Exotic Quantum and Classical Ground States in Geometrically Frustrated Magnets

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I.I T_c T_c 0.9 T_c

Ferromagnet Antiferromagnet 1

 $\mathbf{H} = 2\mathbf{J} \ \Sigma_{ij} \ \mathbf{S_i} \cdot \mathbf{S_j}$



prefer $\uparrow \downarrow$ alignment, but choice of 3rd spin direction is unclear



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Can SOMETIMES lower energy by making a compromise

Geometrical Frustration: *Antiferromagnetism* + *Triangles and Tetrahedra*



2D Triangular Lattice: G.H. Wannier Phys. Rev. 79, 357, 1950.

Ground State of the Ising AF on a Triangular Lattice



Entropy at T=0 is finite ~ 0.34 R

No LRO at any temperature

Geometrical Frustration



* Non-frustrated quantum magnets order in 2D and 3D

Geometrical Frustration



* Non-frustrated quantum magnets order in 2D and 3D



Mean Field Theory predicts a phase transition near T= $|\Theta_{CW}|$, but materials remains disordered to much lower temperatures – <u>Spin Liquid</u>



Geometrical Frustration





Quantum Fluctuations: S=1/2 H Transverse



Low Dimensional Structures







²³⁵U + n →
daughter nuclei +
2-3 n + gammas

neutrons:

no charge s=1/2 massive: mc²~1 GeV

How do we produce neutrons



Fission

- chain reaction
- continuous flow
- 1 neutron/fission



Spallation

- no chain reaction
- pulsed operation
- 30 neutrons/proton

Development of Neutron Science Facilities



(Updated from Neutron Scattering, K. Skold and D. L. Price: eds., Academic Press, 1986)









Clifford G. Shull, MIT, Camebridge, Massachusetts, USA, receives one half of the 1994 Nobel Prize in Physics for development of the neutron diffraction technique.



T < Tc

Neutrons have *mass* so higher energy means faster – lower energy means slower



We can measure a neutron's energy, wavelength by measuring its *speed*



Motivation for Singlet Basis

$$H = J \sum_{\substack{\langle i,j \rangle \\ \vec{S_i} \cdot \vec{S_j} = S_{zi}S_{zj} + S_{xi}S_{xj} + S_{yi}S_{yj}}} \vec{S_i} \cdot \vec{S_j}$$

Neel type state for 2 spins:

Entangled (singlet) state:

$$\begin{array}{l} \langle\uparrow_{i}\downarrow_{j}| \ S_{zi}S_{zj} \mid\uparrow_{i}\downarrow_{j}\rangle = -\frac{1}{4} \\ \langle\uparrow_{i}\downarrow_{j}| \ S_{zi}S_{zj} \mid\uparrow_{i}\downarrow_{j}\rangle = 0 \\ \langle\uparrow_{i}\downarrow_{j}| \ S_{xi}S_{xj} \mid\uparrow_{i}\downarrow_{j}\rangle = 0 \\ \langle\uparrow_{i}\downarrow_{j}| \ S_{yi}S_{yj} \mid\uparrow_{i}\downarrow_{j}\rangle = 0 \\ \langle\uparrow_{i}\downarrow_{j}| \ S_{yi}S_{yj} \mid\uparrow_{i}\downarrow_{j}\rangle = 0 \\ \langle\uparrow_{i}\downarrow_{j}| \ H \mid\uparrow_{i}\downarrow_{j}\rangle = -\frac{1}{4}J \\ (\langle\uparrow_{i}\downarrow_{j}| -\langle\downarrow_{i}\uparrow_{j}|)S_{yi}S_{yj}(\mid\uparrow_{i}\downarrow_{j}\rangle - \mid\downarrow_{i}\uparrow_{j}\rangle) = -\frac{3}{4}J$$

Symmetry in the Ground State



Possible Ground State: Only Nearest Neighbour Bonds

$|\Psi_{\rm sRVB}\rangle =$



"Short Range RVB"

Possible Ground State: All Bond Lengths

$|\Psi_{\rm lRVB}\rangle =$



"Long Range RVB"









in sRVB state, creating 2 spinons costs $\Delta E = +J$, but then the spinons are free to move around by reorganizing singlets



NOTES:

1) Spinons are *deconfined*, but there is a *"spin gap"* ($\Delta E = +J$) 2) Charges are not mobile, only spins - it is like half an electron



SrCu₂(BO₃)₂

- Quasi-2D Cu-BO₃ planes
- Sr ions between planes
- Mott insulator

S=1/2 moments at Cu²⁺ sites arranged in orthogonal dimers on square lattice

Shastry-Sutherland model









DC susceptibility shows singlet ground state – no phase transition in SrCu₂(BO₃)₂

DC susceptibility shows singlet ground state – Spin-Peierls phase transition in CuGeO₃







Triplets split in a magnetic field $\Delta E = g \mu_B H$


SrCu₂(BO₃)₂ enters collective singlet state at low temperature:

Large magnetic fields drive triplets to zero energy; producing steps in the magnetization.



Hard core Boson models for interacting triplets (Miyahara and Ueda, J. Phys. CM 15, R327, 2003)

Plateaus appear at commensurate filling of lattice with triplets

Time of Flight Neutron Scattering Data





Comparison to Theory



S. Haravifard, S. R. Dunsiger, S. El Shawish, B. D. Gaulin, H. A. Dabkowska, M. T. F. Telling, T. G. Perring, and J. Bonča, Phys. Rev. Lett. 97, 247206 (2006)

Variational Calculation (El Shawish and Bonca, PRB 74, 174420, 2006)

Starting point for spin defect around impurity



Starting point *not* an eigenfunction of *H*







Sara Haravivard

Sarah Dunsiger



Renaissance in neutron science with figures of merit increases ~ 100 TOF techniques well suited to exotic magnetism as it measures very effectively across wide dynamic range in Q and E

Shastry-Sutherland singlet ground state in SrCu₂(BO₃)₂: Frustration drives localized triplets + plateaus in triplet density with field

Quenched impurities give spin polarons: extended S=1/2 states which can be measured with neutron scattering



Cubic Pyrochlores:

- Spins on a network of corner-sharing tetrahedra
- **A**₂**Ti**₂**O**₇
- A site is RE ³⁺ (many magnetic possibilities)





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Inelastic neutron scattering on polycrystalline Tb₂Ti₂O₇



 $(\Delta : Ho_2Ti_2O_7 \sim 240 \text{ K}; Dy_2Ti_2O_7 \sim 380 \text{ K})$



Rare Earth moments:

Strong [111] anisotropy



Ferromagnetic exchange:

"Spin Ice": 2 in 2 out

Harris, Bramwell et al, PRL, 79, 2554, 1997

Antiferromagnetic exchange:

All in - All out

$$H = -J \sum_{\langle ij \rangle} \mathbf{S}_{i}^{z_{i}} \cdot \mathbf{S}_{j}^{z_{j}}$$

+ $Dr_{nn}^{3} \sum_{j > i} \frac{\mathbf{S}_{i}^{z_{i}} \cdot \mathbf{S}_{j}^{z_{j}}}{|\mathbf{r}_{ij}|^{3}} - \frac{3(\mathbf{S}_{i}^{z_{i}} \cdot \mathbf{r}_{ij})(\mathbf{S}_{j}^{z_{j}} \cdot \mathbf{r}_{ij})}{|\mathbf{r}_{ij}|^{5}}$



B.C. den Hertog and M.J.P. Gingras, PRL, 84, 3430, 2000.



Spin Ice Ground State indicated by Shottky-like anomaly at T_{SI} ~ 2 K in both Dy₂Ti₂O₇ and Ho₂Ti₂O₇

Missing entropy is the same as for proton disorder in solid H₂O, hence "Spin Ice"

> *A.P. Ramirez et al, Nature, 399, 333, 1999*





Spin Ice Ground State in Ho₂Ti₂O₇



J. P. Clancy, J. P. C. Ruff, S. R. Dunsiger, Y. Zhao, H. A. Dabkowska, J. S. Gardner, Y. Qiu, J. R. D. Copley, T. Jenkins, and B. D. Gaulin, Phys. Rev. B 79, 014408 (2009)

Diffuse Scattering Evolves with T on scale of $\sim 17~K$





Spin Ice correlations due to enforced "ice rules" (2 spins in – 2 spins out / tetrahedron) gives rise to "Zone Boundary Scattering"

Evolve on scale of $T_{SI} \sim 2 K$



Application of weak [1-10] magnetic field breaks system up into α and β chains.

Polarizable α-[1-10] chains (parallel to field)

Perpendicular β-[110] chains



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[1-10] Magnetic field should decompose pyrochlore lattice into polarized α chains (red chains) and decoupled qausi-1D β chains (blue chains).



Ferro-ordering of α-chains





1D-correlations along β-chains



J. P. Clancy, J. P. C. Ruff, S. R. Dunsiger, Y. Zhao, H. A. Dabkowska, J. S. Gardner, Y. Qiu, J. R. D. Copley, T. Jenkins, and B. D. Gaulin, Phys. Rev. B 79, 014408 (2009)

Geometric Frustration in 3D: The Pyrochlore Lattice



Corner-Sharing Tetrahedra

Geometric Frustration in 3D: The Pyrochlore Lattice



Corner-Sharing Tetrahedra









B.C. den Hertog and M.J.P. Gingras, PRL, 84, 3430, 2000.

but, Tb₂Ti₂O₇ doesn't order at low T (~ 20 mK)



I/T_I relaxation rate muSR

Low T powder neutron diffraction





Ordered phase(s) appear(s) on application of H // 110







Low field (002) phase persists to very high T > 25 K

High field (112) phase exists on expected $T_N \sim 2 K$





K. C. Rule, J. P. C. Ruff, B. D. Gaulin, S. R. Dunsiger, J. S. Gardner, J. P. Clancy, M. J. Lewis, H. A. Dabkowska, I. Mirebeau, P. Manuel, Y. Qiu, and J. R. D. Copley, Phys. Rev. Lett. 96, 177201 (2006)



One Transition in Zero Field



Five Transitions in Non-Zero Field



Ho₂Ti₂O₇ vs Tb₂Ti₂O₇ Static Spin Ice vs Dynamic Spin Liquid (H=0)



Magnetic Structure Factors appear complementary


Tb₂Ti₂O₇ displays evidence for magnetization plateau in [111] magnetic field: Signature of spin ice state - quantum spin ice?



Kate Ross

Pat Clancy

Jacob Ruff

 Rare earth titanates: playground for exotic frustrated ground states
 Ho₂Ti₂O₇: Canonical spin ice 1D decomposition in [110] field; kagome ice + plateau in [111] field
 Tb₂Ti₂O₇: enigmatic spin liquid [110] field induced LRO; evidence for plateau in [111] field - quantum spin ice?



Temperature dependence of 1-triplet excitation and 2-triplet excitation are identical and follow the complement of the dc-susceptibility (1-χ).







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Spin Excitations: Spinons in IRVB

in IRVB state, creating 2 spinons can cost NO energy because long bonds are bound by distant neighbor exchange only



NOTE: 1) Spinons are *gapless* and *deconfined*







Motivation for Singlet Basis









Where do quantum spins come from?

3d⁷ big CEF effect: low spin state



Where do quantum spins come from?





The (111) planes of the Pyrochlore structure are interleaved Kagome and triangular planes



002 inelastic, H=1T, T=0.4K

[0,0,L]

002 inelastic, H=2T, T=0.4K



Geometric Frustration in 2D



Sometimes the system will fluctuate between these degenerate states, increasing entropy: NO Long Range Order







Time-of-flight neutron scattering and 2D spin correlations in 3D crystals

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cMaster University Francis Xavier U ST VS, ORNL





Z*A*

Geometric Frustration in 2D





prefer anti-ferromagnetic alignment



Low temperature powder

neutron diffraction from

Tb₂Ti₂O₇

Counts (10³ / 6 hrs)

Frustration in three dimensions:

The cubic pyrochlore structure; A network of corner-sharing tetrahedra





Tb₂Ti₂O₇ . Spin Liquid



µSR Studies of Magnetic Ground States in:

 $\begin{array}{c} Tb_2Ti_2O_7\\ Tb_2Mo_2O_7\\ Y_2Mo_2O_7 \end{array} \end{array} \\$



Tb₂Mo₂O₇ : Spin Liquid and Spin Glass

Y₂Mo₂O₇. Spin Glass