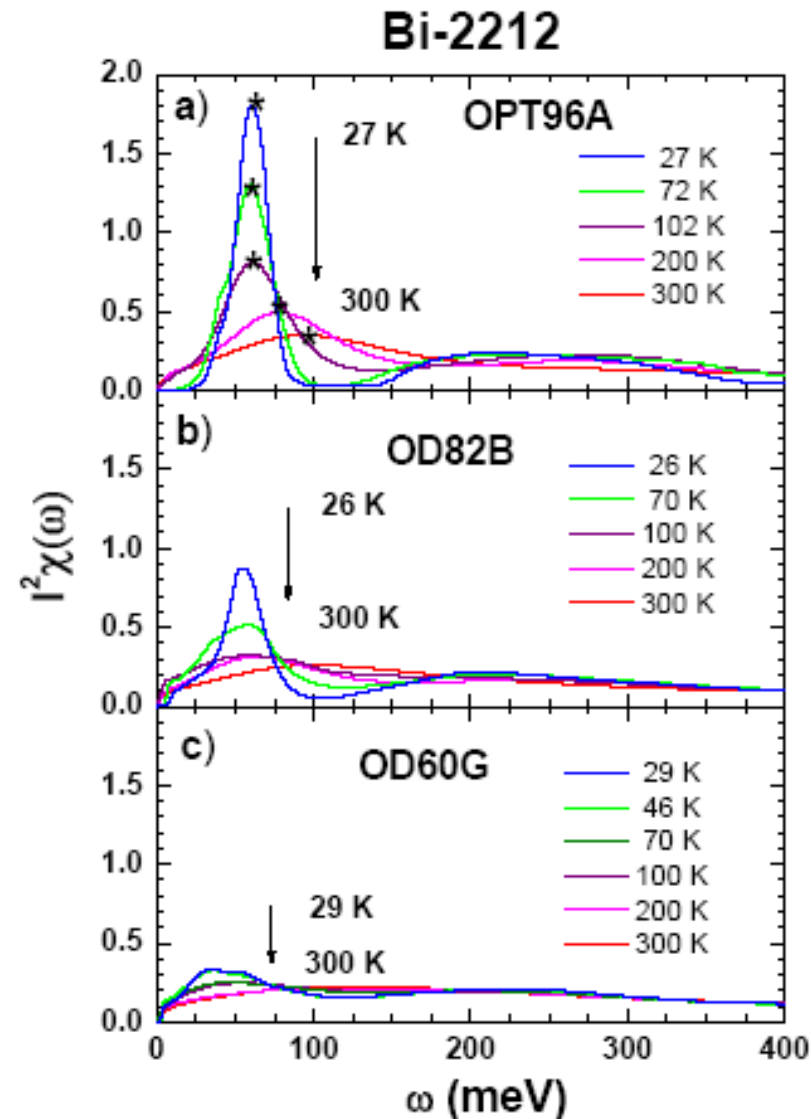


Temperature and Doping evolution of Electron-Boson Spectral Function

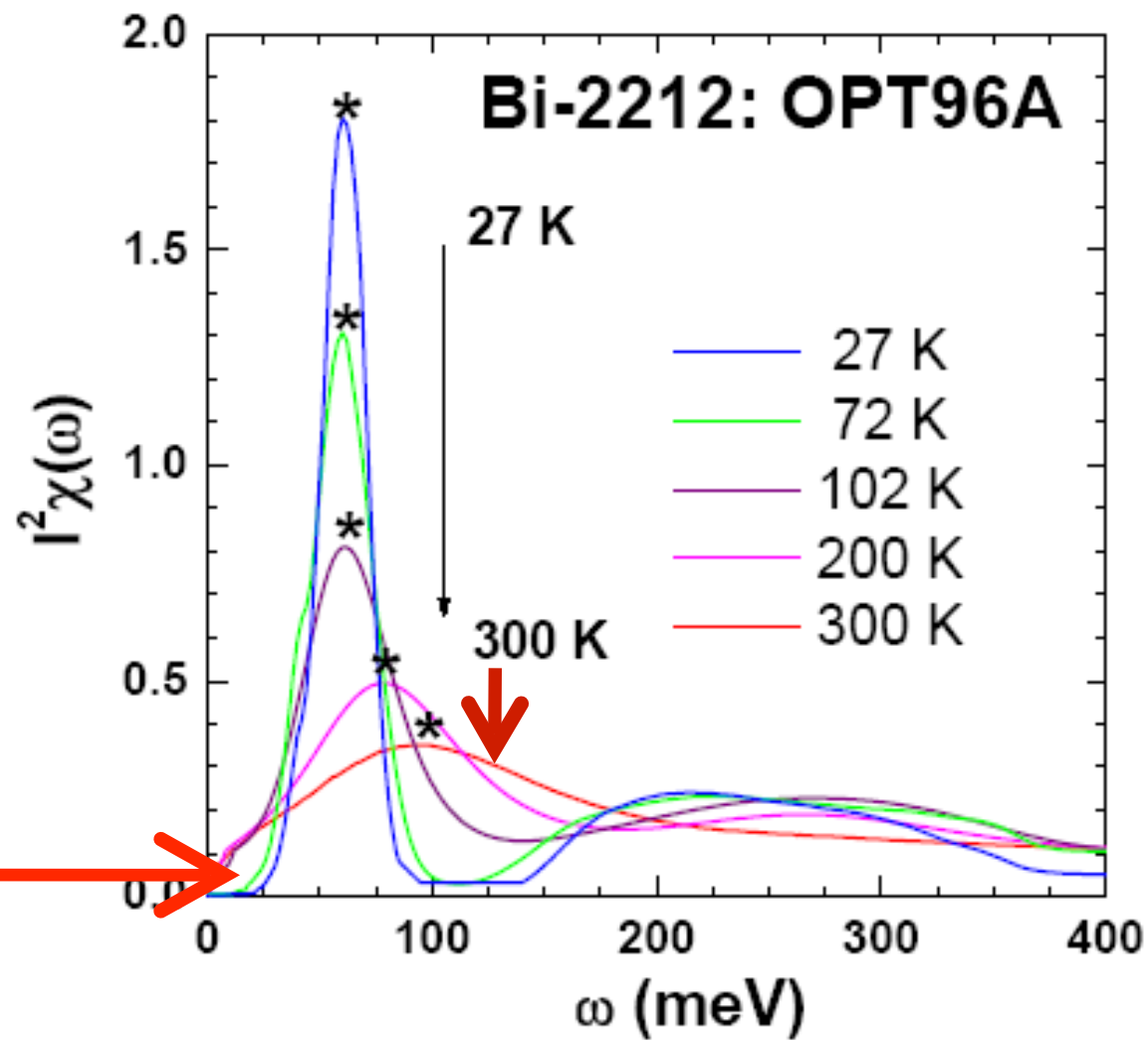
Sharp peak goes away with increasing temperature and doping



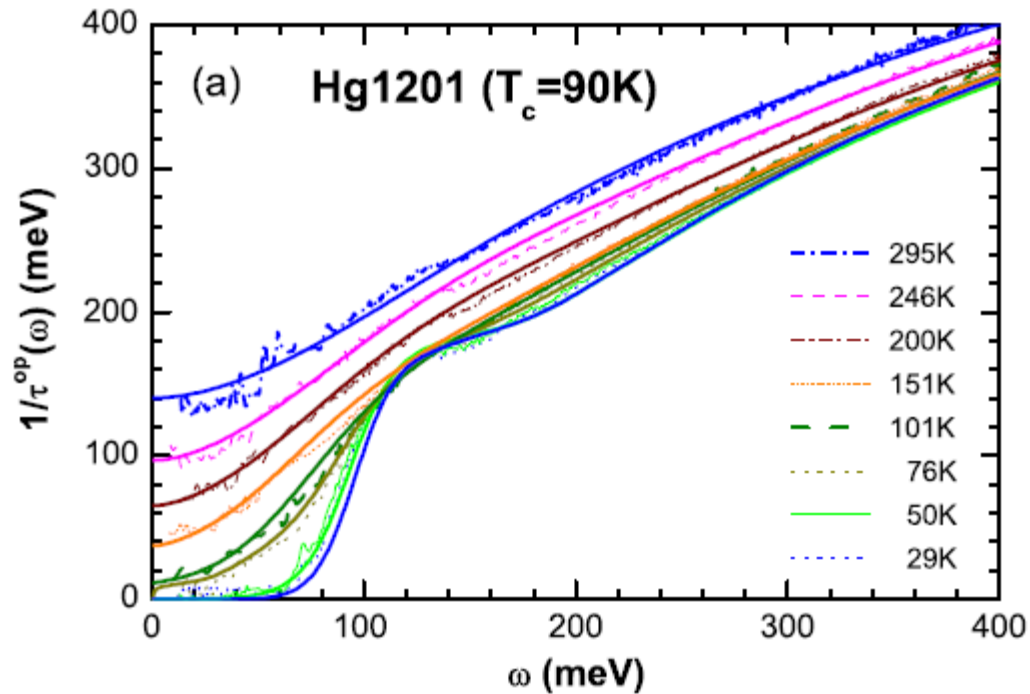
Hwang et al, PRB 75, 144506 [2007]



Redistribution of spectral weight with temperature in OP doped Bi2212

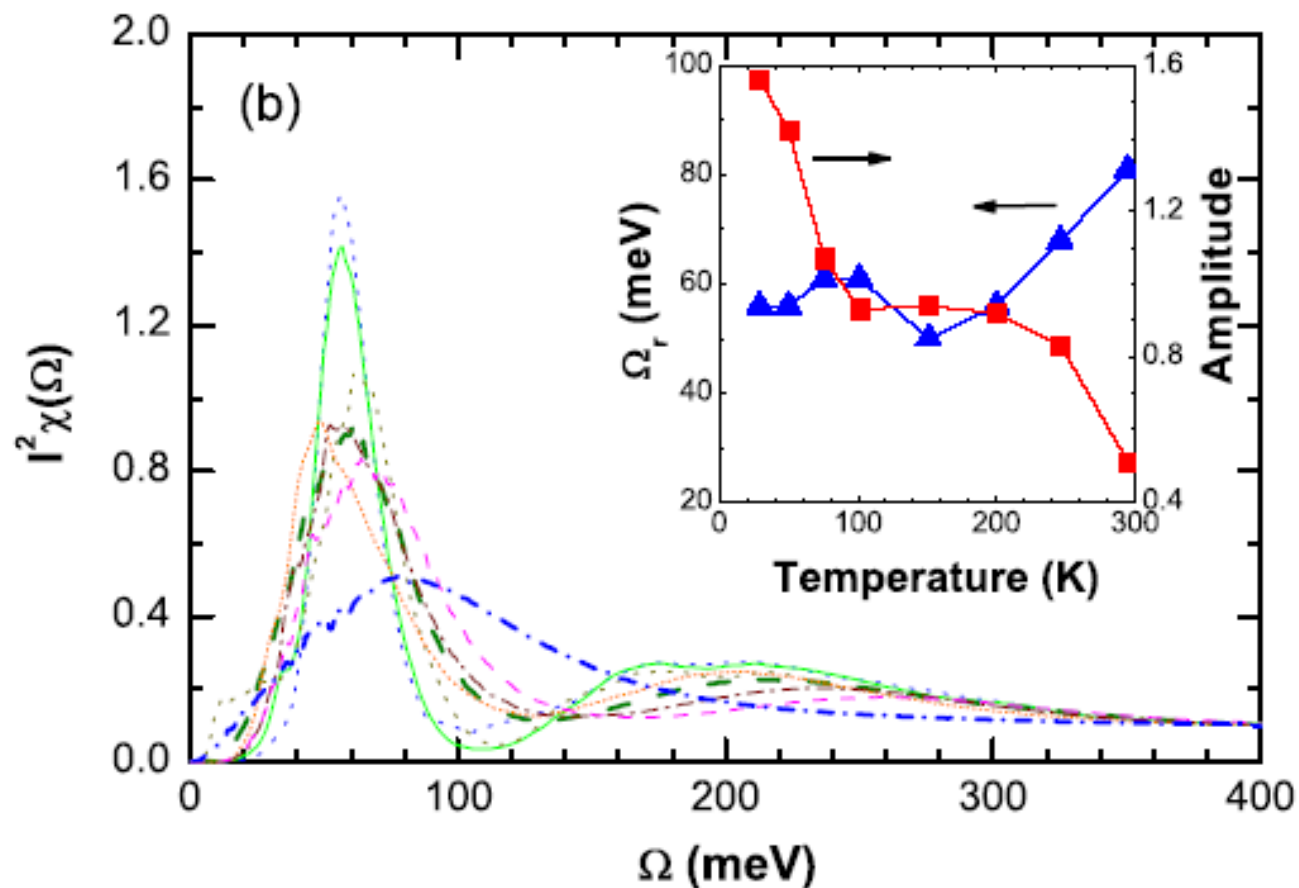


O₄+, T_c=90K



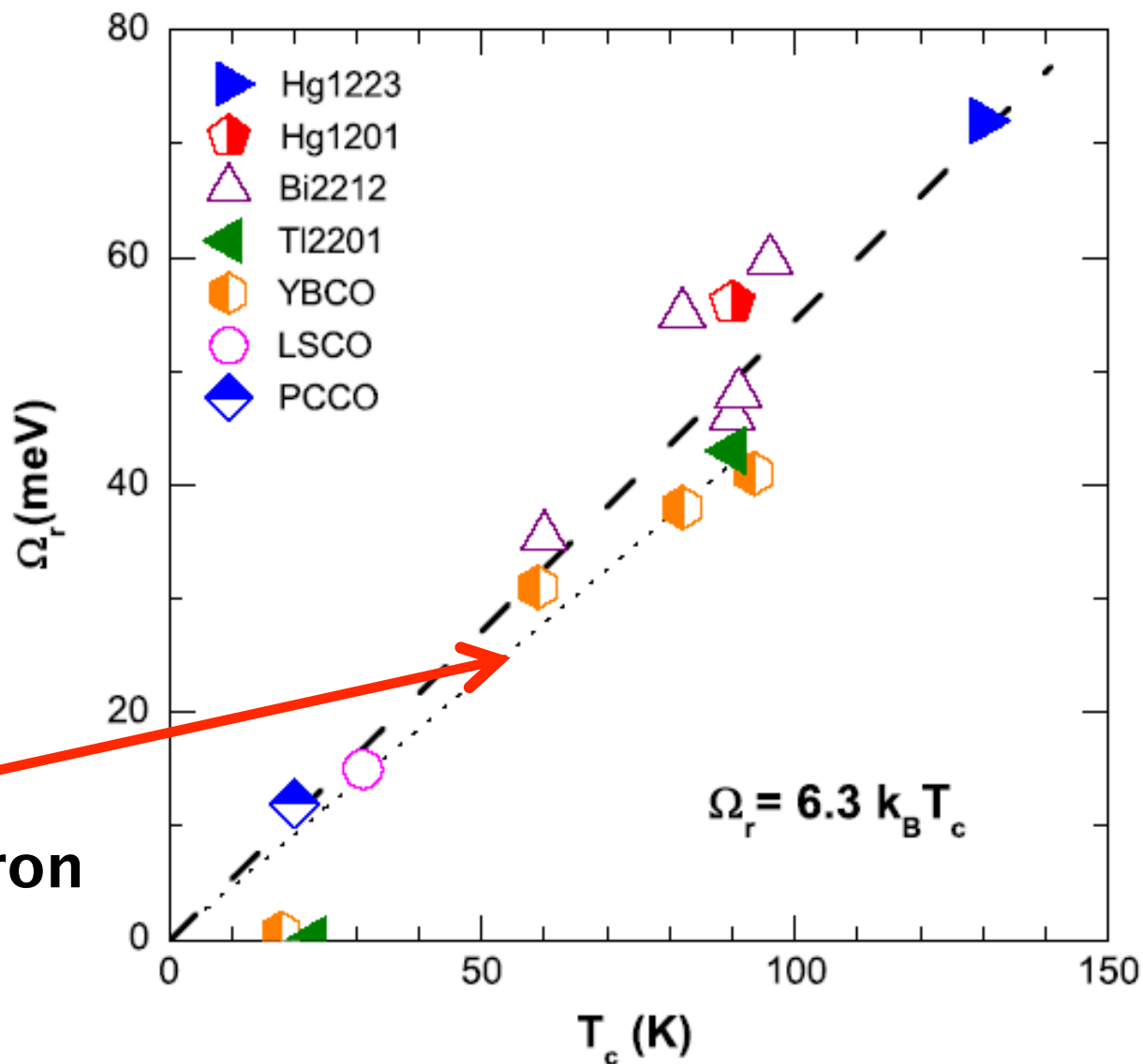
Optical scattering
rate

Same temperature evolution as Bi2212 Resonance energy moves up and amplitude is reduced with increasing temperature



Spectral density extends to high energy and evolves with temperature

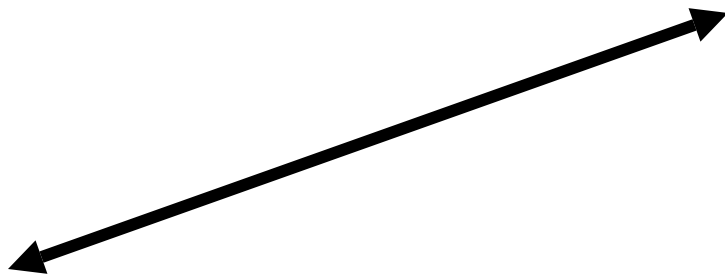
Summary of optical resonance energy vs T_c



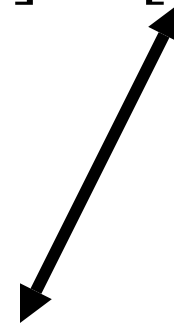
From neutron scattering

Electron spectral density

$$A(\mathbf{k}, \omega) = \frac{1}{\pi} \frac{\Sigma''(\mathbf{k}, \omega)}{[\omega - \varepsilon_{\mathbf{k}} - \Sigma'(\mathbf{k}, \omega)]^2 + [\Sigma''(\mathbf{k}, \omega)]^2}$$



Real part self energy



Imaginary part self energy

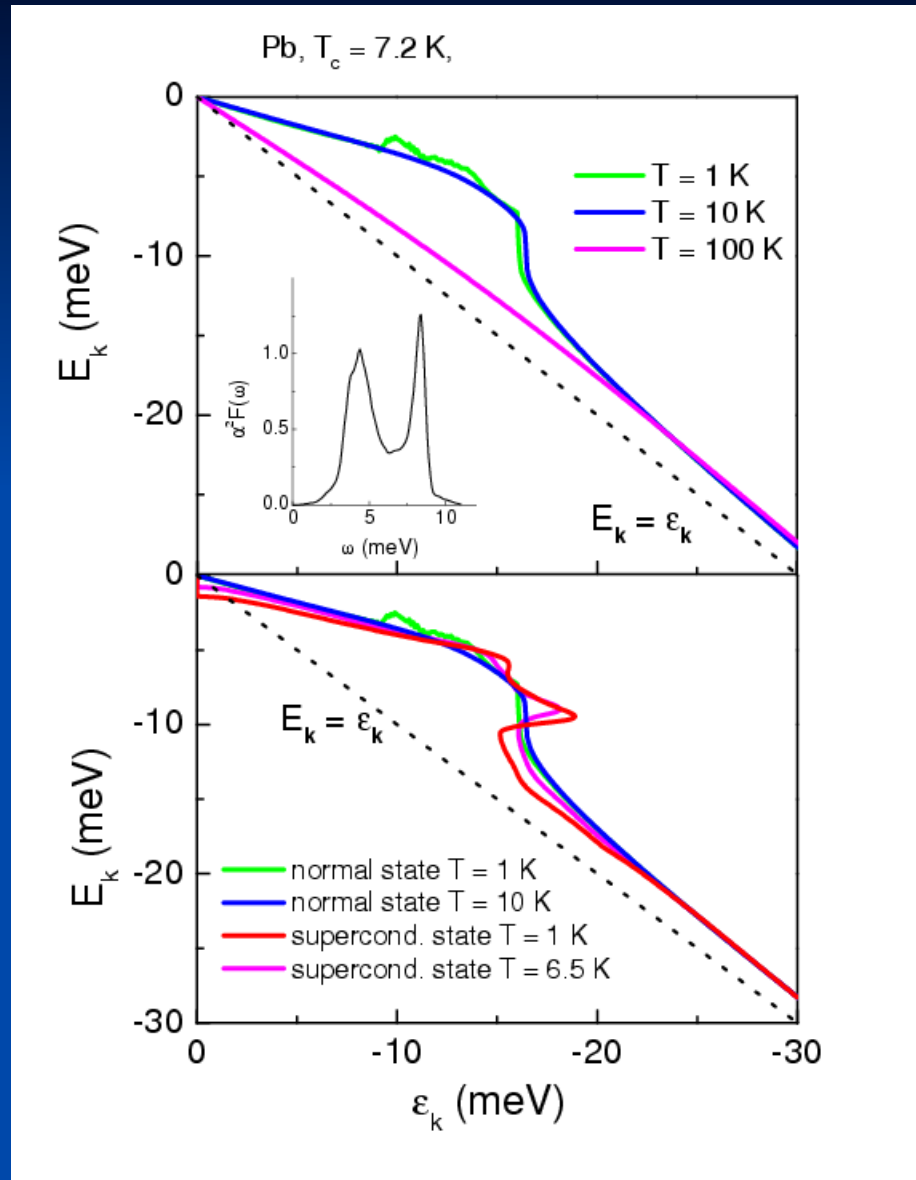
ARPES measures this function at negative energy only

Self energy $\Sigma(\mathbf{k}, \omega)$ gives renormalizations

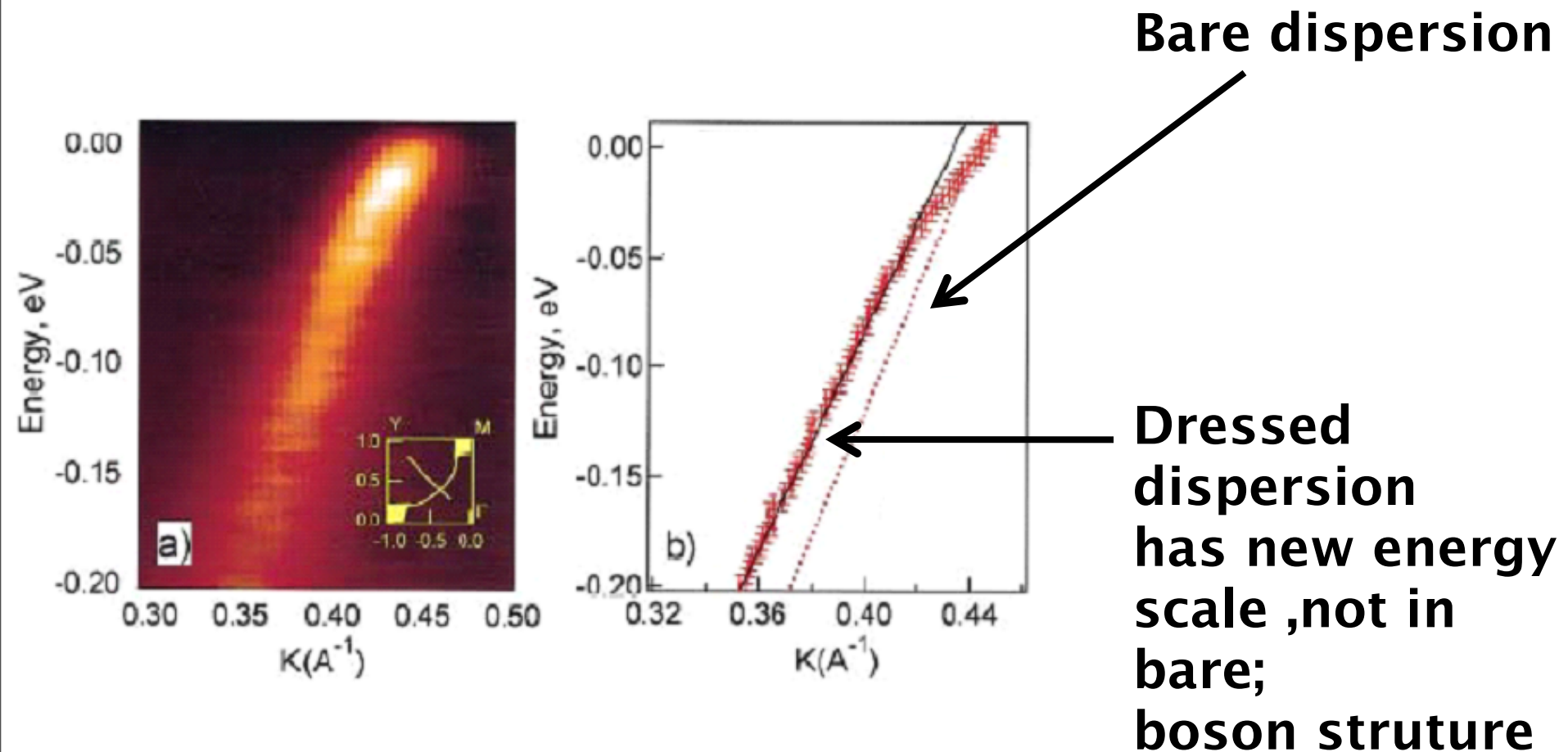
Renormalized dispersion in Pb: Electron-phonon interaction

$$E_{\mathbf{k}} = \varepsilon_{\mathbf{k}} + \text{Re}[\Sigma(E_{\mathbf{k}})]$$

Schachinger et al PRB 67,
214508 [2003]

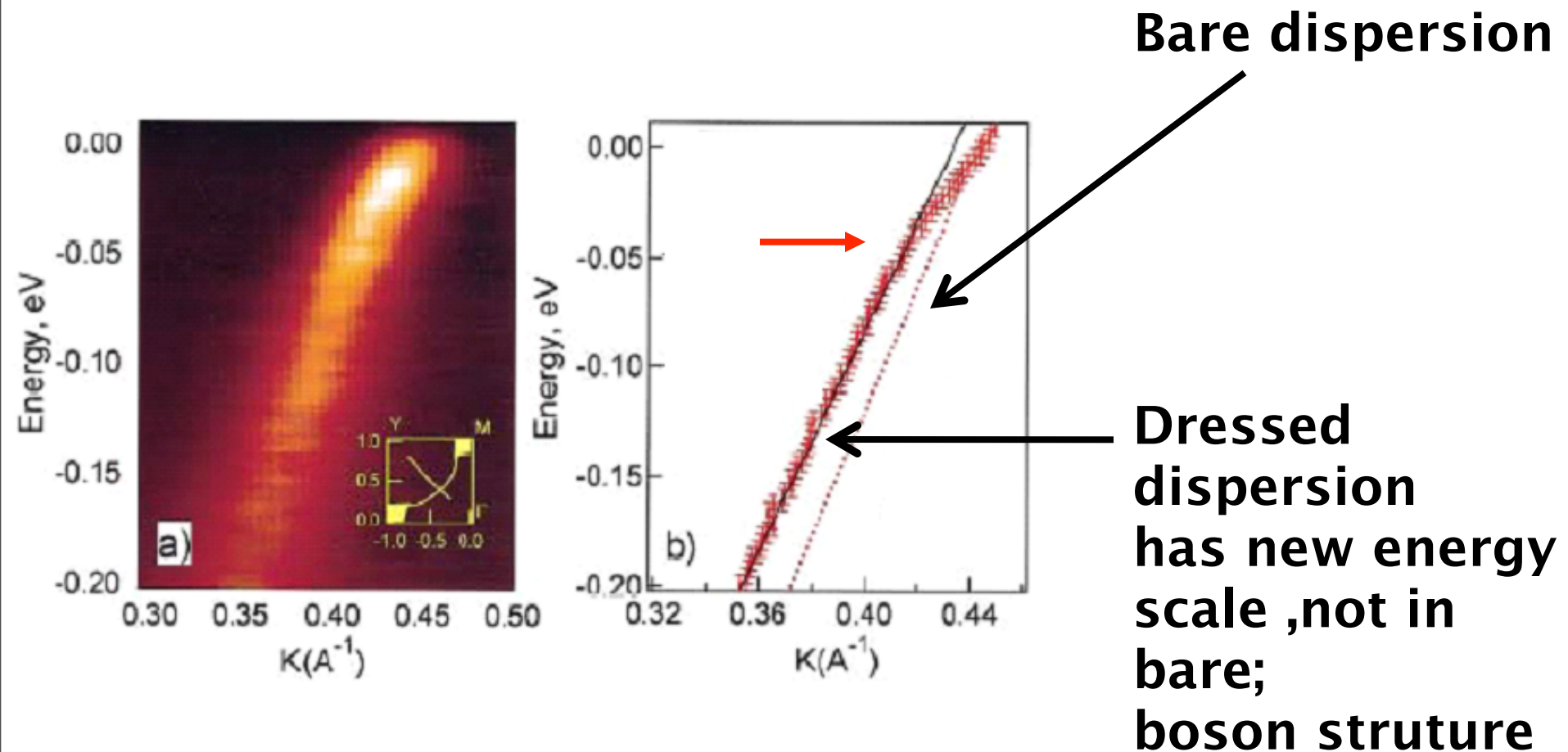


The Kink in ARPES

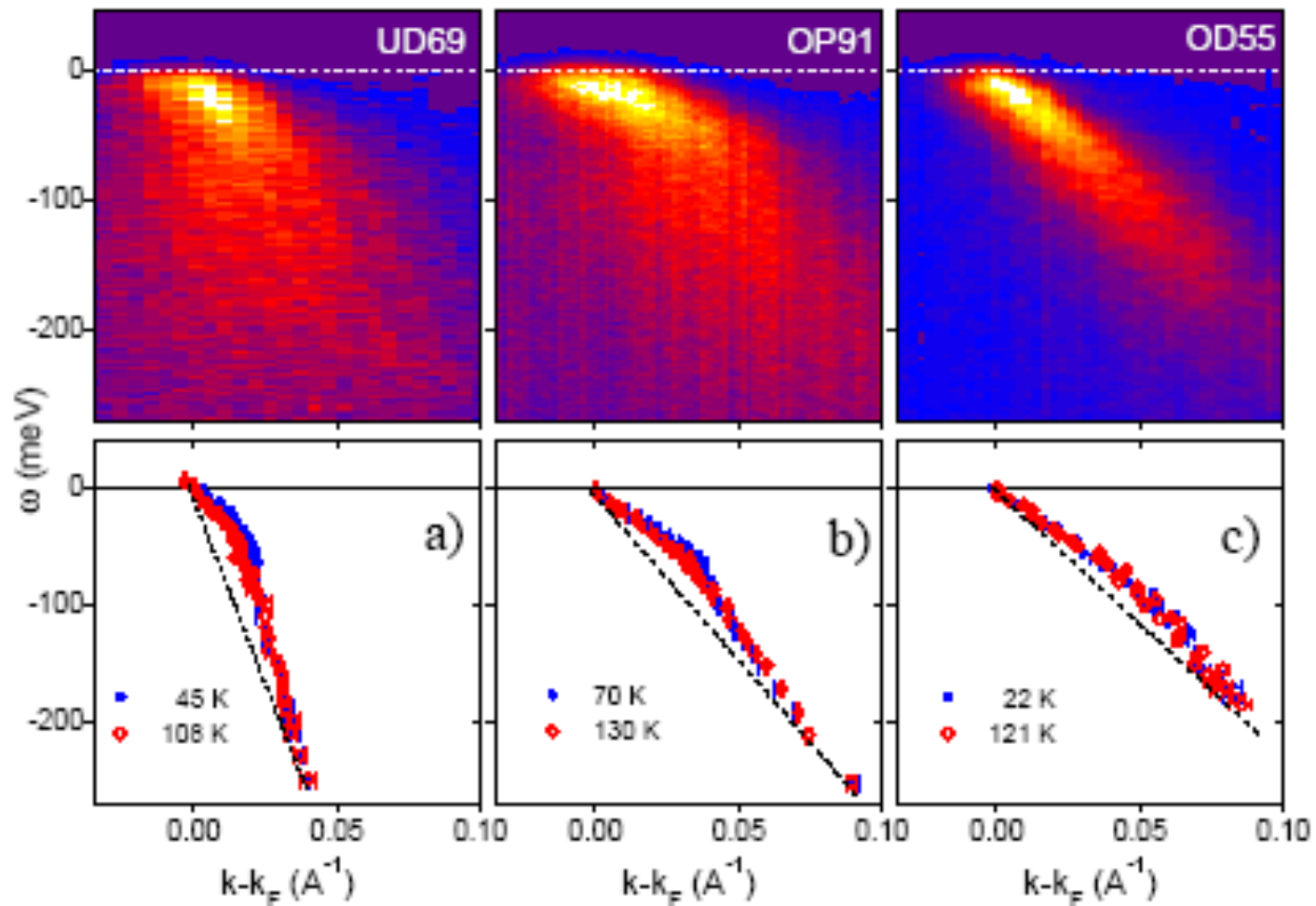


Bogdanov, Lanzara et. al. Phys. Rev. Lett. 85,2581 [2000]

The Kink in ARPES



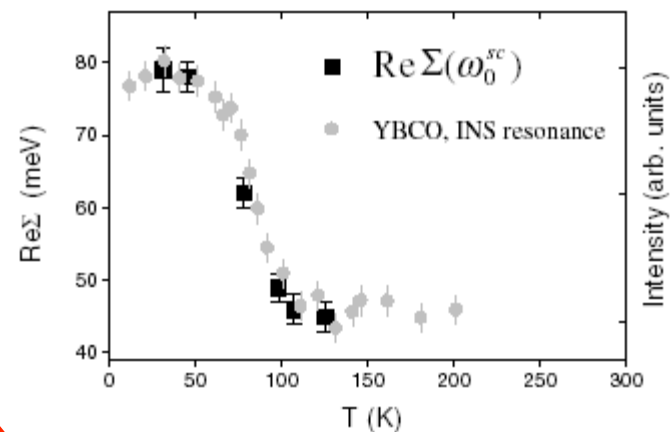
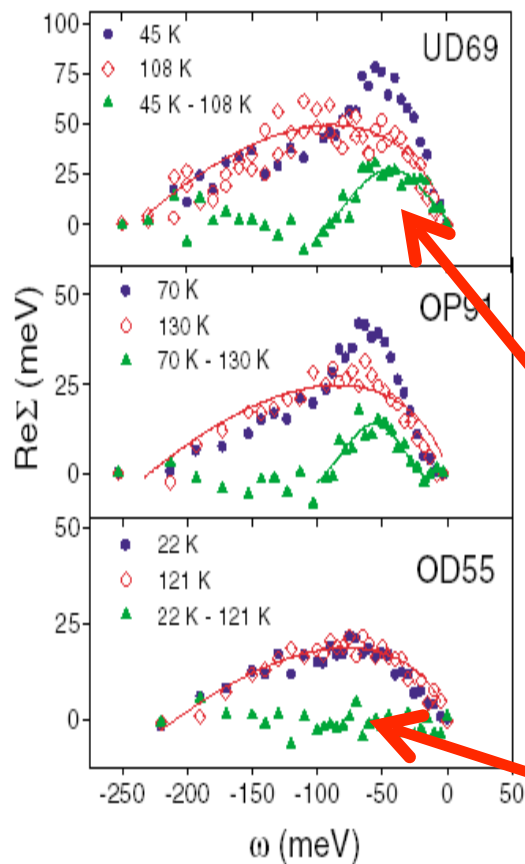
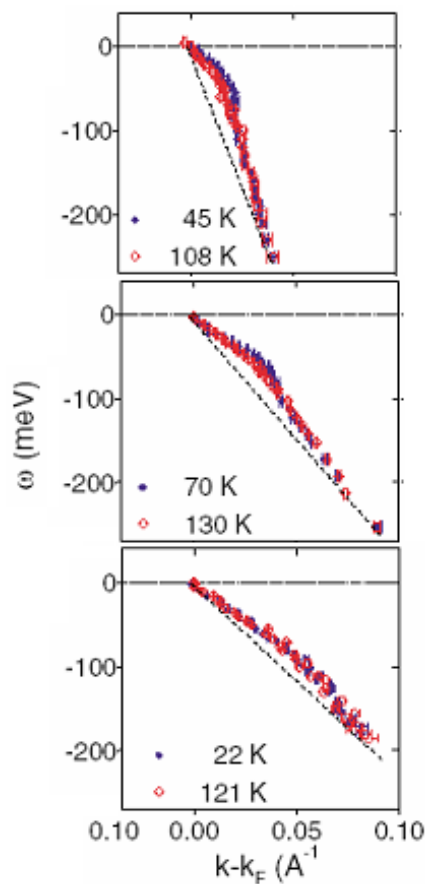
Bogdanov, Lanzara et. al. Phys. Rev. Lett. 85,2581 [2000]



Johnson et. al. PRL 87,177007 [2001]

Bi-2212 near the nodal region

$\Sigma^{qp}_1(\omega)$ Quasiparticle self energy



$$\Omega \sim 6.0 k_B T_c$$

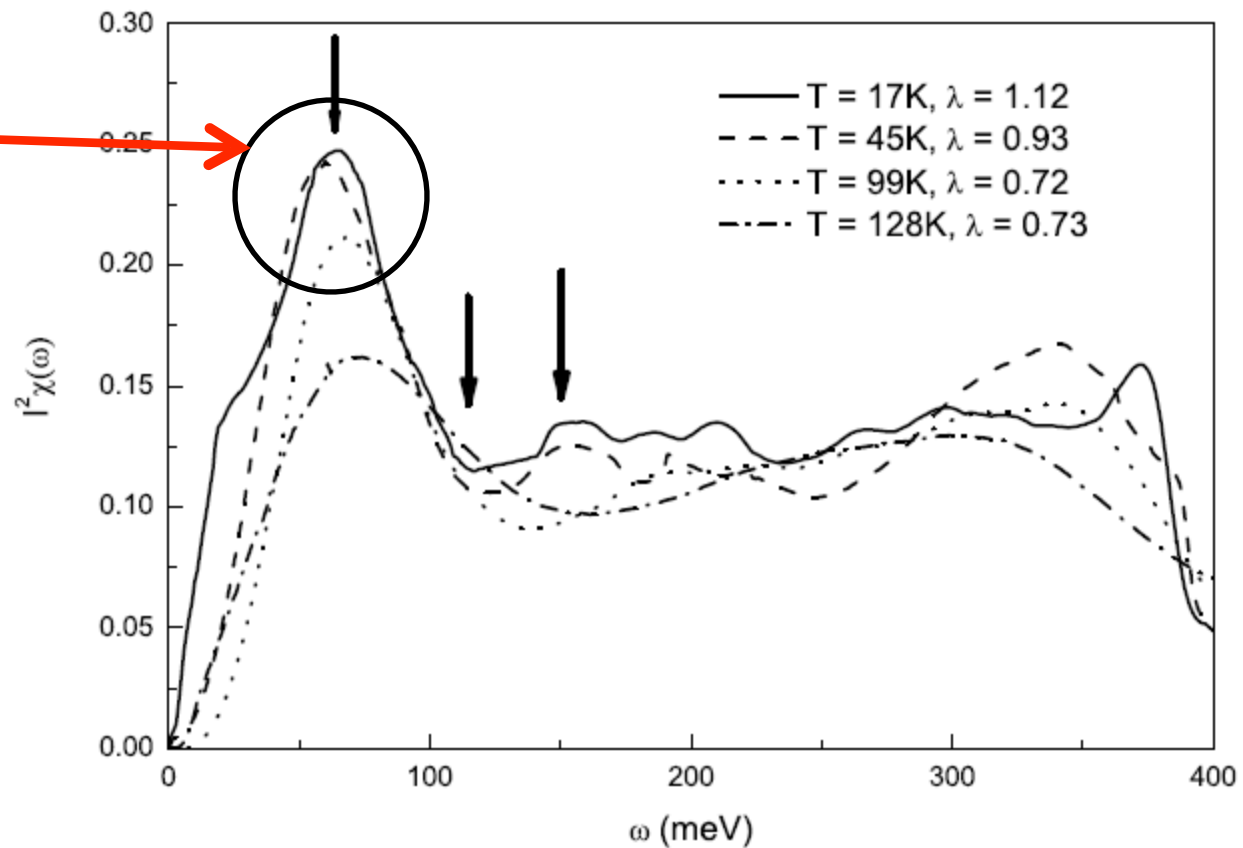
peak

no peak

$$E_{\mathbf{k}} = \varepsilon_{\mathbf{k}} + \text{Re}[\Sigma(E_{\mathbf{k}})]$$

Schachinger – Carbotte , inversion [PRB 77,094524 [2008]
of nodal direction ARPES data in Bi2212 by X. J. Zhou–gro

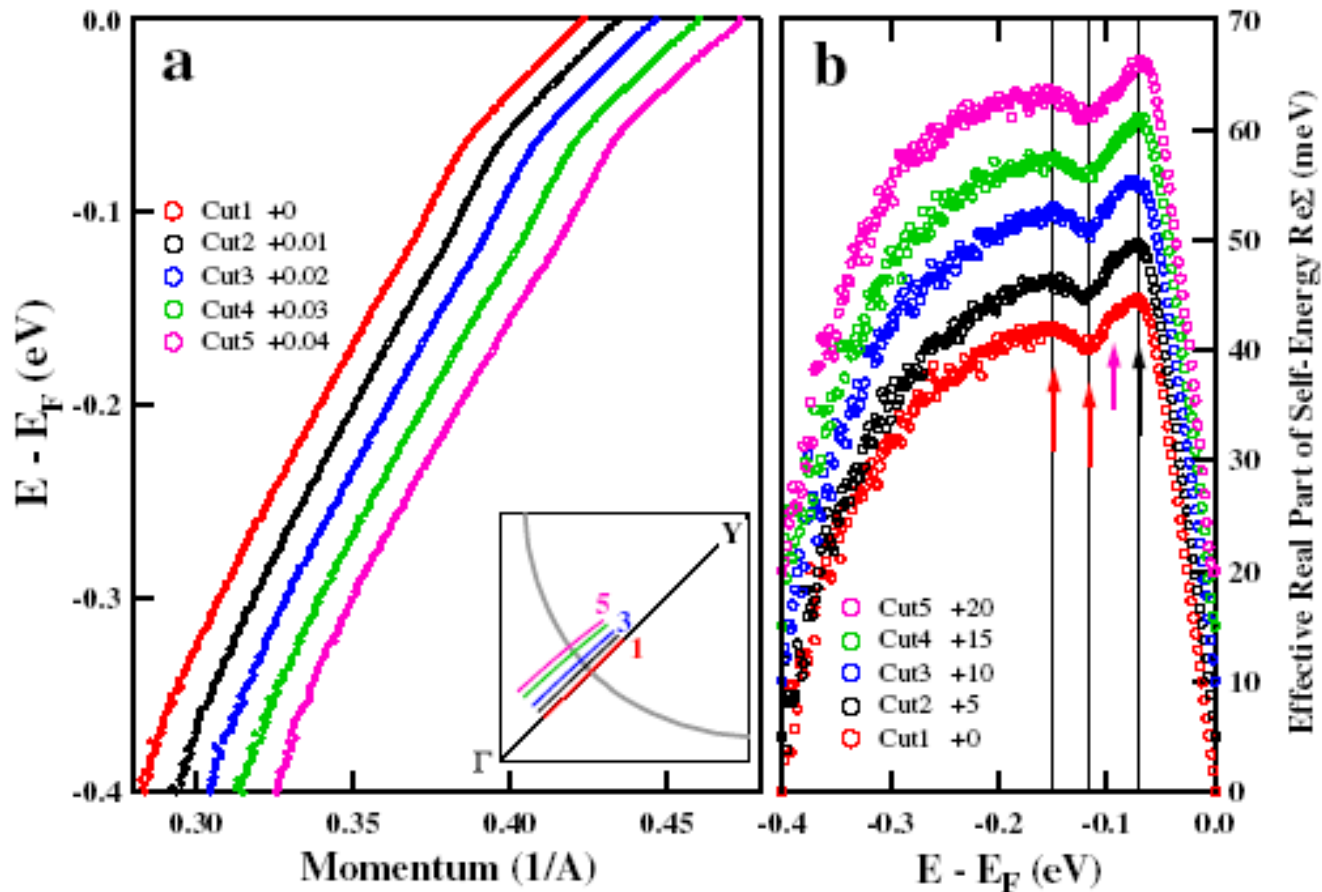
Small phonon
contribution?



Spectrum extends to high energy and evolves with temperature

Features at 70,115,150 meV
by authors as not phonons

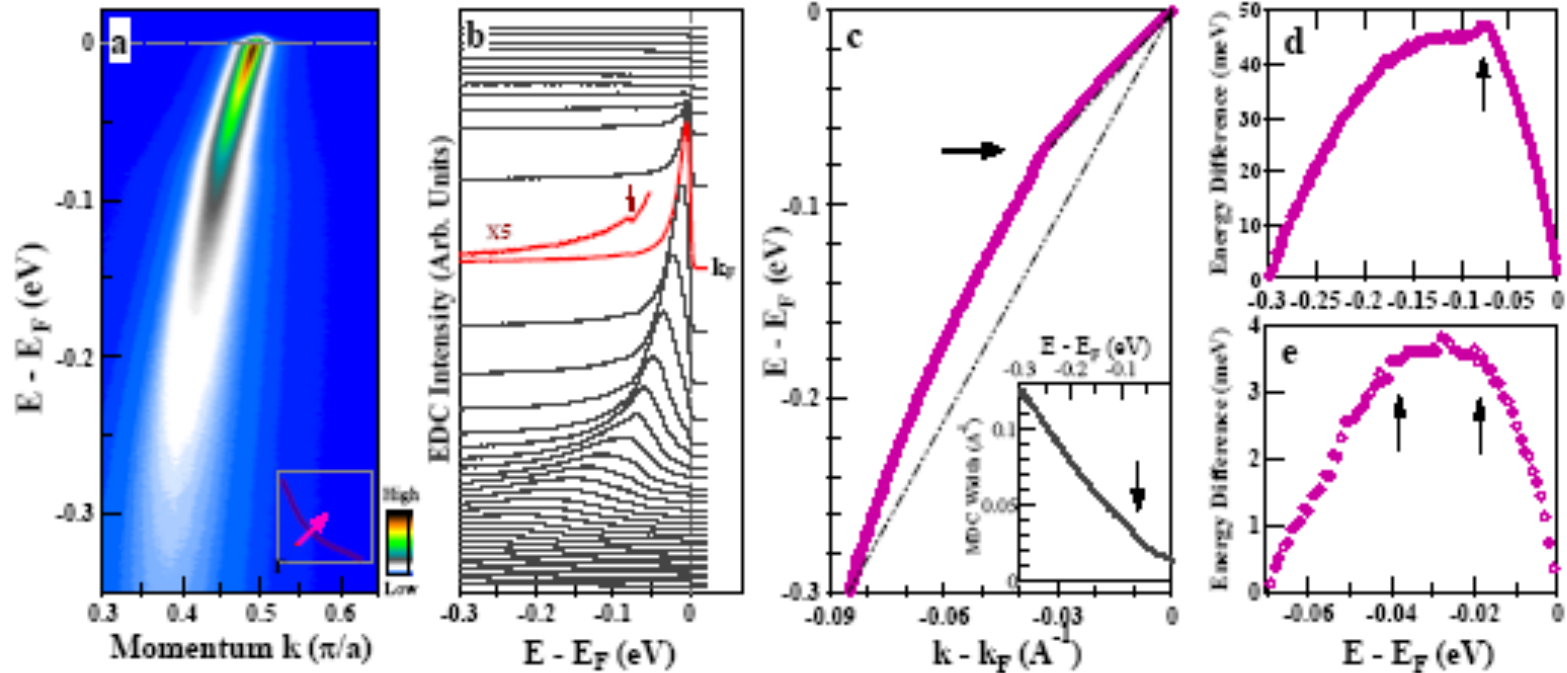
interpreted



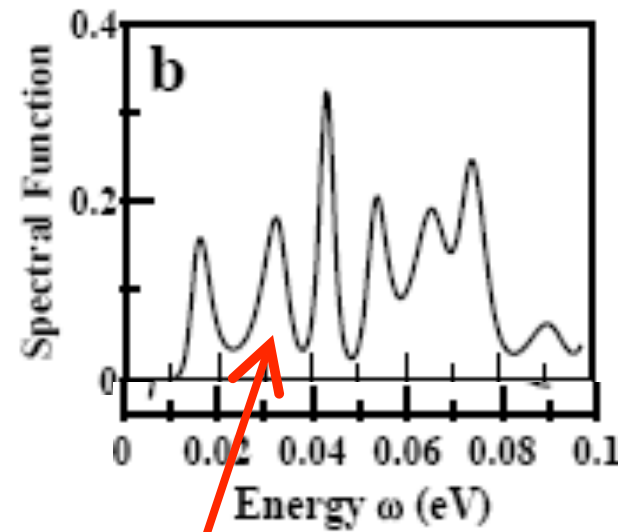
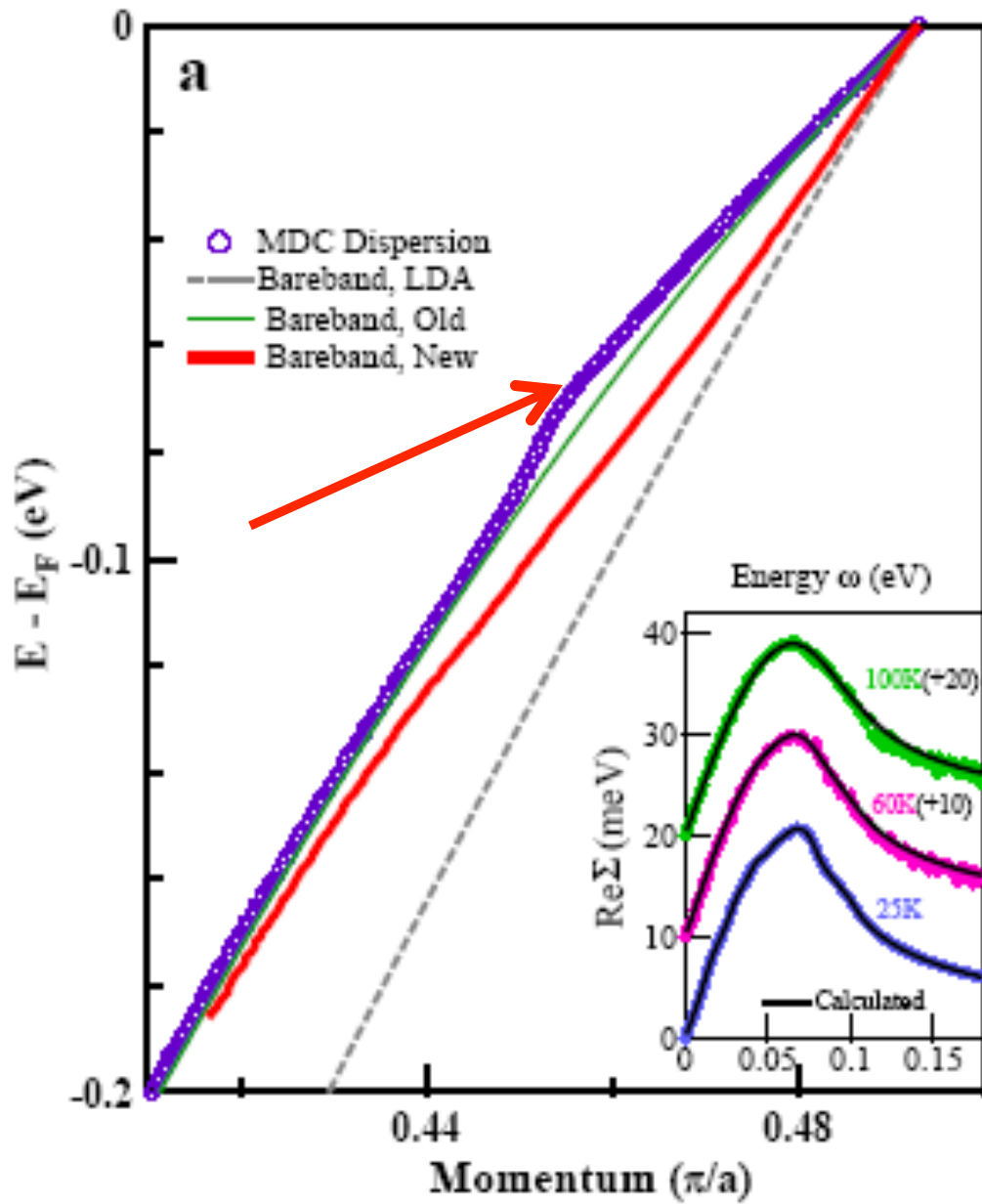
Momentum resolution 0.004 \AA^{-1} $T_c=91\text{K}$, energy resolution 0.56 meV

On the other hand

L. Zhao et. al. [X. J. Zhou-group] APS Portland 2010



heavily over-doped [BI PB] 2SR 2CU O6+ , $T_c = 5$ K
 Coupling to multiple phonon modes at 70 meV, $\lambda =$



?

Heavily over-doped
[Bi Pb] Sr Cu O

Interpreted by
authors
as Phonons

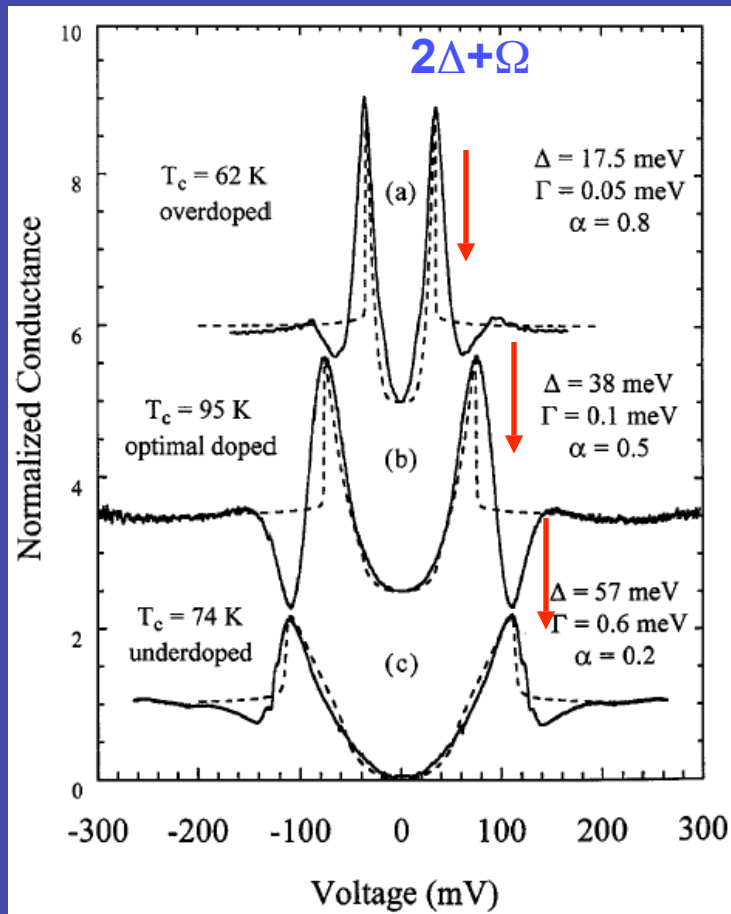
Tunneling measures the electron density of states $N[\omega]$ DoS

For superconducting-normal junction it is $N[\omega]$ that enters

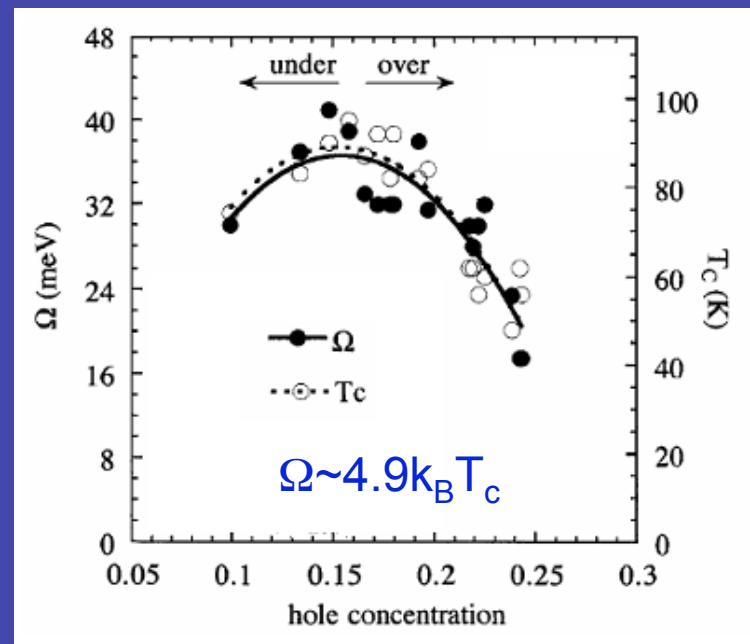
For superconductor-superconductor junctions it's a convolution of the two DoS

peak-dip-hump in Tunneling

Bi-2212 (SIS junction)



Tunneling \leftrightarrow INS




OD62

OPT95

UD74

Zasadzinski *et al.* Phys. Rev. Lett. **87**, 067005 (2001)

Four different techniques give very similar results for the energy of boson peak



ARPES gives about	6
Tunneling gives about	5
Optics gives about $8/1.43 =$ about	5.5
Neutron gives about	5.4

All in units of the critical temperature

**Good agreement about resonance but
its origin remains controversial**

**Extent to high energy of spectral density
and its evolution with temperature indicate
spin fluctuations**

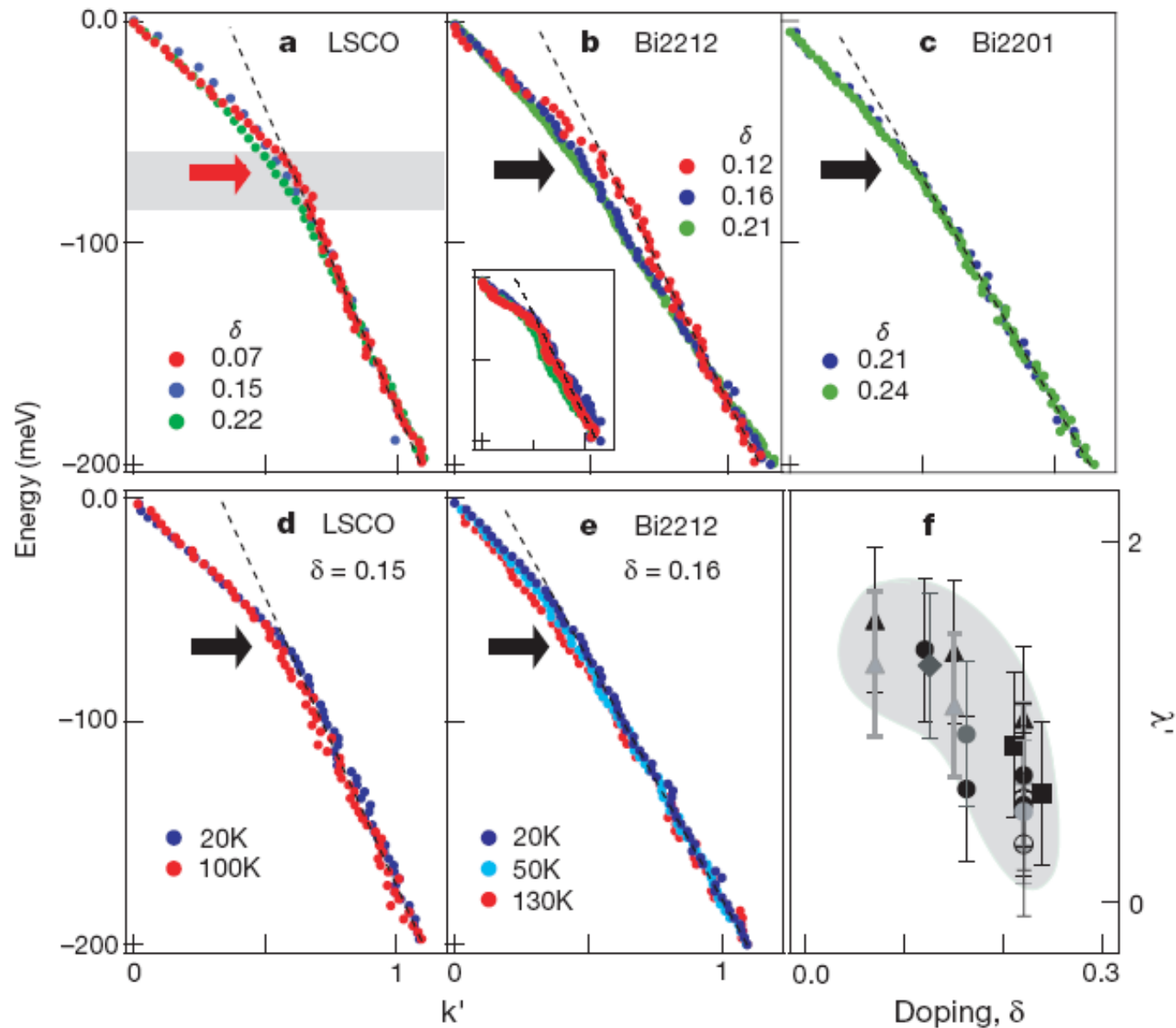
**Probably both are involved
What do you think?**

**We have come a long way!
wonderful journey of discoveries**

BUT it is not over yet

END

Lanzara et. al. Nature ,421, 510 [2001]



Signatures of underlying Electron Boson spectrum seen in

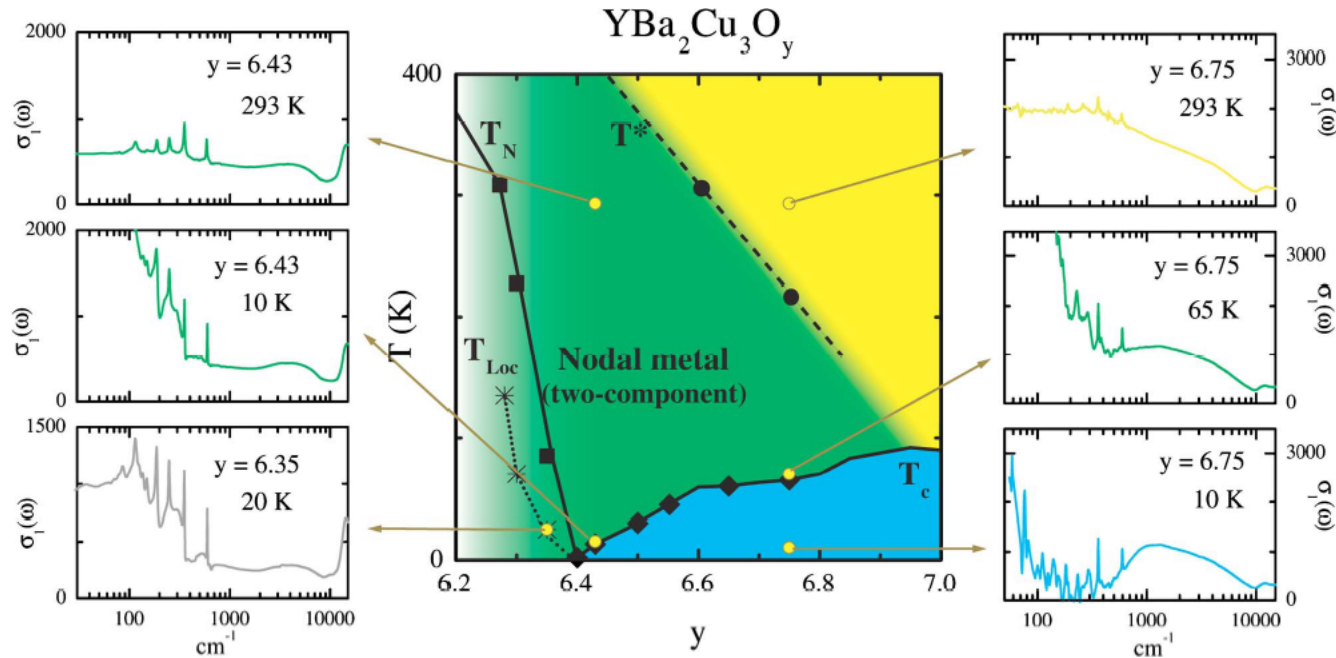
Neutron scattering **INS**

Angular resolved photo emission **ARPES**

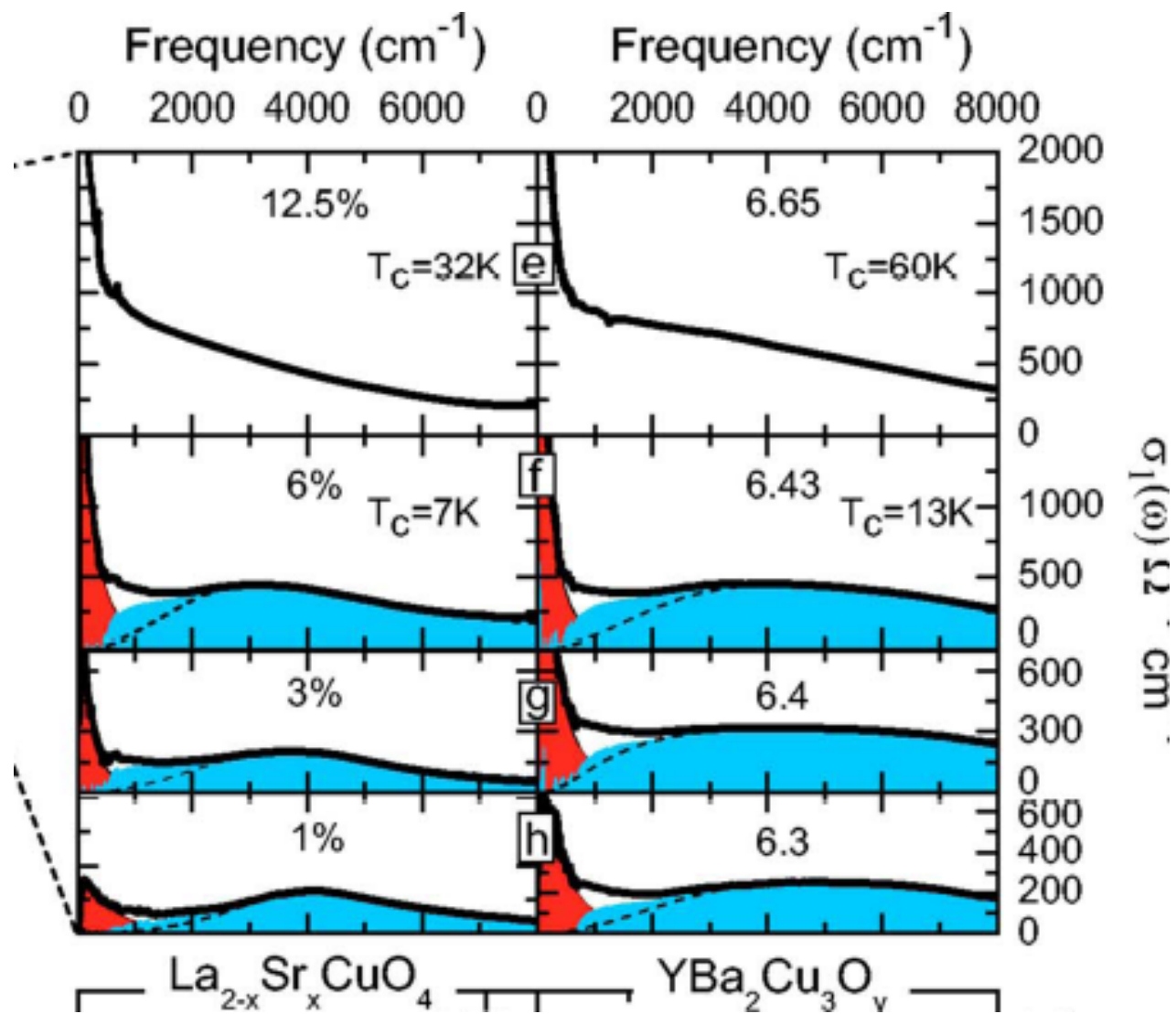
Tunneling **SIS junctions**

Optics **IR-spectroscopy**

Coherence in underdoped state and beyond

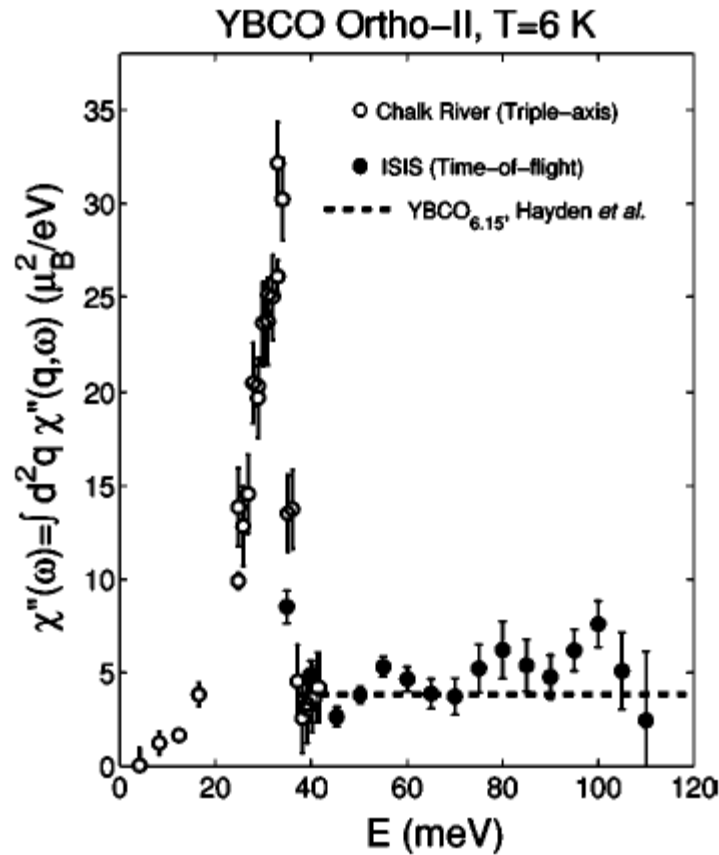


Padilla et. al. [Basov-group] PRB 72,060511 [2005]



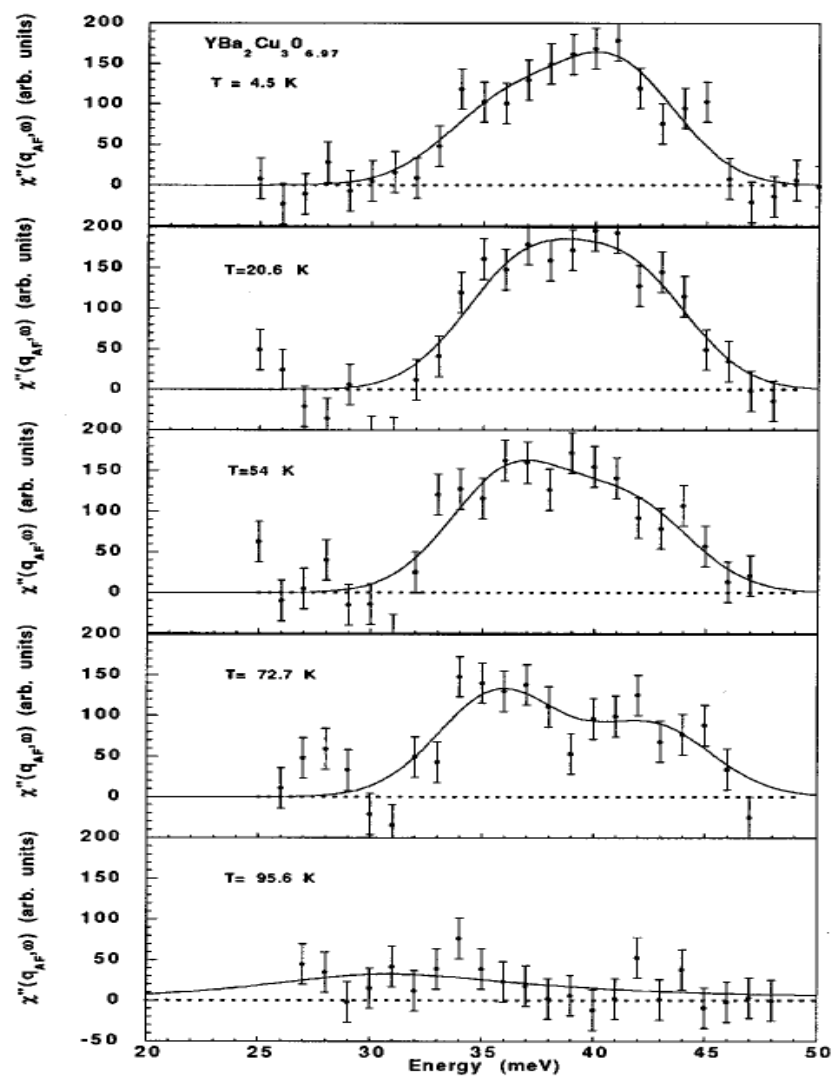
Stock et.al. PRB 71,024522 [2005]

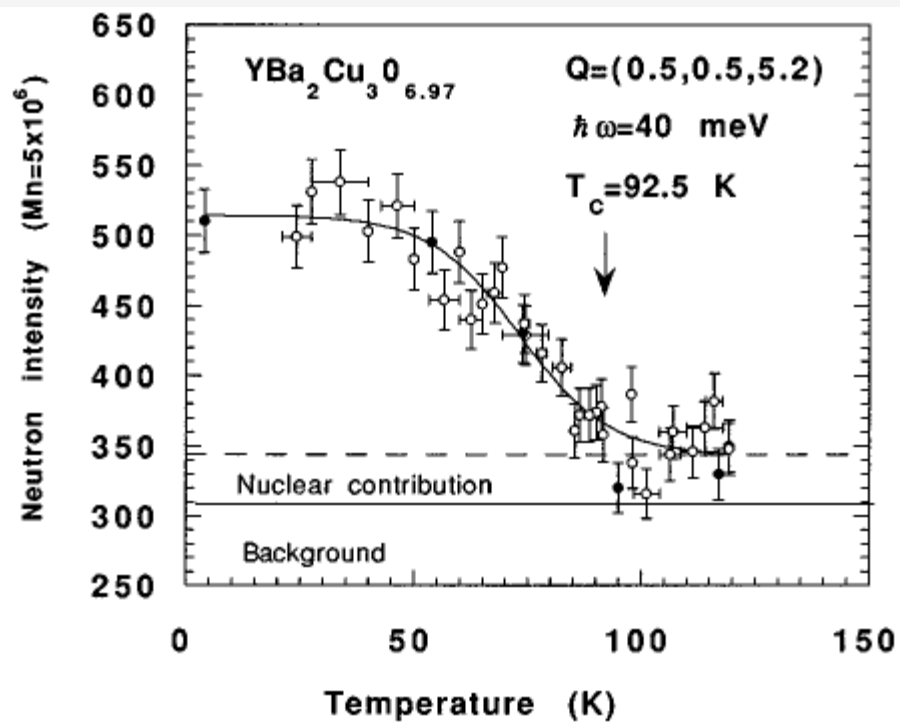
Full and empty
chains alternate



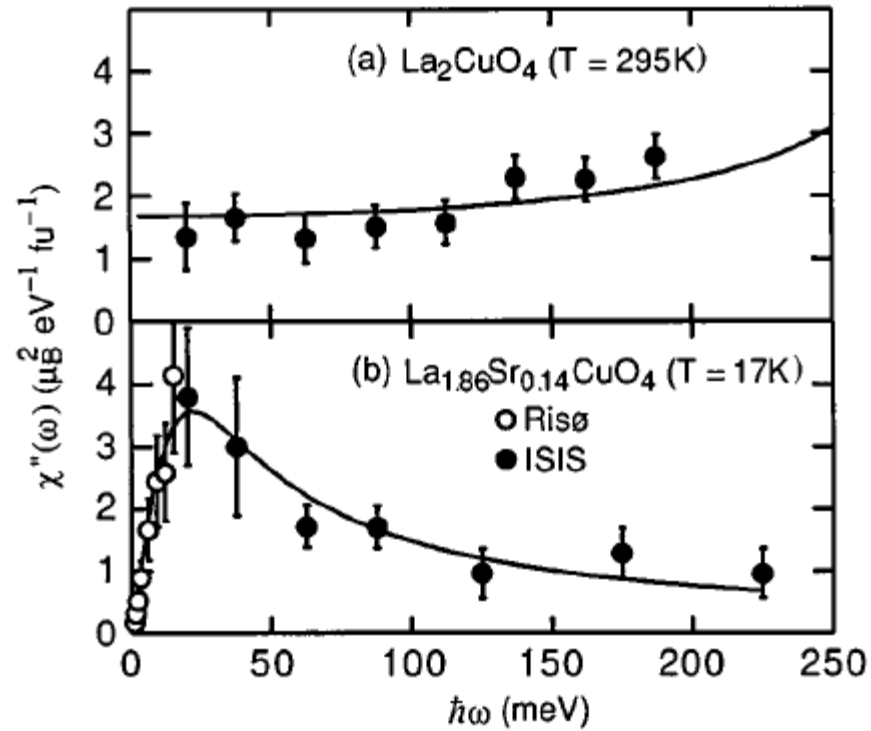
Local spin susceptibility from spin polarized neutron scattering

Bourges PRB
53,876 [1996]





Hayden et.al. <PRL 76,1344 [1996]



Local spin susceptibility from polarized spin neutron scattering

Electron-phonon spectral density in Pb

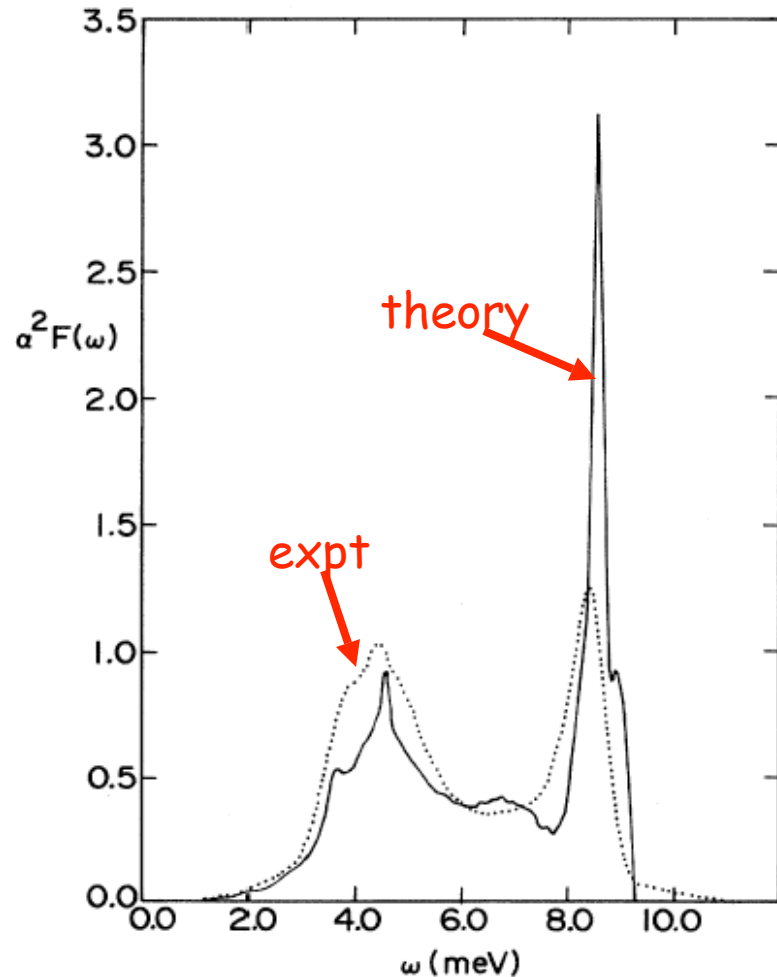
Tunneling:

McMillan and Rowell, Phys. Rev. Lett. **14**, 108 (1965).

Calculations:

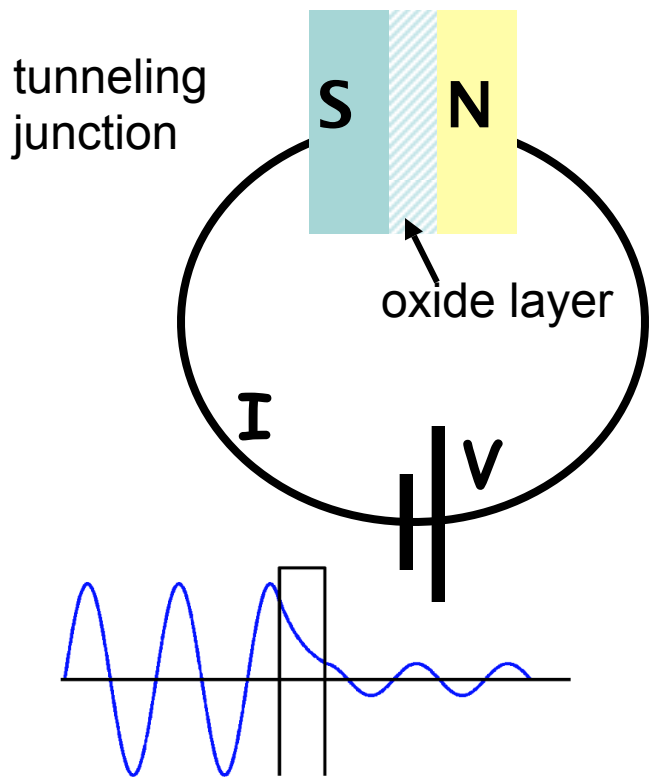
Tomlinson and Carbotte, Phys. Rev. B **13**, 4738 (1976).

This function contains all of the complicated information on electron and phonon dynamics and the coupling between them that enters superconductivity.



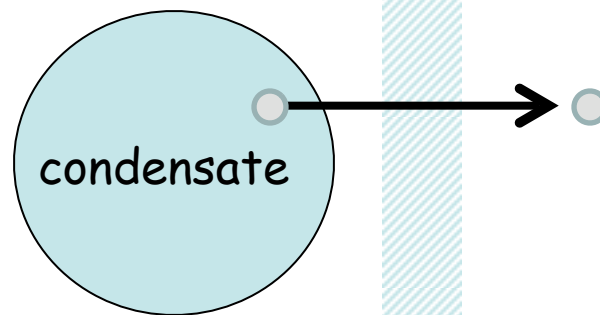
Remarkable condensation of required information!

I-V characteristics of a tunneling junction



Gap in superconducting state means no current till V is equal or greater than Δ

Superconducting (S) Normal (N)



Takes energy Δ

