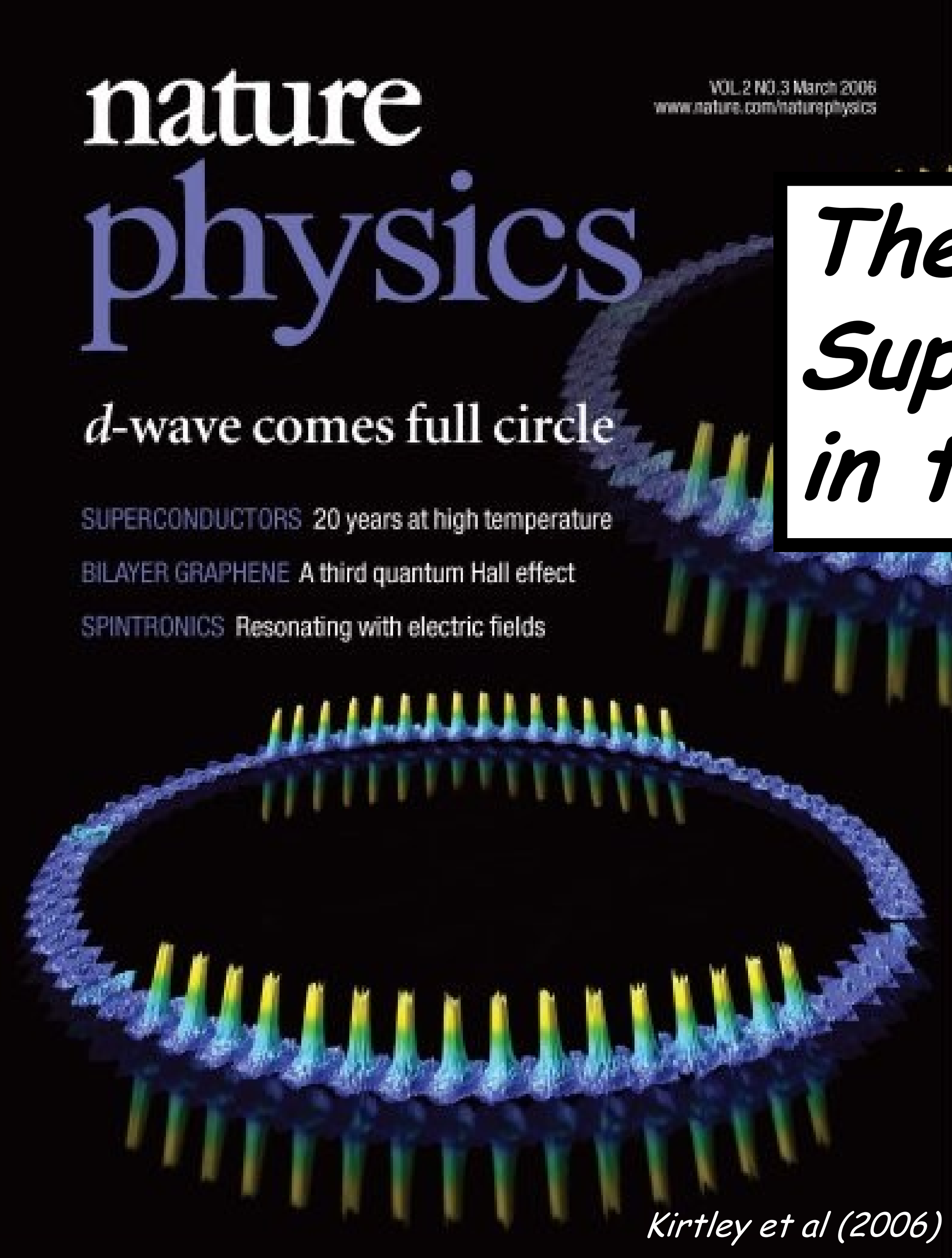
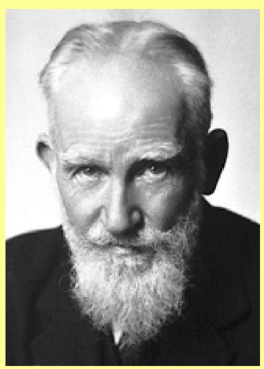


Kirtley et al (2006)

The Nature of Superconductivity in the Cuprates

Elisabeth Nicol
University of Guelph

1957 Bardeen, Cooper and Schreiffer solved the problem of superconductivity



"Science … never solves a problem without creating ten more"

G.B. Shaw

Nobel Prize (Literature, 1925)

Adapted from I. Vekhter

1957 Bardeen, Cooper and Schreiffer solved the problem of superconductivity

High Tc Cuprates

Heavy Fermions

Organics

Pnictides

Noncentrosymmetric SCs

Magnesium Diboride

Doped-Fullerenes

Strontium Ruthenate

1957 Bardeen, Cooper and Schreiffer solved the problem of superconductivity

BCS Theory:

- ***electrons pair***
- ***spin singlet***
- ***s-wave symmetry***
- ***electron-phonon interaction***

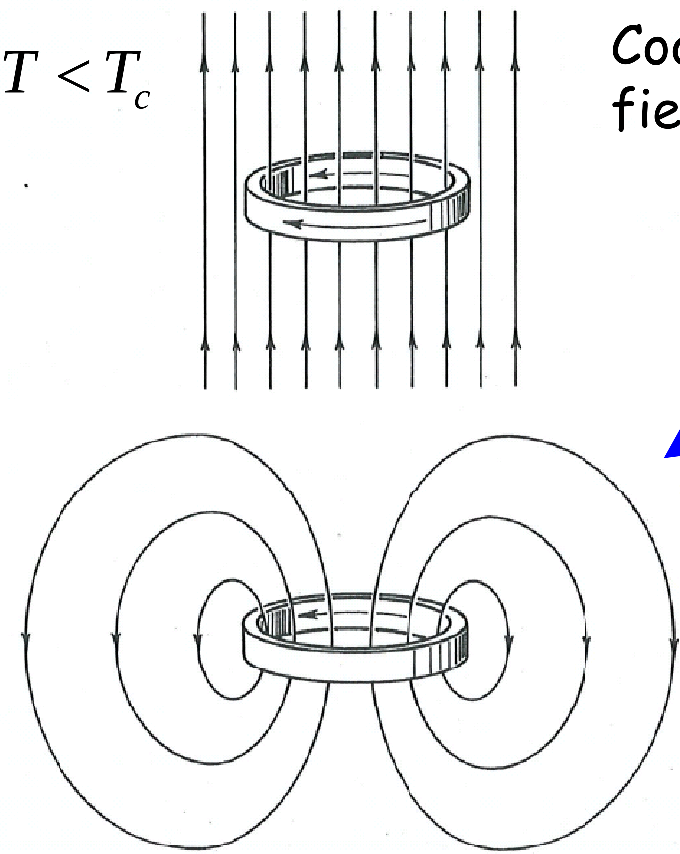
Is it the same or different for cuprate superconductors? How do we know?

How do we know that electrons pair?

Flux quantization in a superconducting ring

$T < T_c$

Cool in magnetic field



then remove magnetic field

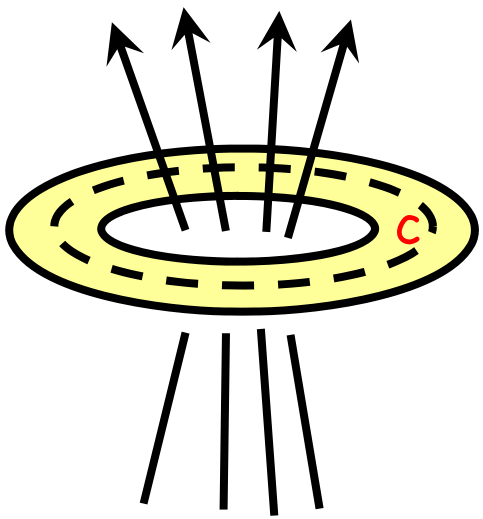
Flux quantized in units of hc/q

q=2e for Cooper pair

Picture from Eisberg and Resnick (1974, Wiley)

How do we know that electrons pair?

Flux quantization in a superconducting ring



$$\oint_c \vec{j} \cdot d\vec{l} = 0 \qquad \psi = |\psi_0| e^{i\phi}$$

$$\vec{j} = |\psi_0|^2 \frac{2e}{m}\left(\hbar \vec{\nabla}\phi - \frac{2e}{c}\vec{A}\right)$$

charge 2e
Cooper pair

$$\frac{\hbar c}{2e}\underbrace{\oint_c \vec{\nabla}\phi \cdot d\vec{l}}_{2\pi n} = \underbrace{\oint_c \vec{A}\cdot d\vec{l}}_{\Phi}$$

phase is single-valued

$$\Rightarrow \quad \boxed{\Phi = \frac{\hbar c}{2e} 2\pi n = n\Phi_0}$$

$$\boxed{\Phi_0 = \frac{hc}{2e}}$$

Ref: Marcel Franz's Lecture CifarSS 2009
Ashcroft and Mermin Chap. 24, p. 749

The charge carriers are paired!

Flux within a superconducting loop must be quantized in units of hc/q

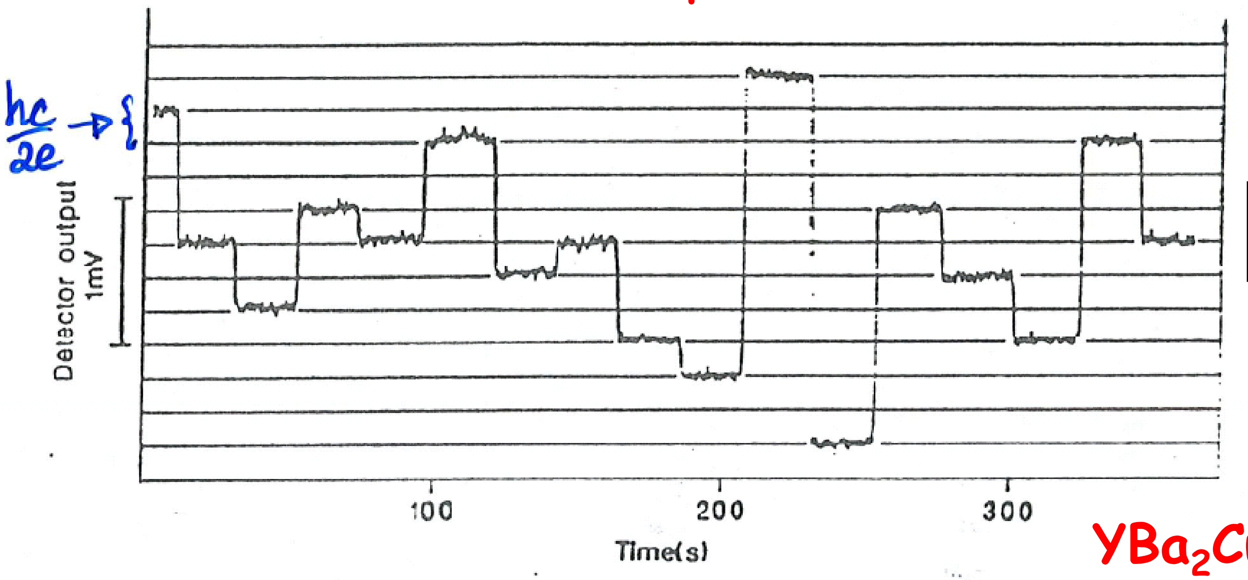


Fig. 2 Output of the r.f.-SQUID magnetometer showing small integral numbers of flux quanta jumping in and out of the ring.

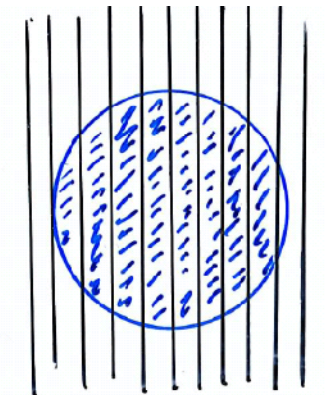
Find q=2e

$YBa_2Cu_3O_{7-\delta}$ (YBCO)

C.E. Gough et al. Nature ***326****, 855 (1987).*

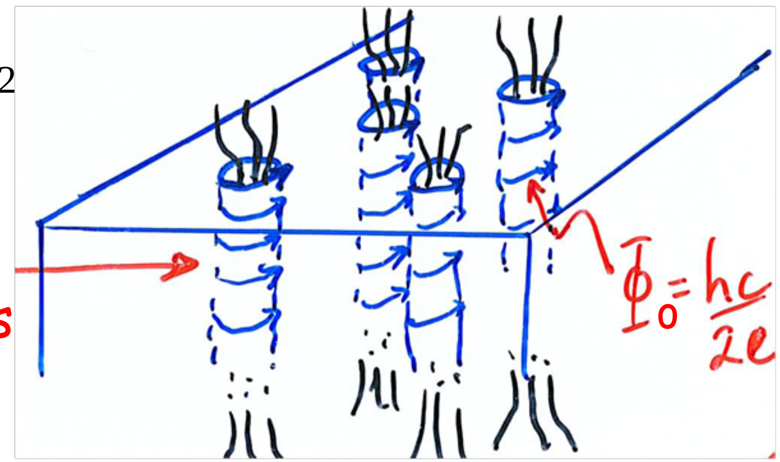
Another way of measuring flux quantum:

The Vortex State



$$H_{c1} < H < H_{c2}$$

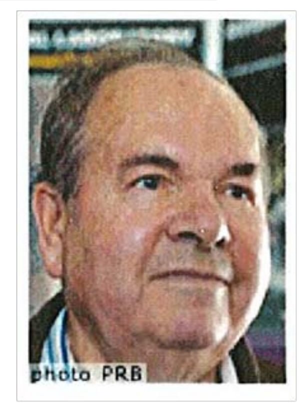
Tubes of magnetic flux called vortices



$$\Phi_0 = \frac{hc}{2e} = 2.07 \times 10^{-7}\,\mathrm{Gcm}^2$$

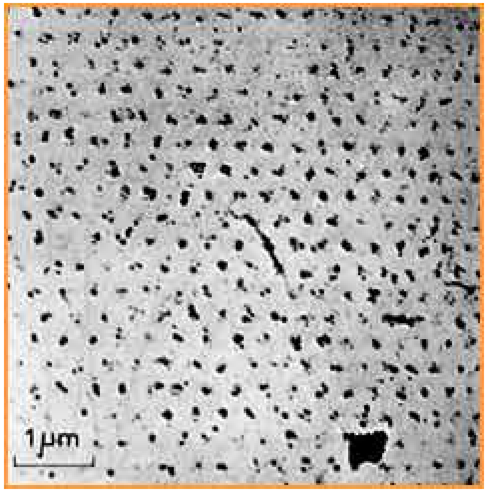
$$(2.07 \times 10^{-15}\,\mathrm{Tm}^2)$$

Predicted by A.A. Abrikosov in 1957
(Nobel Prize - 2003)



Conventional Superconductors

Magnetic or Bitter Decoration



Pb-In

Essmann and Trauble. Phys. Lett. ***24A****, 526 (1967).*

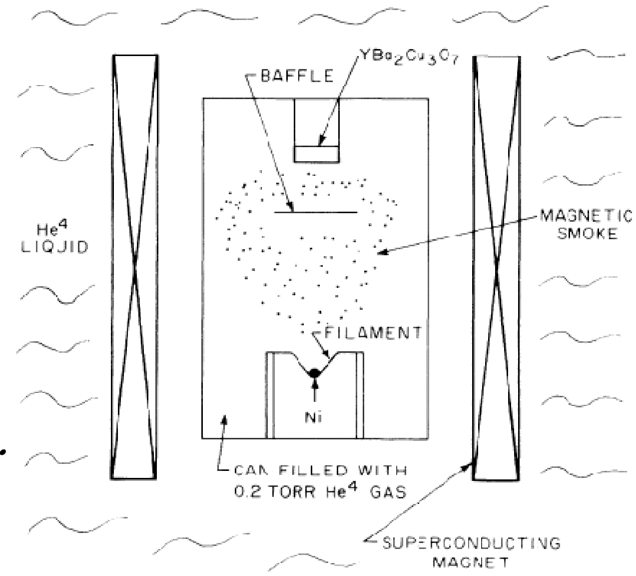


FIG. 1. Sketch of the decoration apparatus.

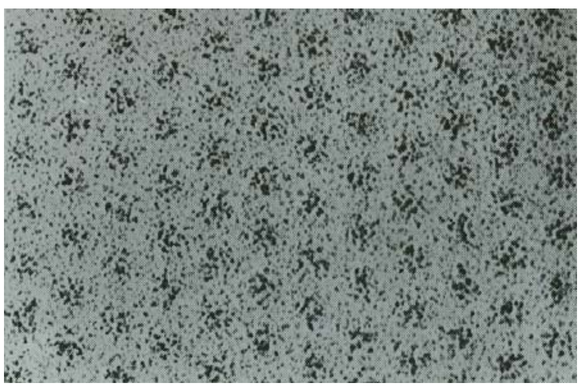
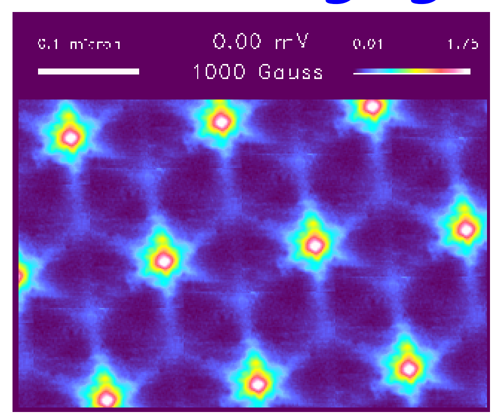


Fig. 34.5 Ashcroft and Mermin

STM imaging

$NbSe_2$



Hess et al. PRL ***62****, 214 (1989).*

Magnetic Decoration

In cuprates the vortex lattice
is more disordered

Gammel et al. PRL **59**, 2592 (1987).

YBCO

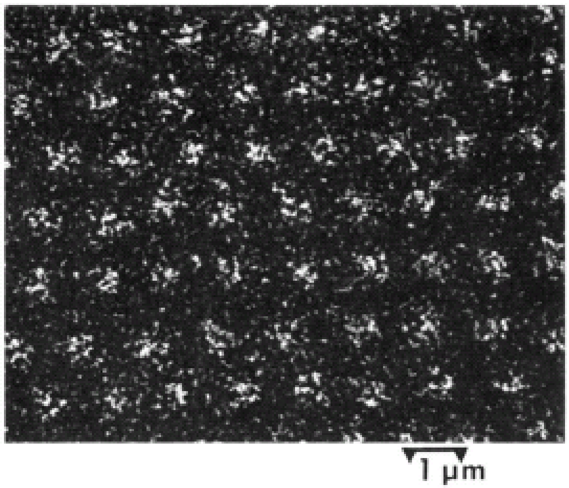
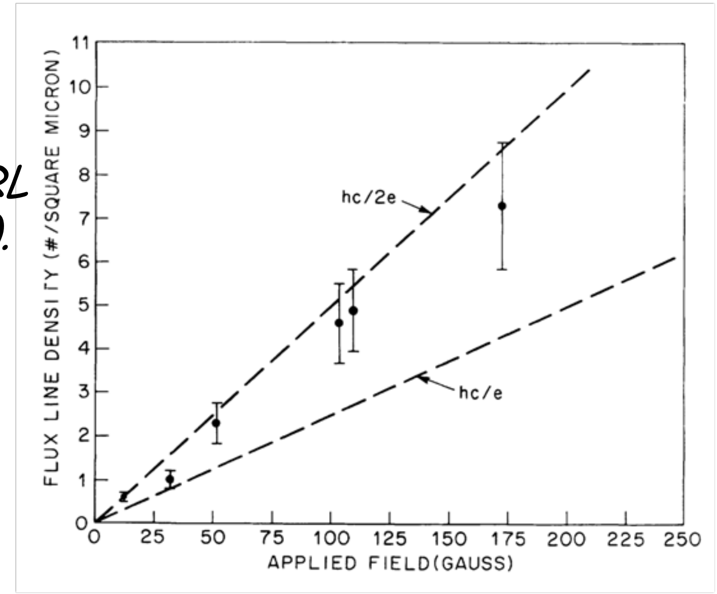
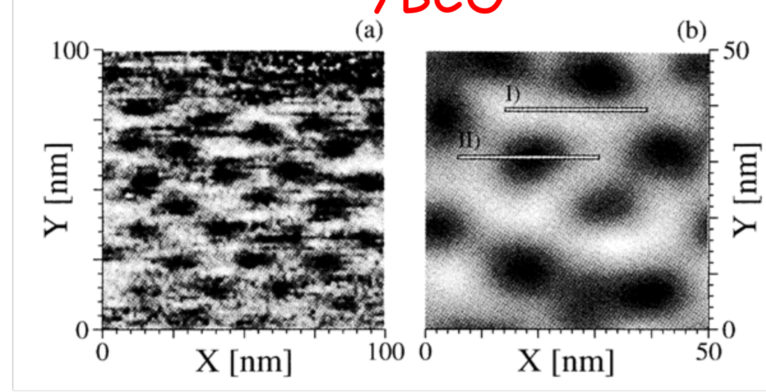


FIG. 2. Flux spots in a $YBa_2Cu_3O_7$ sample decorated after cooling in a field of 13 G.



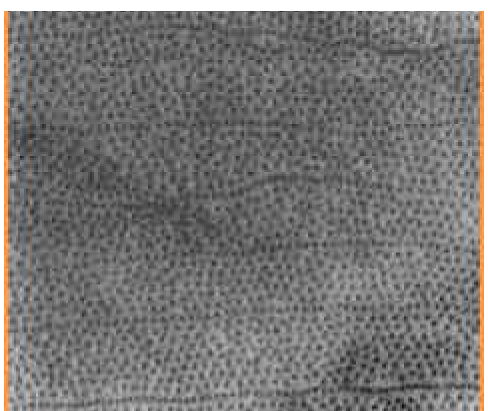
STM imaging

YBCO



Maggio-Aprile et al. PRL **75**, 2754 (1995).

BISCCO



Bolle et al. PRL **66**, 112 (1991).

→ ***charge pairing!***

What is the symmetry of the pair wave function?

Recall for two spin ½ fermions

$$\Psi(\mathbf{r}_1,\mathbf{r}_2,s_1,s_2)=\underbrace{\psi(\mathbf{r}_1-\mathbf{r}_2)}_{\text{spatial}}\underbrace{\chi(s_1,s_2)}_{\text{spin}}\Rightarrow F_{\mathbf{k}}\chi(s_1,s_2)$$

$$F_{\mathbf{k}}=\Delta_{\mathbf{k}}/2E_{\mathbf{k}}$$

$$E_{\mathbf{k}}=\sqrt{\tilde{\varepsilon}_{\mathbf{k}}^{2}+\Delta_{\mathbf{k}}^{2}}$$

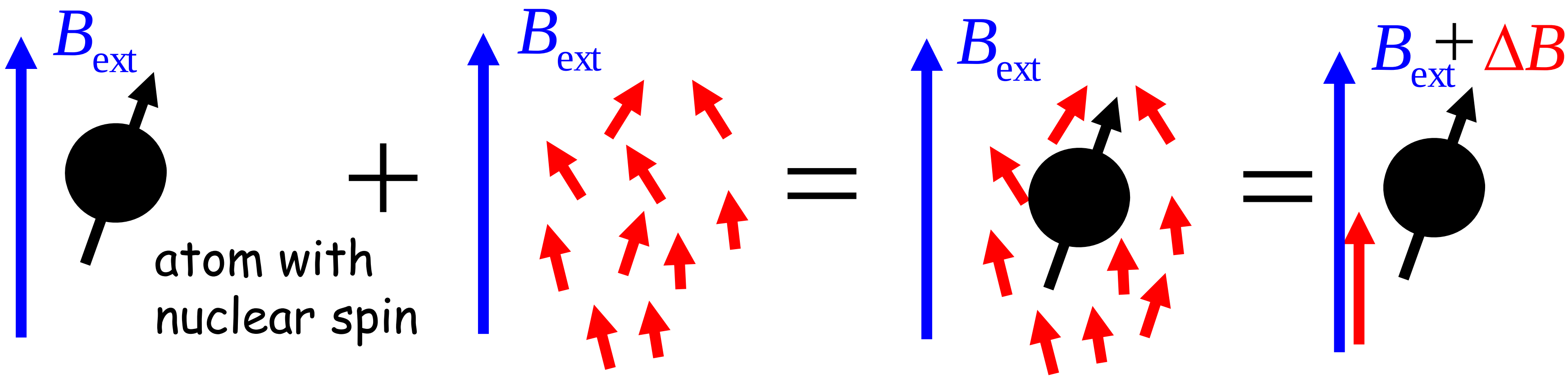
Requiring overall antisymmetry with respect to exchange of two particles

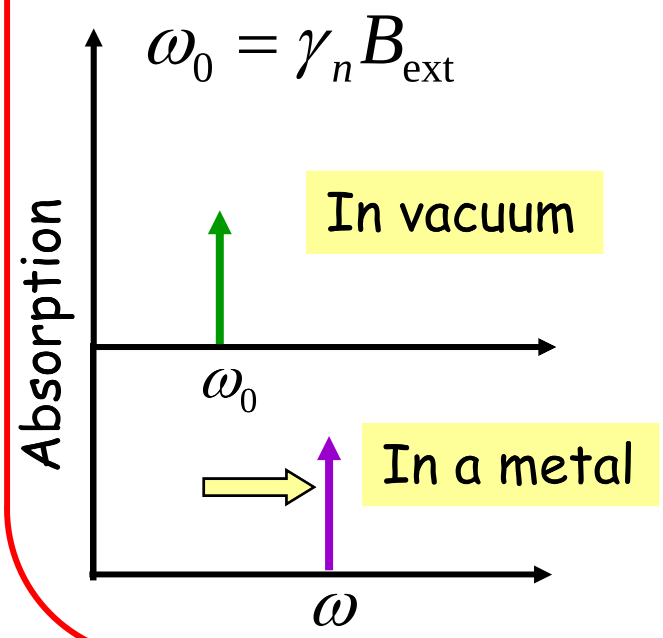
S=0 singlet $\chi=|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle$ → Spatially symmetric $(\Delta_{\mathbf{k}}=\Delta_{-\mathbf{k}})$
L=0 (**s-wave**), L=2 (**d-wave**), etc

S=1 triplet $\chi=|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle,|\uparrow\uparrow\rangle,|\downarrow\downarrow\rangle$

→ Spatially antisymmetric $(\Delta_{\mathbf{k}}=-\Delta_{-\mathbf{k}})$
L=1 (**p-wave**), L=3 (**f-wave**), etc

NMR Knight shift can measure pair spin state



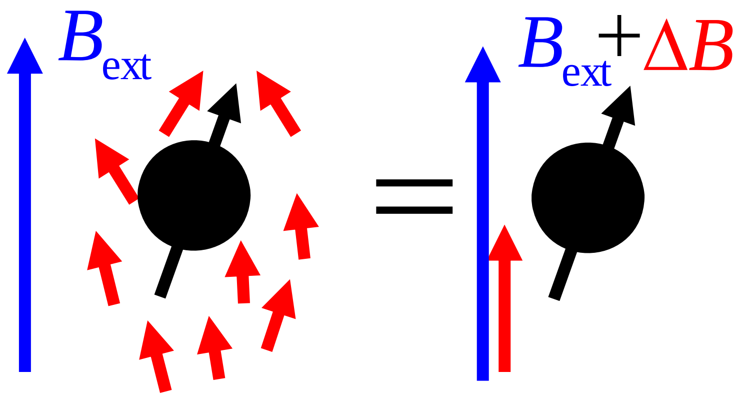


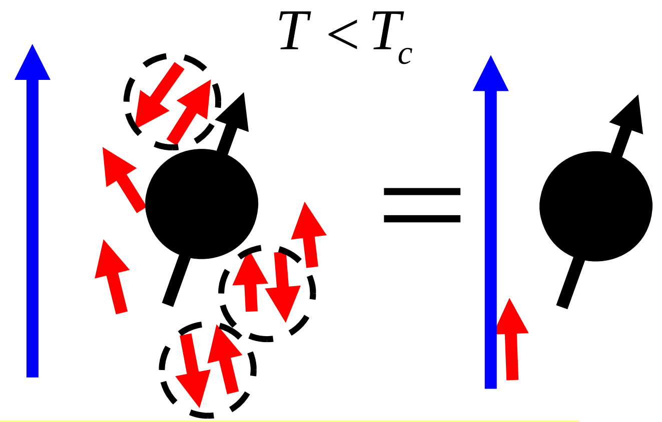
Electrons polarized by B_{ext} to give extra ΔB felt by the atom through hyperfine interaction. This causes a shift in the resonance frequency.

$$\omega = \gamma_n (B_{ext} + \Delta B)$$

$$= \gamma_n B_{ext}(1 + K)$$

Knight Shift



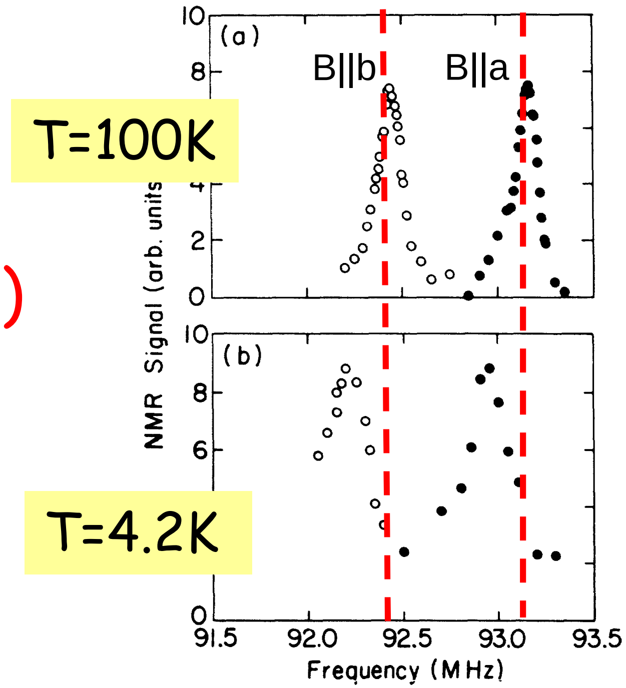


Normal state: polarizability of electrons provides a frequency shift

Superconducting state: less shift if electrons pair in spin singlet (S=0) and therefore do not contribute to ΔB

YBCO Cu(1)

T_c=90K

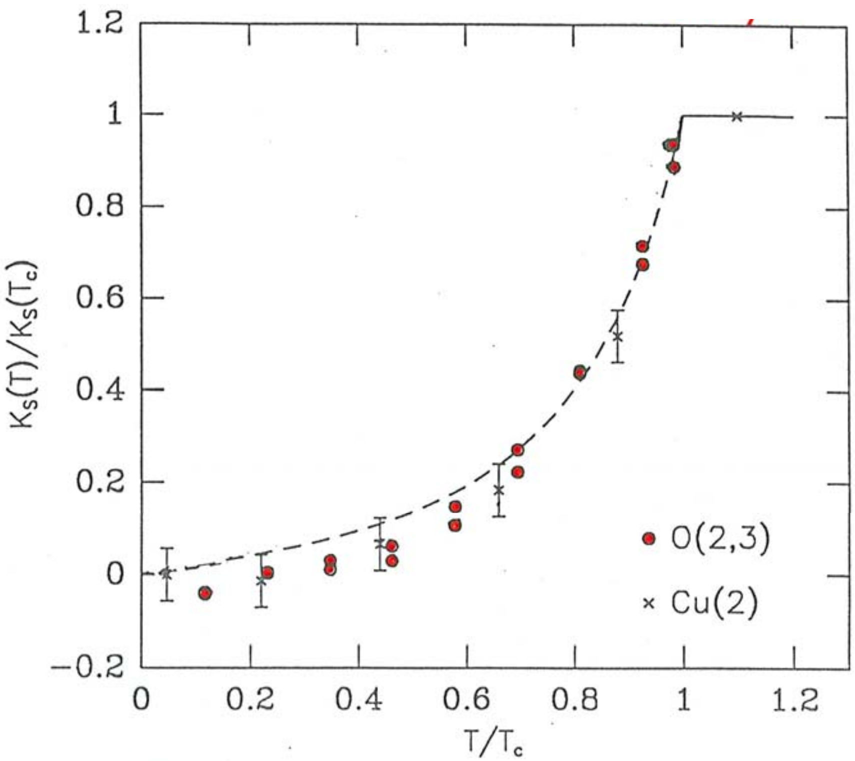


S.E. Barrett et al.
*PRB **41**, 6283 (1990)*

The pairing is a spin singlet state!

The Knight shift is proportional to the electron spin susceptibility χ.
χ->0 as T->0, implies electrons paired such that total spin is 0.

Knight Shift
in NMR



YBCO

*Data of S.E. Barrett et al. PRB **41**, 6283 (1990);*
*M. Takigawa et al. PRB **39**, 7371 (1989); Physica **162-164C**, 853 (1989)*

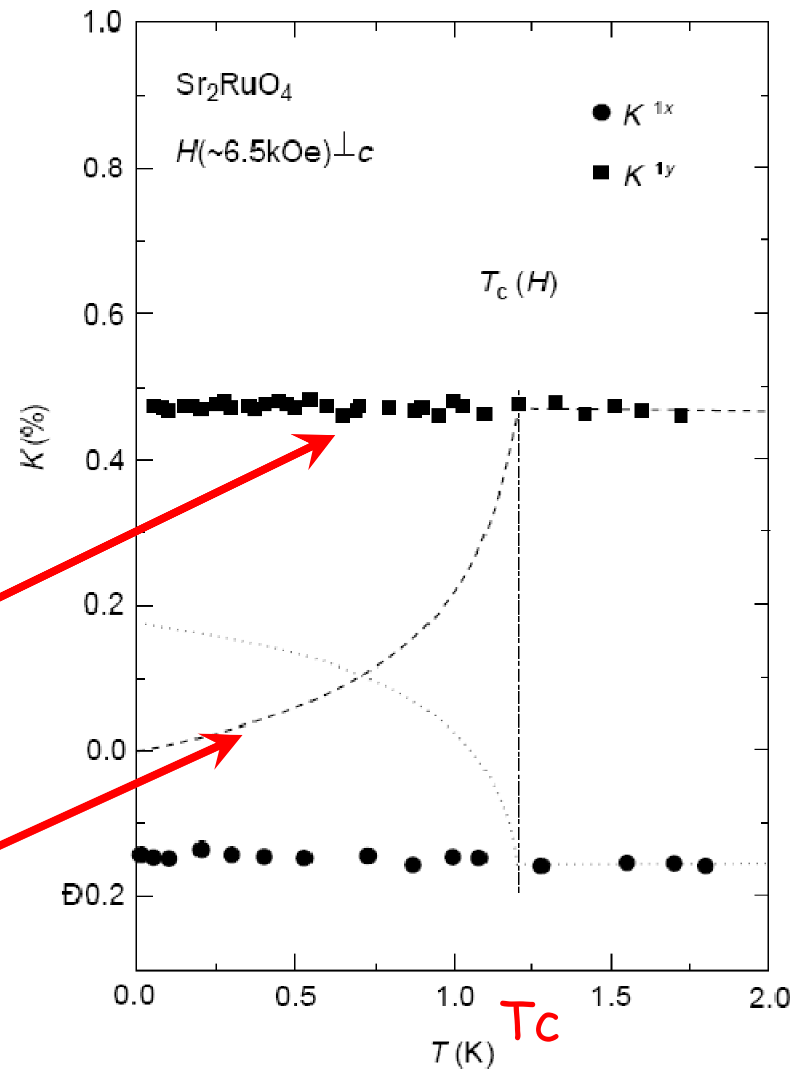
Plot from Scalapino, Physics Reports **250**, 329 (1995)

Spin-triplet superconductivity in Sr_2RuO_4 identified by ^{17}O Knight shift

K. Ishida*, H. Mukuda*, Y. Kitaoka*, K. Asayama*, Z. Q. Mao†‡, Y. Mori† & Y. Maeno†‡

* *Department of Physical Science, Graduate School of Engineering Scienc Osaka University, Toyonaka, Osaka 560-8531, Japan*
† *Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan*
‡ *CREST, Japan Science and Technology Corporation, Kawaguchi, Saitama 332-0012, Japan*

Nature **396**, 658 (1998).

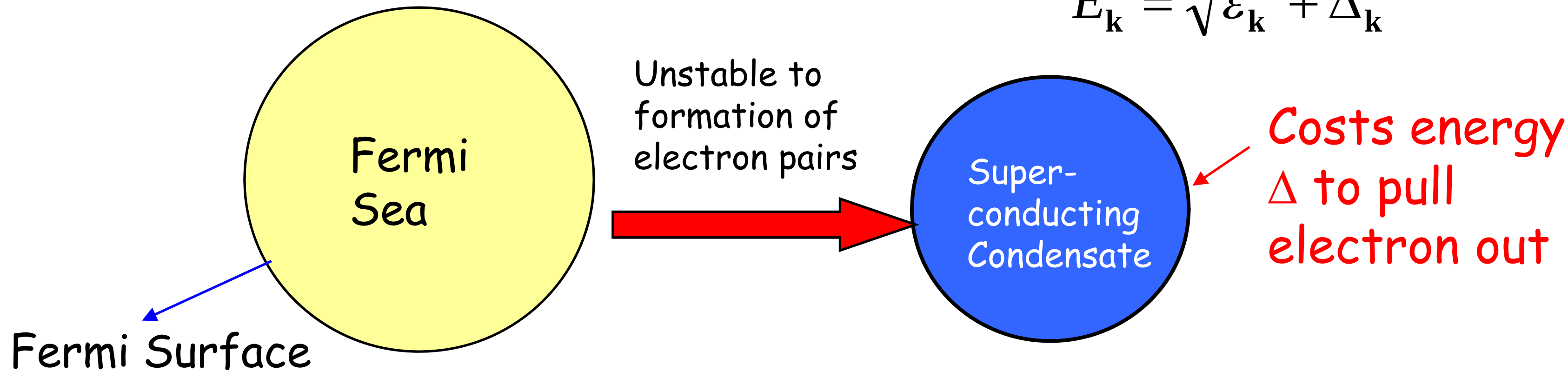


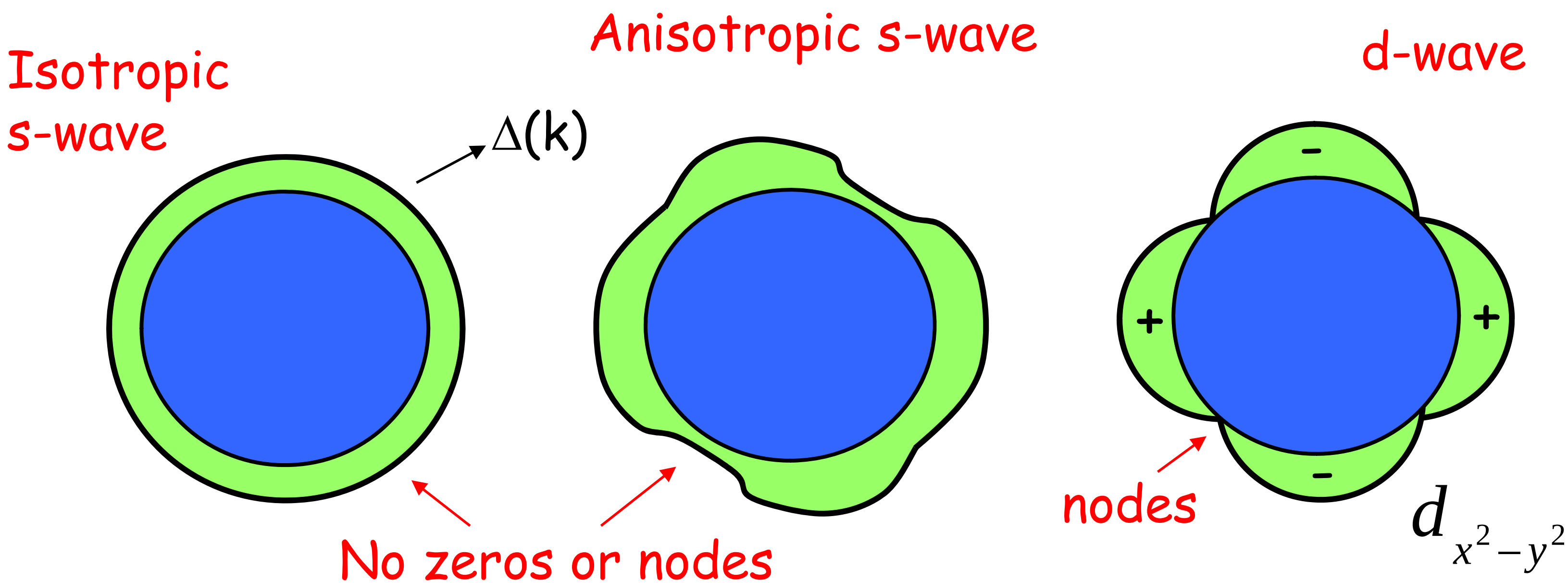
Triplet is flat

Singlet decays below Tc

Big Question: What is the symmetry of the energy gap or order parameter?

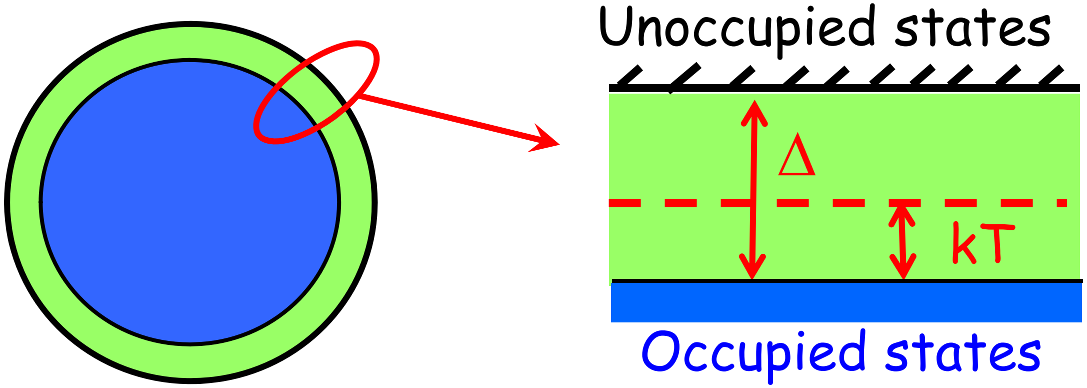
$$E_{\mathbf{k}} = \sqrt{\tilde{\varepsilon}_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$$





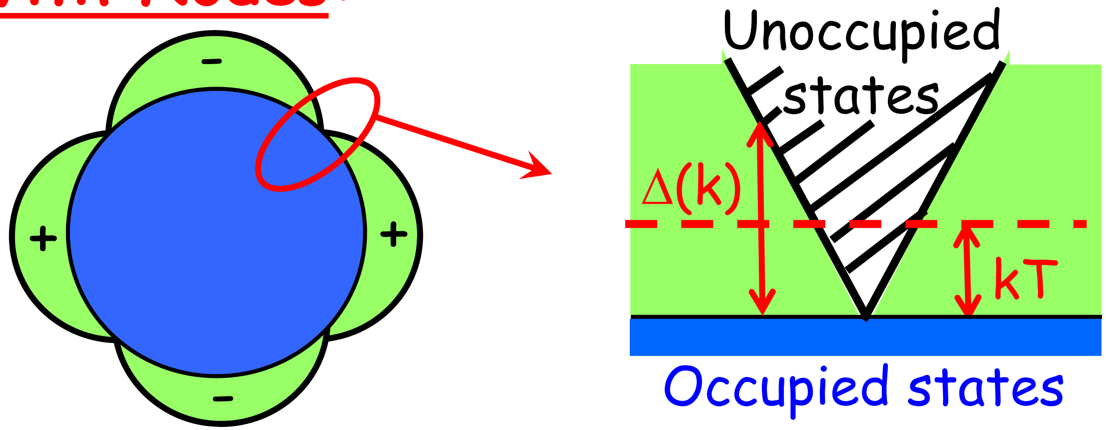
Differences between a finite gap and a gap with nodes

Finite Gap:



Exponential activation at low T:
$C \sim \exp(-\Delta/T)$

With Nodes:



Power laws in T:

$C \sim T^2$

Eg., d-wave $\Delta(k)=\Delta_o \cos(2\phi)$

Density of States and Tunneling

s-wave

$$\Delta_{\mathbf{k}} = \Delta_0$$

transformation of variable

$$E = \sqrt{\tilde{\varepsilon}^2 + \Delta_0^2}$$

$$\int N(0)\,d\tilde{\varepsilon} \rightarrow \int N(0)\frac{|E|}{\sqrt{E^2 - \Delta_0^2}}\,dE$$

for $|E|>\Delta$,
0 otherwise

$$\Rightarrow \quad \frac{N(E)}{N(0)} = \mathrm{Re}\left\{\frac{|E|}{\sqrt{E^2 - \Delta_0^2}}\right\}$$

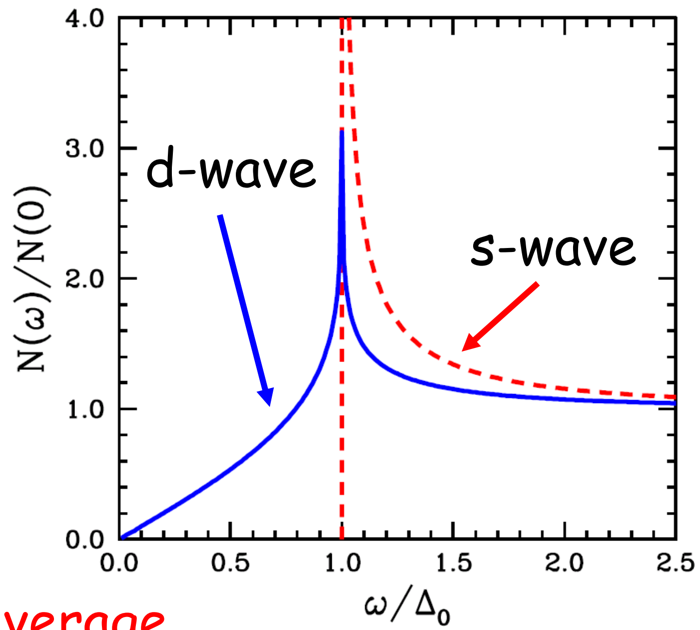
superconducting density of states $N(E)$ in s-wave

d-wave

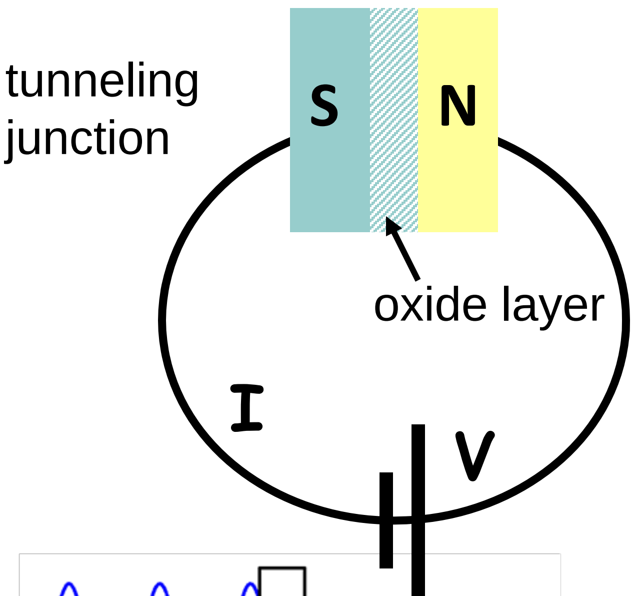
$$\Delta_{\mathbf{k}} = \Delta_0 \cos(2\phi)$$

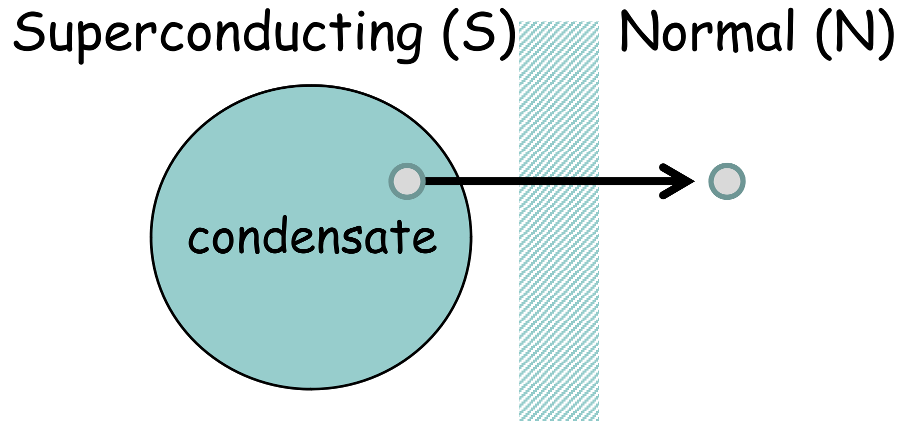
$$\Rightarrow \quad \frac{N(E)}{N(0)} = \left\langle \mathrm{Re}\left\{\frac{|E|}{\sqrt{E^2 - \Delta_{\mathbf{k}}^2}}\right\}\right\rangle$$

angular average



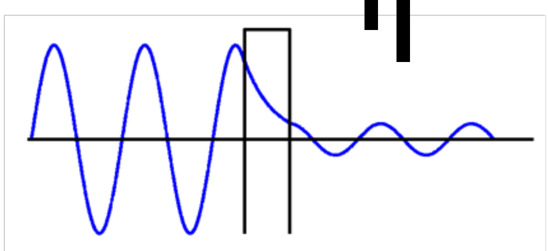
I-V characteristics of a tunneling junction



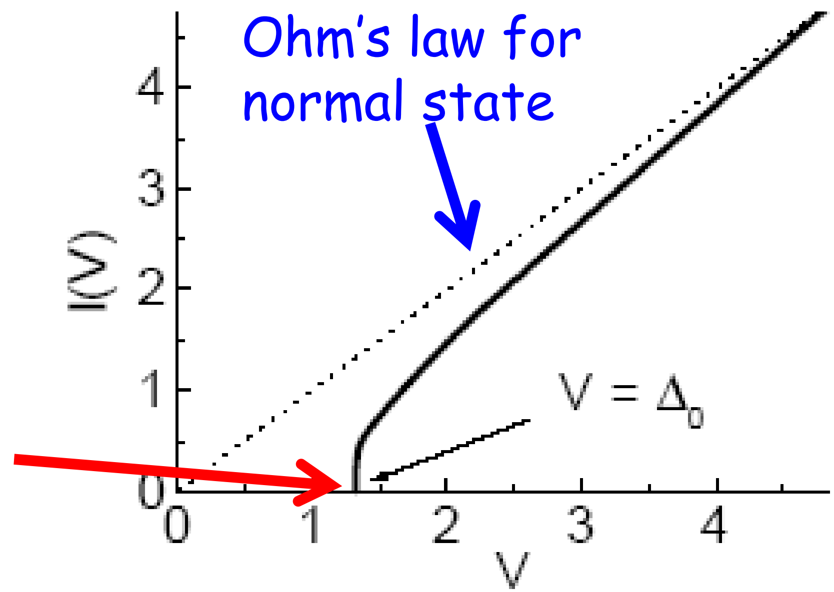


s-wave

Takes energy Δ



Gap in superconducting state means no current till V is equal or greater than Δ



dI/dV at low temperature

s-wave

Pb

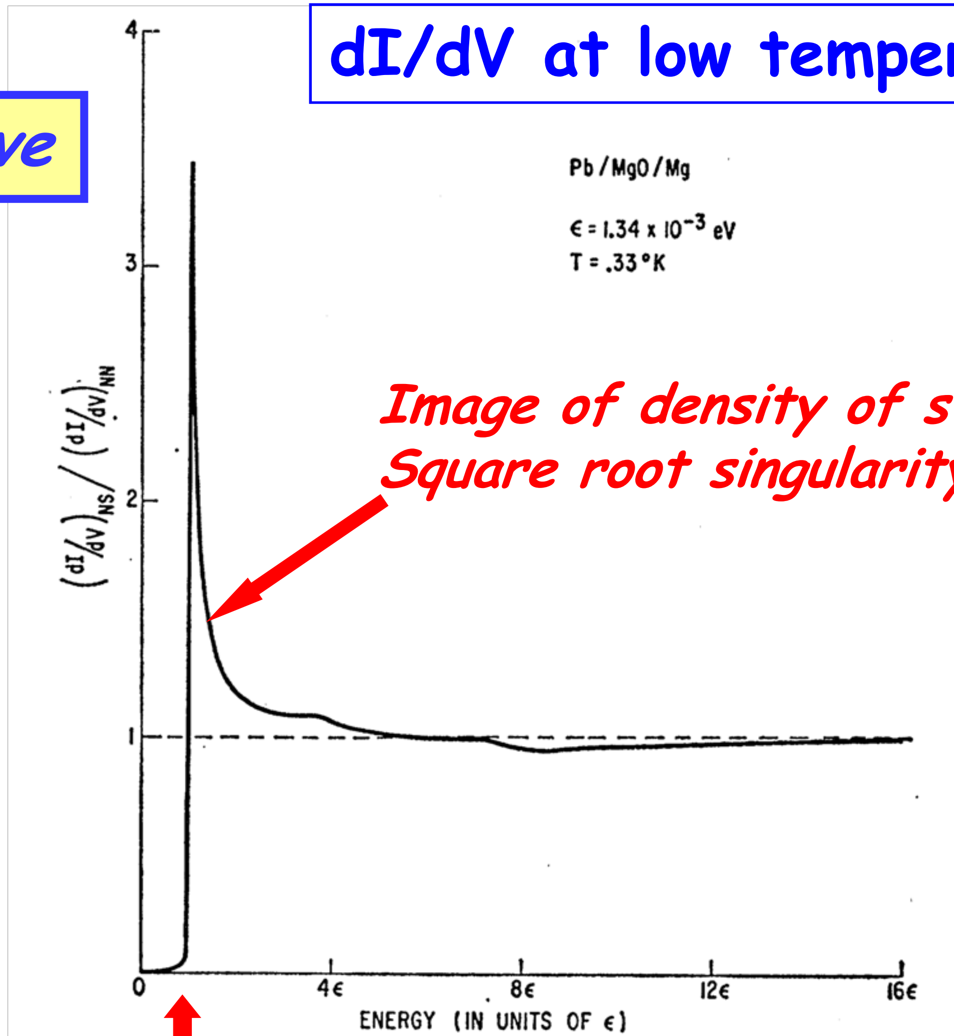
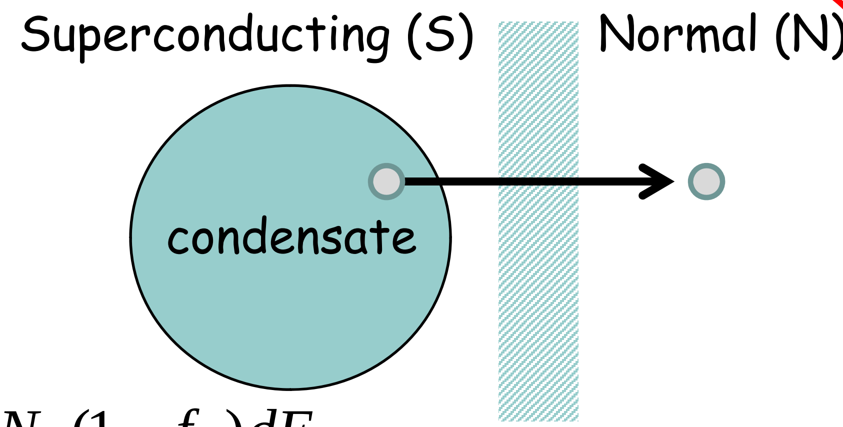


Image of density of states!
Square root singularity in s-wave

Energy gap Δ_0

Giaever, Hart, and Megerle, Phys. Rev. ***126****, 941 (1962).*

How does the dI/dV give the Density of States?



$$I_{tot} = I_{S\to N} + I_{N\to S}$$

$$\propto \int N_S f_S N_N (1-f_N)dE - \int \underbrace{N_N f_N}_{\text{occupied}} \underbrace{N_S (1-f_S)}_{\text{unoccupied}} dE$$

$$\propto \int N_S N_N (f_S - f_N) dE$$

$N_S(E)$

$N_N(E+V) = N(0)$

$f(E)$

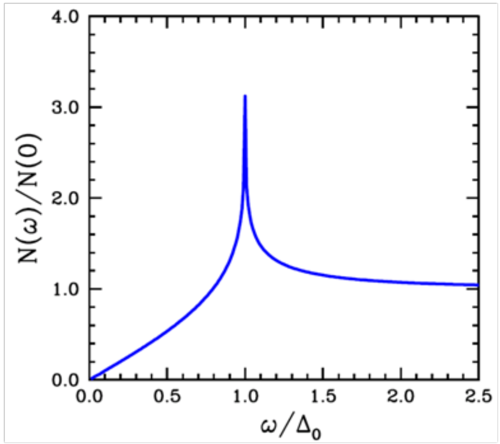
$f(E+V)$

$$I(V) \propto \int dE N_S(E)\left[f(E) - f(E+V)\right]$$

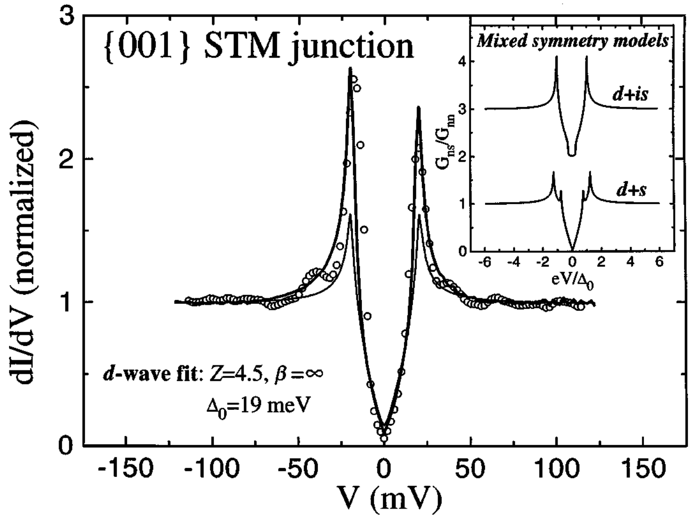
$$\frac{dI(V)}{dV} \propto \int dE N_S(E)\left[-\frac{df(E+V)}{dV}\right]$$

at T=0 $\dfrac{dI(V)}{dV} \propto N_S(V)$

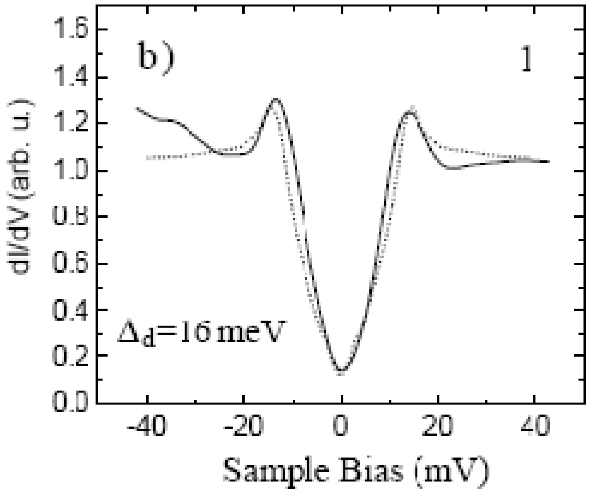
d-wave



*Wei et al., PRL **81**, 2542 (1998).*



STM on YBCO single crystal

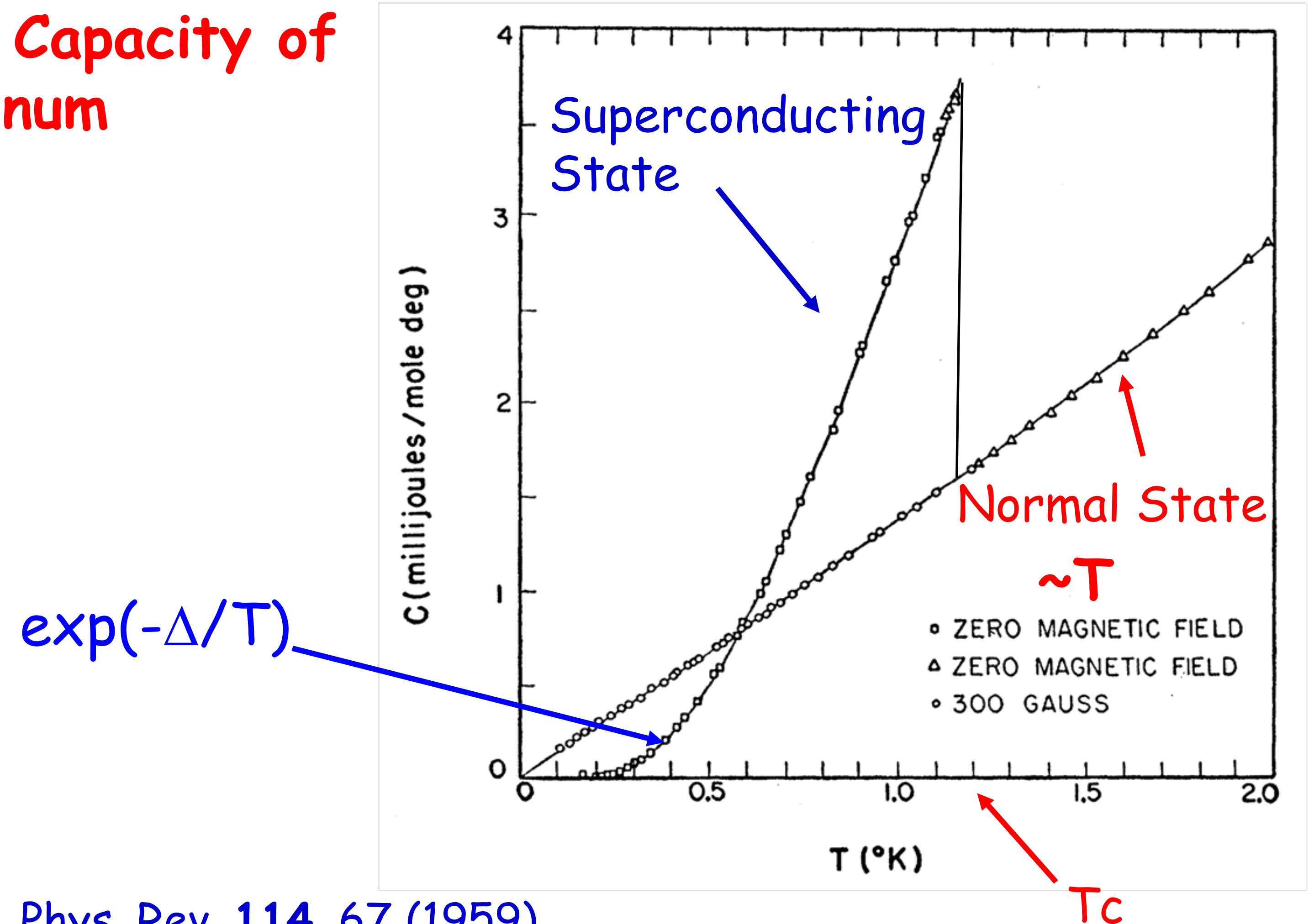


STM on YBCO thin film

*Sharoni et al., EPL **54**, 675 (2001).*

Classic BCS s-wave gap

Heat Capacity of Aluminum



Phillips, Phys. Rev. **114**, 67 (1959)

A Reminder about Heat Capacity

$$C_V = \frac{dU}{dT}$$

$$U = \int N(\varepsilon) f(\varepsilon) \varepsilon d\varepsilon$$

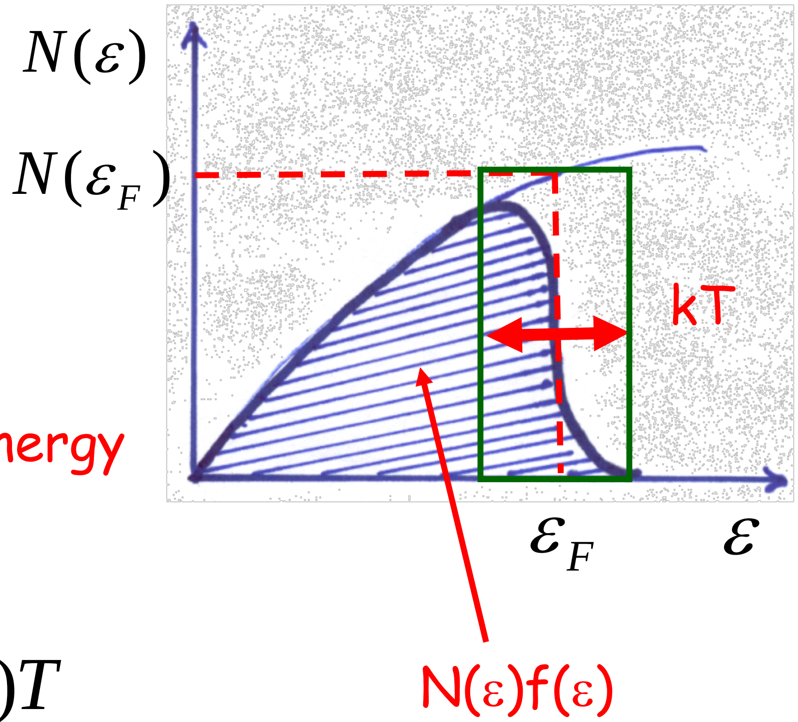
Free Electrons:

of electrons

$$U \approx [N(\varepsilon_F)kT]kT$$

energy

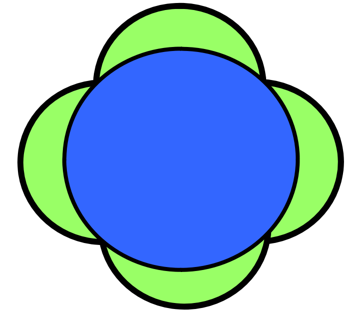
$$\approx N(\varepsilon_F)T^2$$

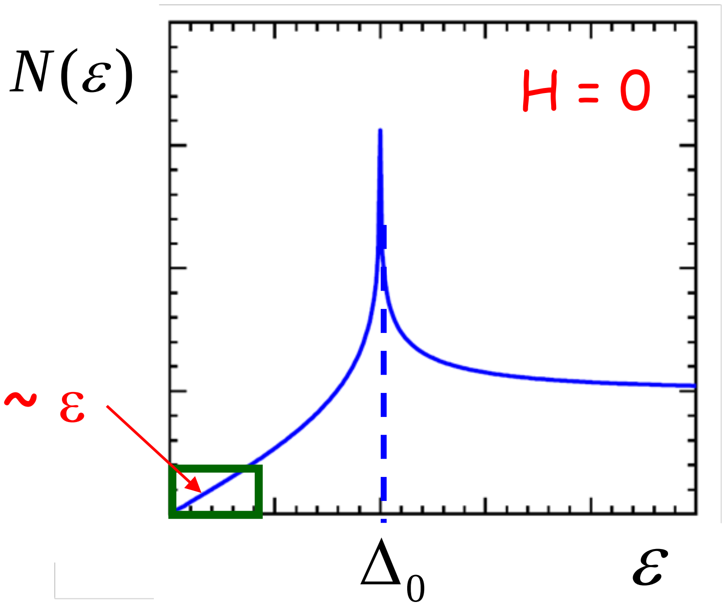


$$C_V = \frac{dU}{dT} \propto N(\varepsilon_F)T$$

Gap Anisotropy: the case of nodes

Eg.: d-wave superconductor $\Delta(k) = \Delta_0 \cos(2\phi)$



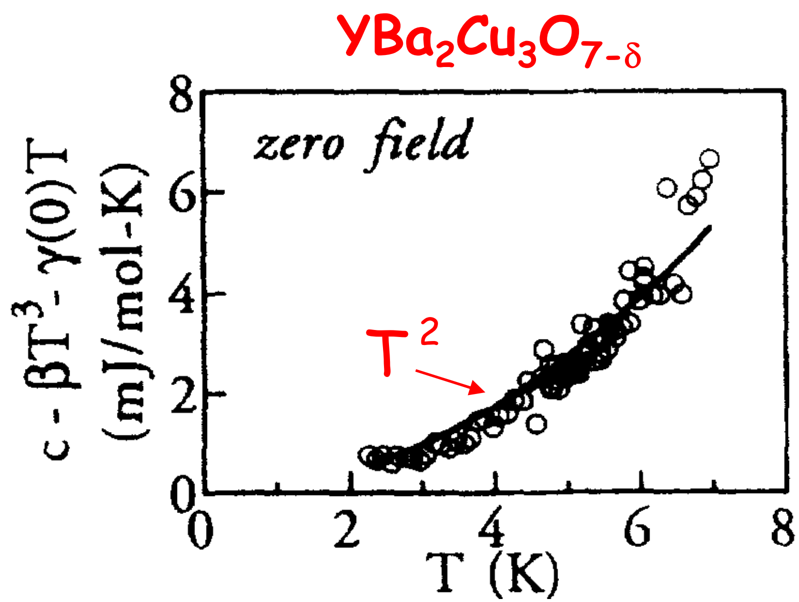


$$U \approx N(\varepsilon)T^2 \approx (kT)T^2$$

ε ~ kT

$$C_V \propto T^2$$

$$(T < \Delta_0)$$

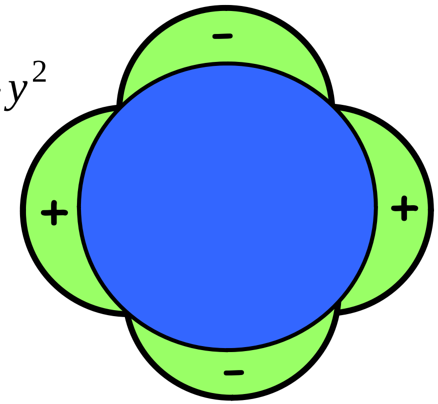


K. Moler et al., PRL **73**, 2744 (1994).

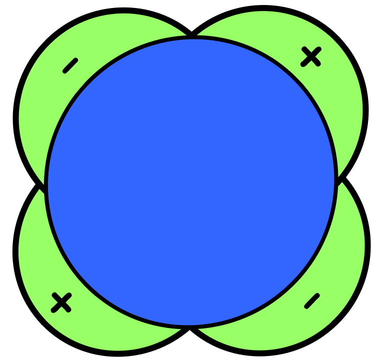
Power laws point to nodes in the gap.

But can we know where the nodes are?

$d_{x^2-y^2}$

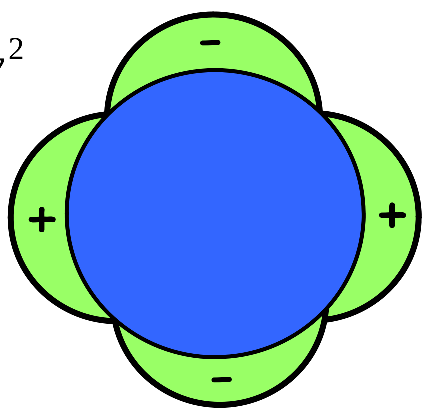


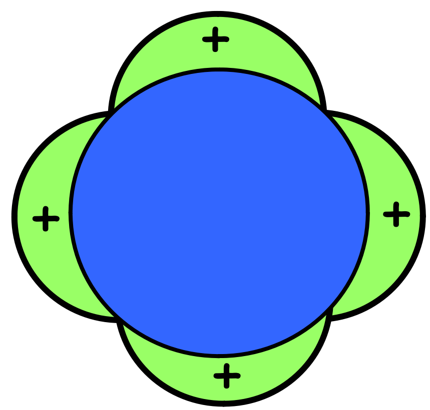
d_{xy}



Can we be sure that there is a sign change?

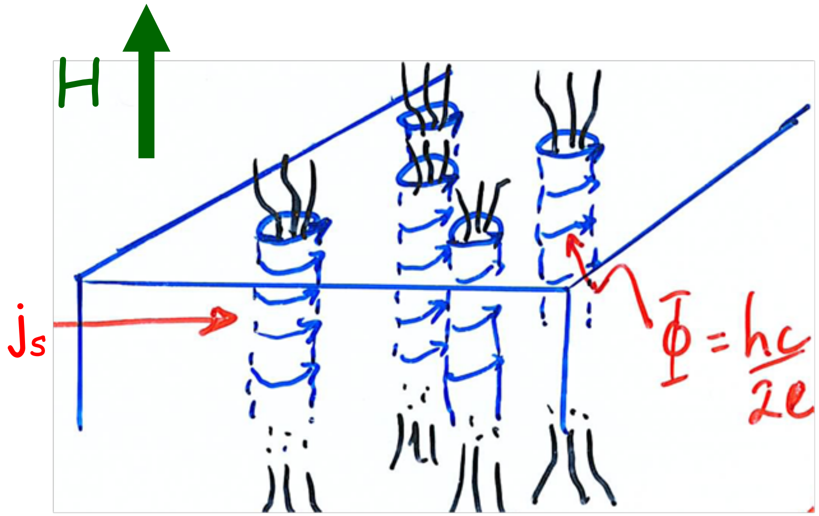
$d_{x^2-y^2}$





Can we know where the nodes are?

- Use a magnetic field!



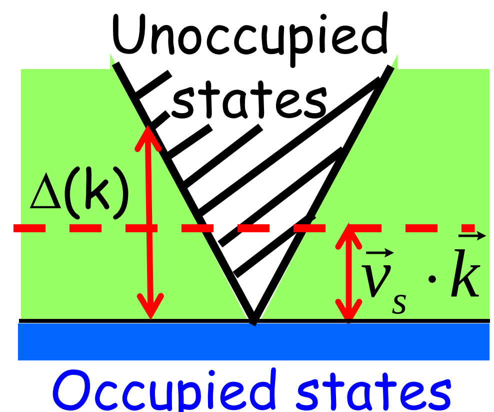
Vortex State

- circulating supercurrents

$$\vec{j}_s = 2en\vec{v}_s$$

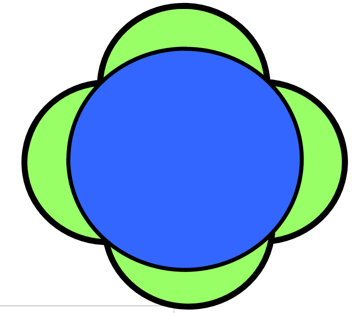
- quasiparticles in presence of supercurrents have a Doppler shift to their energy: $E'_{\vec{k}} = E_{\vec{k}} + \vec{v}_s \cdot \vec{k}$

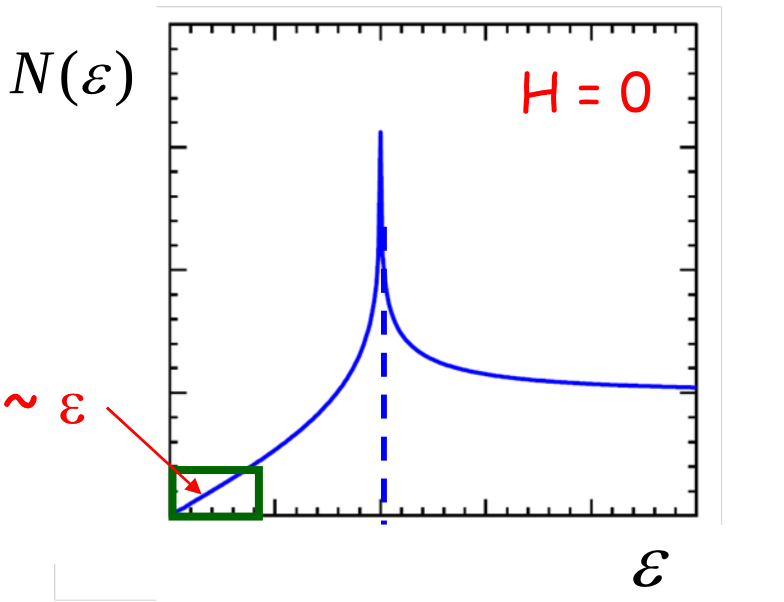
- now magnetic field H replaces T
- $\vec{v}_s$ depends on H
- H can be orientated in different directions

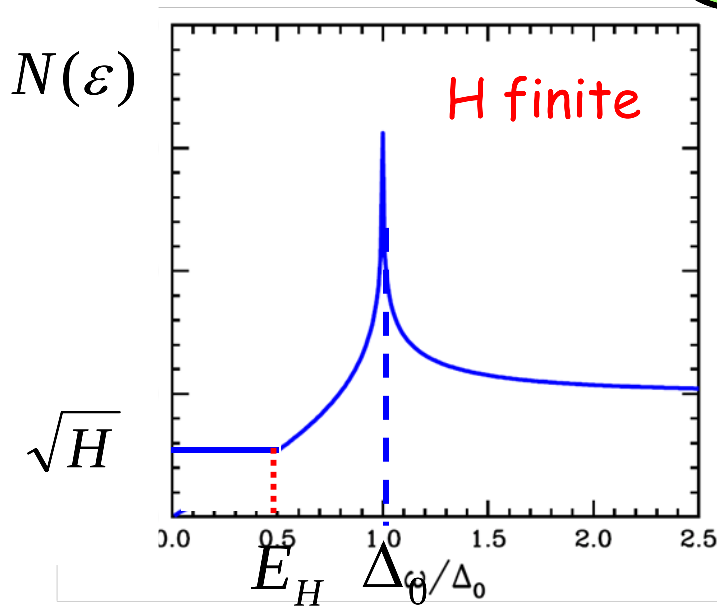


Effect of a Magnetic Field:

Eg.: d-wave superconductor $\Delta(k) = \Delta_0 \cos(2\phi)$







$$U \approx N(\varepsilon)T^2 \approx (kT)T^2$$

ε ~ kT

$$C_V \propto T^2$$

$$(T < \Delta_0)$$

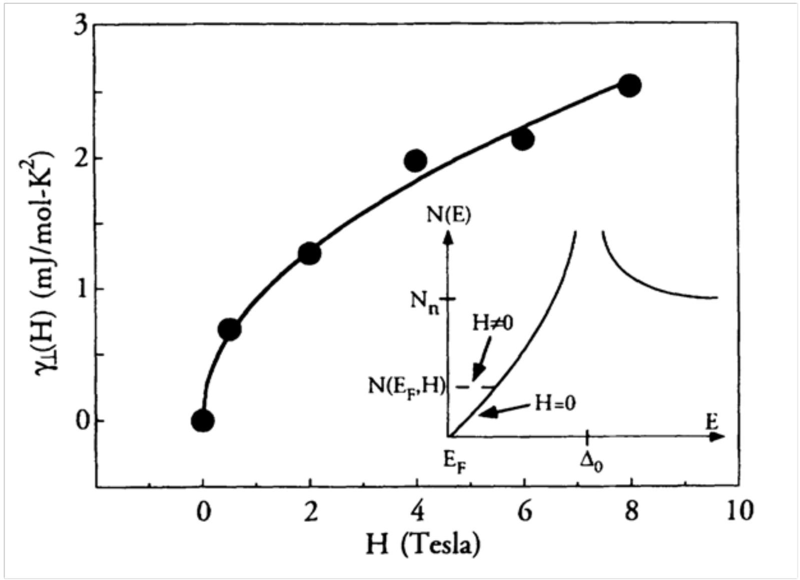
$$U \approx N(\varepsilon)T^2 \approx \sqrt{H}T^2$$

$$C_V \propto T\sqrt{H}$$

$$(T < E_H)$$

H || c: Evidence for Nodes and d-wave Gap

$\frac{C}{T}$



$YBa_2Cu_3O_{7-\delta}$

K. Moler et al., PRL **73**, 2744 (1994).

[And other groups, too!]

$$C \propto T\sqrt{H}$$

Volovik, JETP Lett. **58**, 469 (1993).

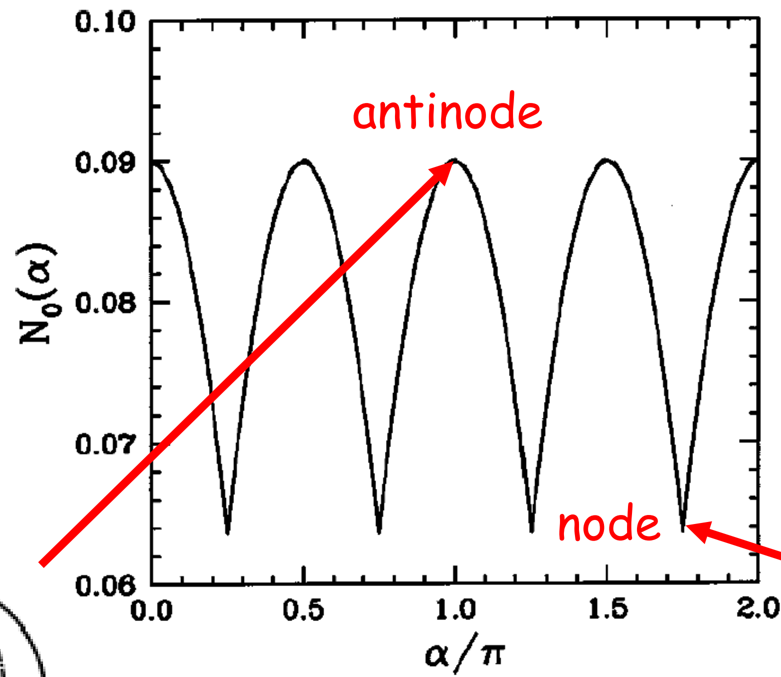
Theory

H in a-b plane:

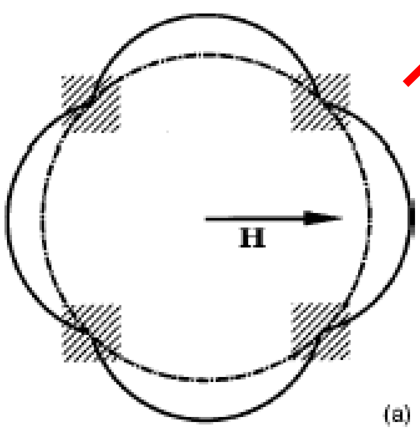
$$N_0(H,\alpha) \propto \sqrt{H}\,\max(|\sin\alpha|,|\cos\alpha|)$$

Oscillations in residual density of states as H rotated in plane!

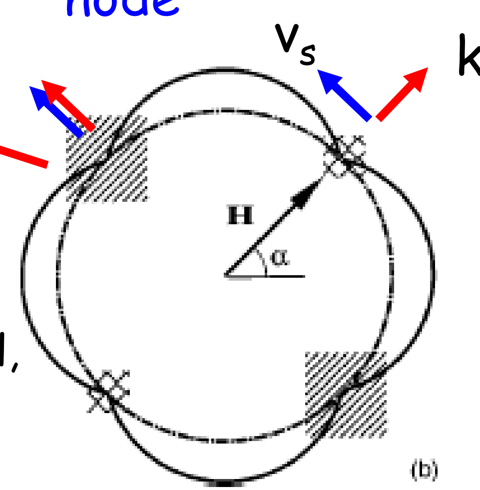
$$C \propto N_0(H,\alpha)T$$



Field in direction of antinode



Field in direction of node

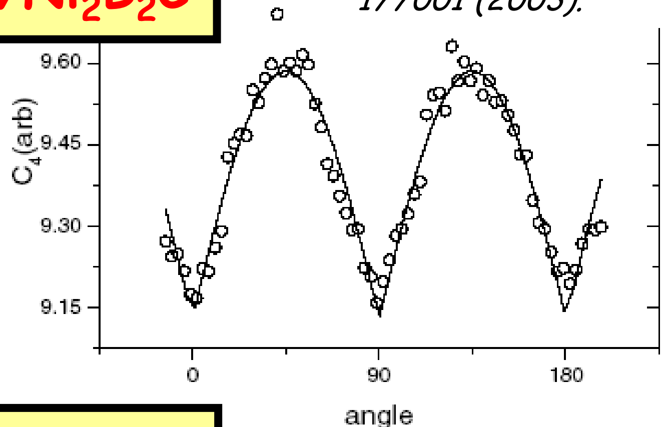


Vekhter, Hirschfeld, Carbotte, Nicol, PRB **59**, R9023 (1999);
Vekhter, Hirschfeld, Nicol, PRB **64**, 064513 (2001)

Experimental Observation of Oscillations in Specific Heat!

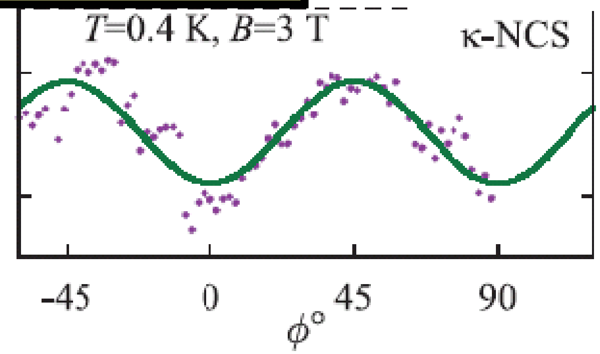
YNi_2B_2C

T. Park et al., PRL **90**, 177001 (2003).



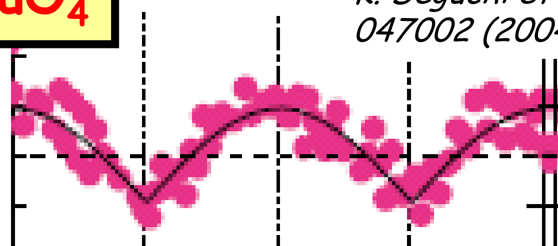
κ-$(ET)_2Cu(NCS)_2$

Malone et al., arXiv:0905.2342



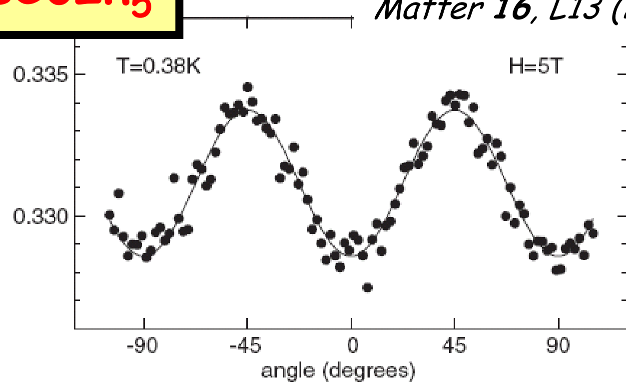
Sr_2RuO_4

K. Deguchi et al., PRL **92**, 047002 (2004)



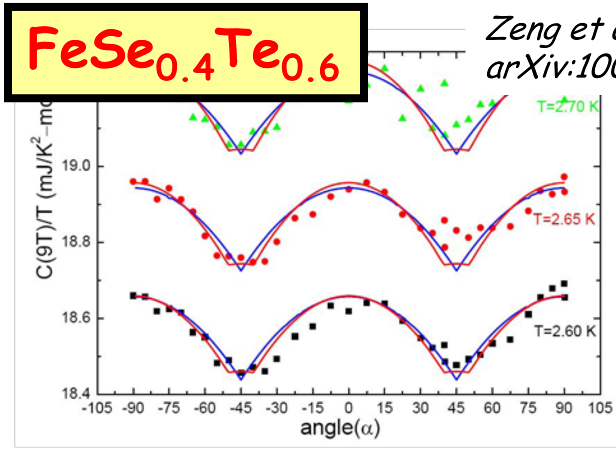
$CeCoIn_5$

H. Aoki et al., J. Phys.: Condens. Matter **16**, L13 (2004).



$FeSe_{0.4}Te_{0.6}$

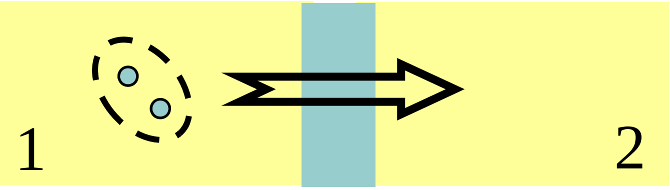
Zeng et al., arXiv:1004.2236.



... but not in the cuprates

Phase sensitive experiments for determination of sign change and orientation

Use ***Josephson Effect*** - Cooper pairs can tunnel to another superconductor

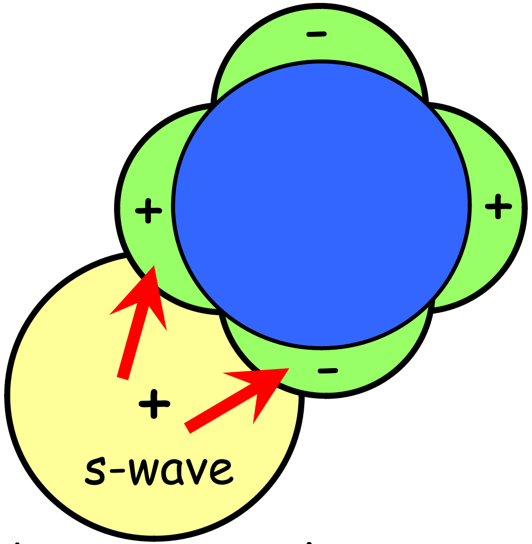


$$\Psi_1 = |\Delta| e^{i\phi_1} \qquad \Psi_2 = |\Delta| e^{i\phi_2}$$

$$I = \frac{\hbar e^* A}{mL} |\Delta|^2 \sin(\phi_2 - \phi_1) = I_0 \sin(\phi_2 - \phi_1)$$

Supercurrent due to phase difference between superconductors

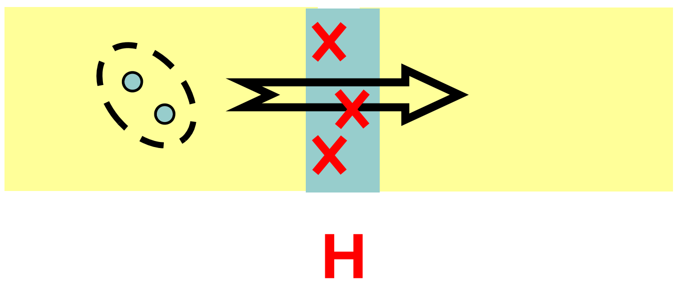
Predicted by Josephson in 1962, Nobel Prize 1973



Use this idea to tunnel into different regions

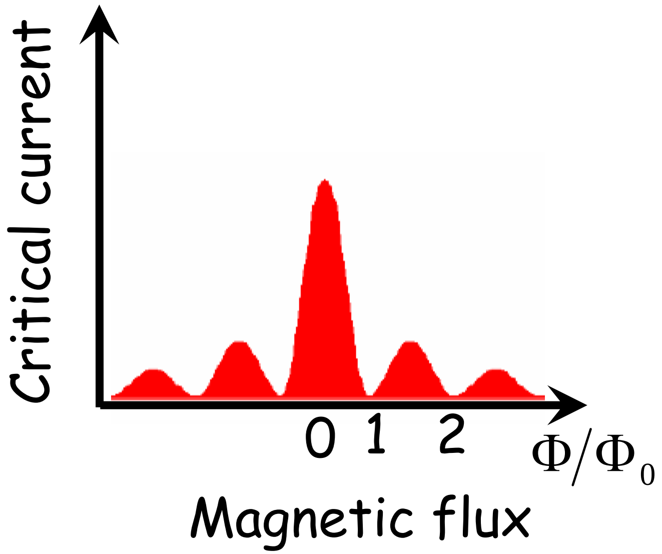
One more thing about Josephson junctions:

Applying a magnetic field through junction barrier



$$I_c(x) = I_0(x)\sin\gamma$$

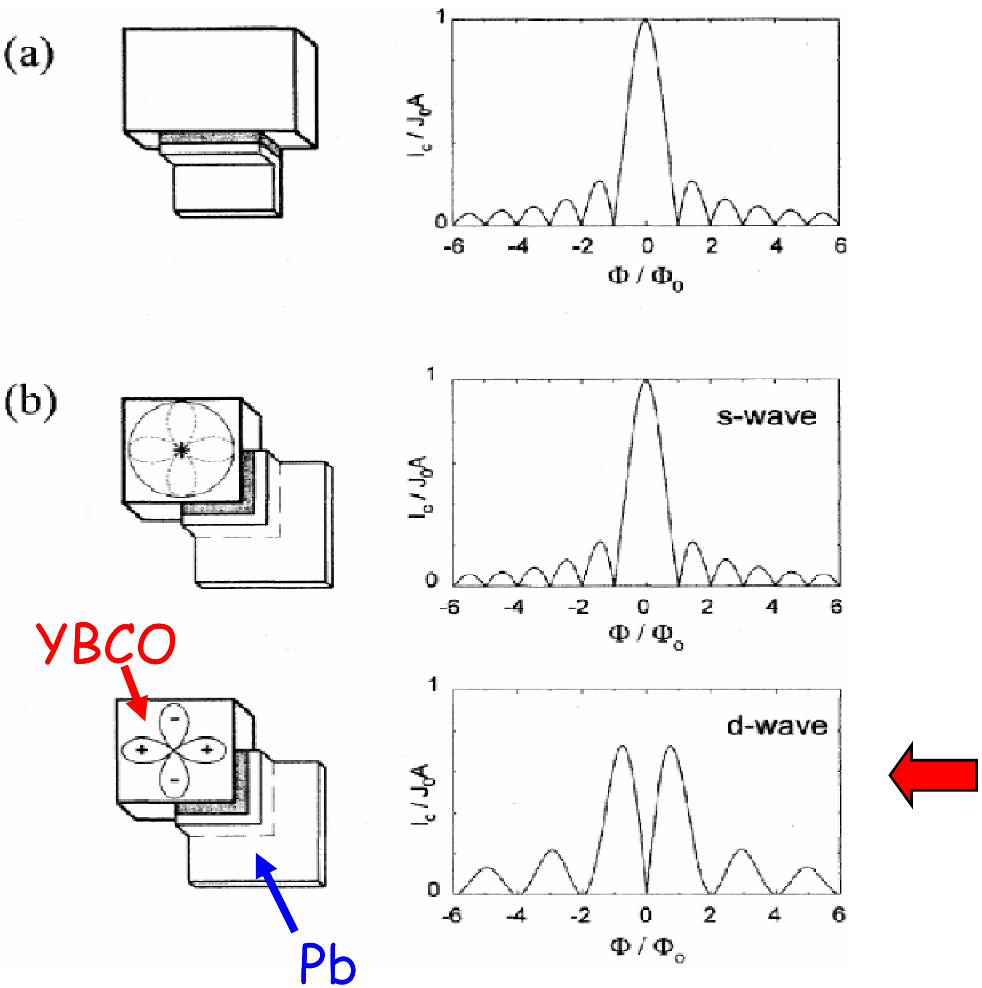
$$\gamma = \int d\vec{l}\cdot\left[\vec{\nabla}\phi - \frac{2e}{\hbar c}\vec{\mathbf{A}}\right]$$

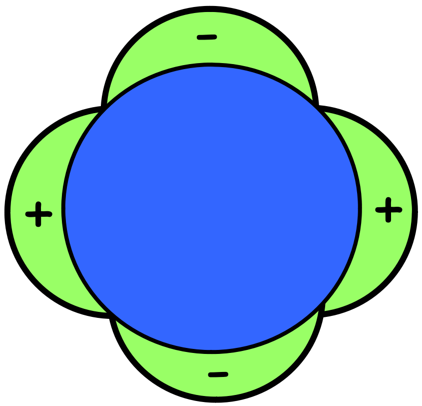


Fraunhofer pattern analogous to single slit diffraction

$$I_c(\Phi) = I_0\left|\frac{\sin(\pi\Phi/\Phi_0)}{(\pi\Phi/\Phi_0)}\right|$$

Evidence for phase of order parameter





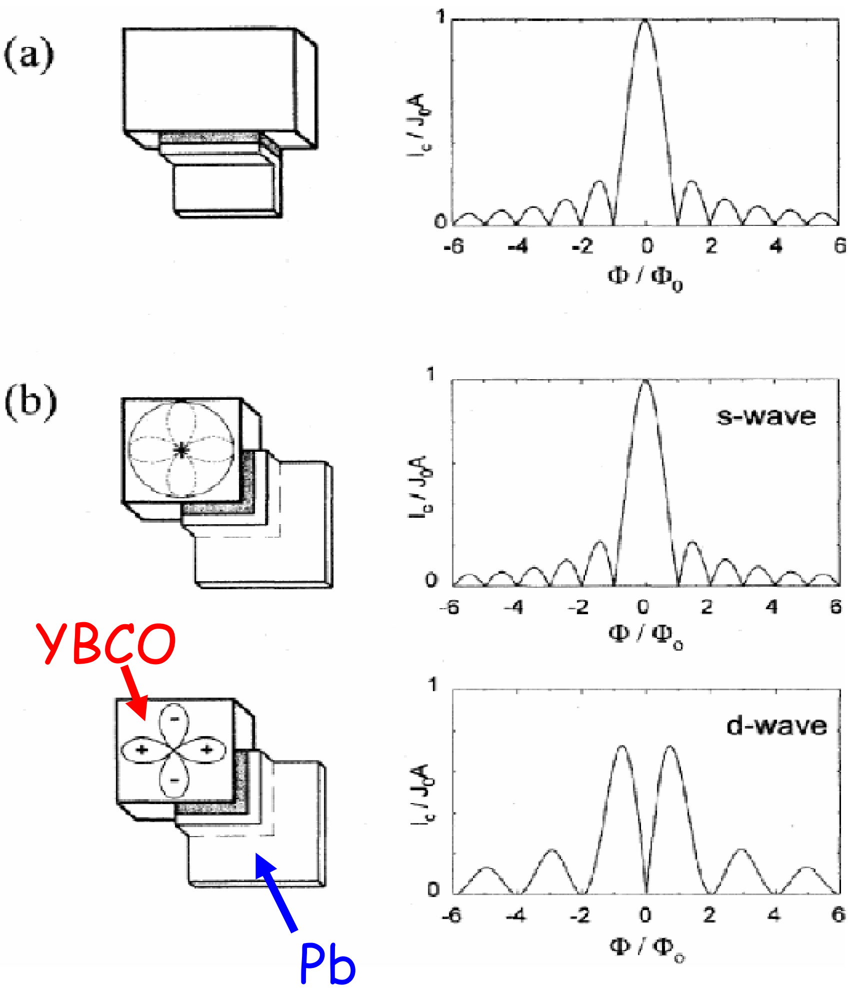
Phase difference of $\Delta\phi$ on the right and $\Delta\phi+\pi$ on bottom gives net current:

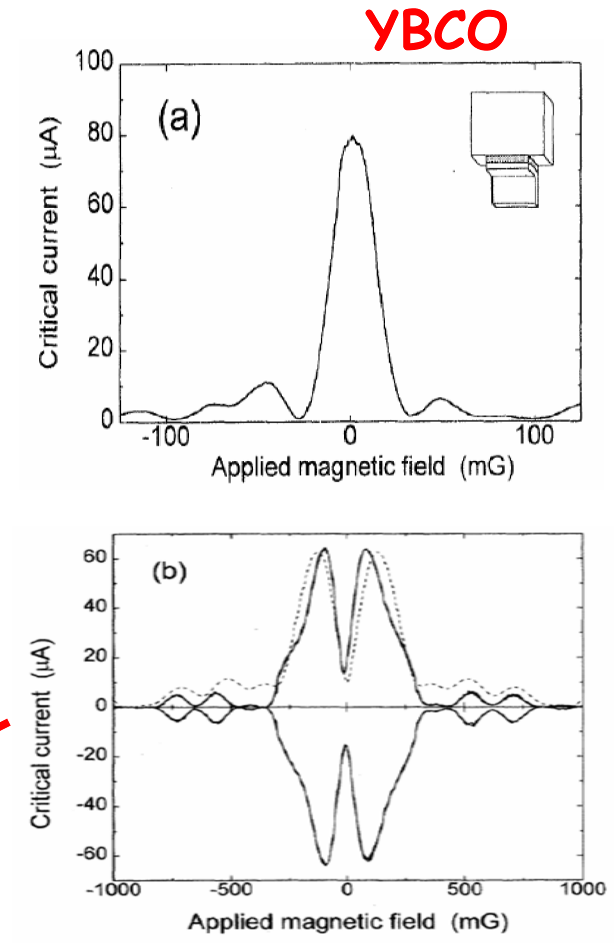
$$I_c \propto \sin(\Delta\phi) + \sin(\Delta\phi + \pi) = 0$$

D.A. Wohlmann et al.
*PRL **71**, 2134 (1993); PRL **74**, 797 (1995)*

Adapted from I. Vekhter

Evidence for phase of order parameter





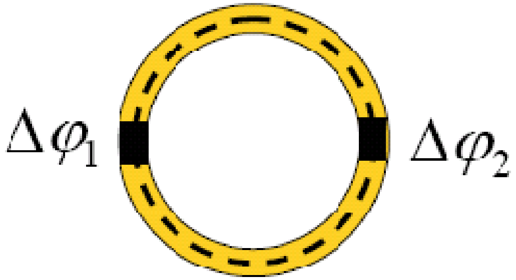
sign change! d-wave!

D.A. Wohlmann et al.
*PRL **71**, 2134 (1993); PRL **74**, 797 (1995)*

Adapted from I. Vekhter

Spontaneous generation of flux in π–rings

No applied magnetic field



Recall phase is single-valued

$$\oint_c \vec{\nabla}\phi \cdot d\vec{l} + \Delta\varphi_1 - \Delta\varphi_2 = 2\pi n$$

Make a ring where $\Delta\varphi_1 - \Delta\varphi_2 = \pi$ **called a π–ring**

then $\oint_c \vec{\nabla}\phi \cdot d\vec{l} = \pi$ instead of 2π

$$\Rightarrow \quad \Phi = \oint_c \vec{A} \cdot d\vec{l} = \frac{\Phi_0}{2}$$

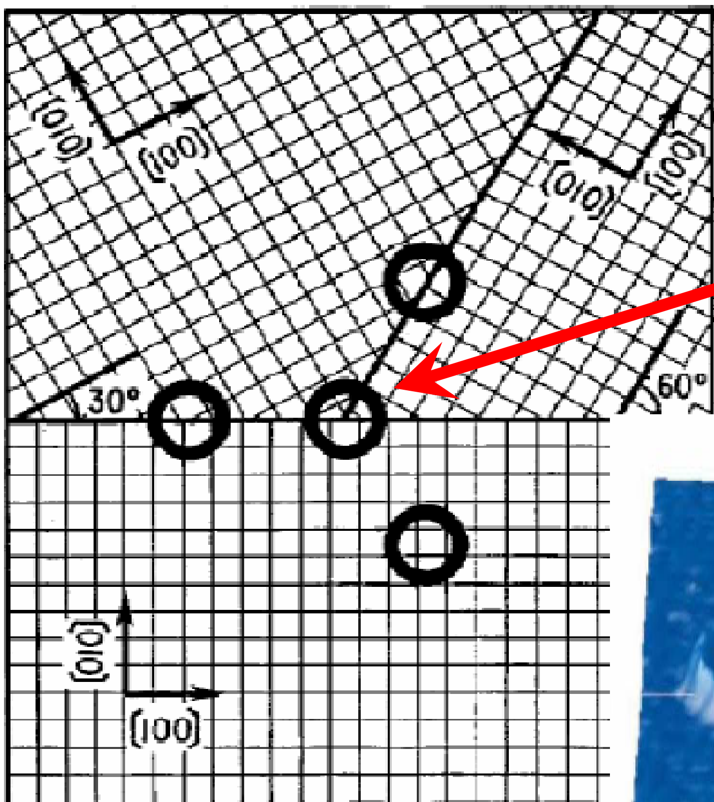
spontaneously generated magnetic flux

Bulaevskii et al. 1977, Geshkenbein and Larkin 1986, Sigrist and Rice, 1992

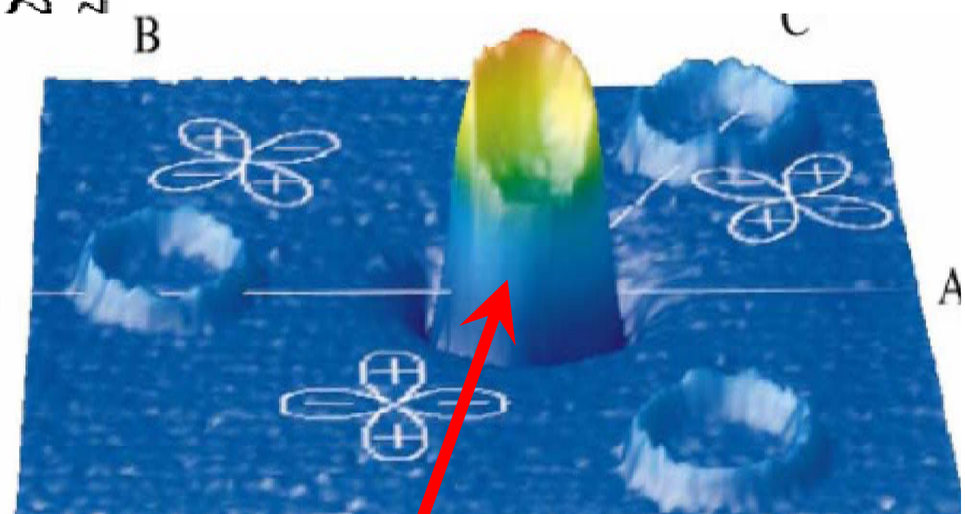
*Tsuei and Kirtley, RMP **72**, 969 (2000)*

Adapted from I. Vekhter

Scanning SQUID microscopy



YBCO ring patterned on a tricrystal with 3 grain boundaries or junctions



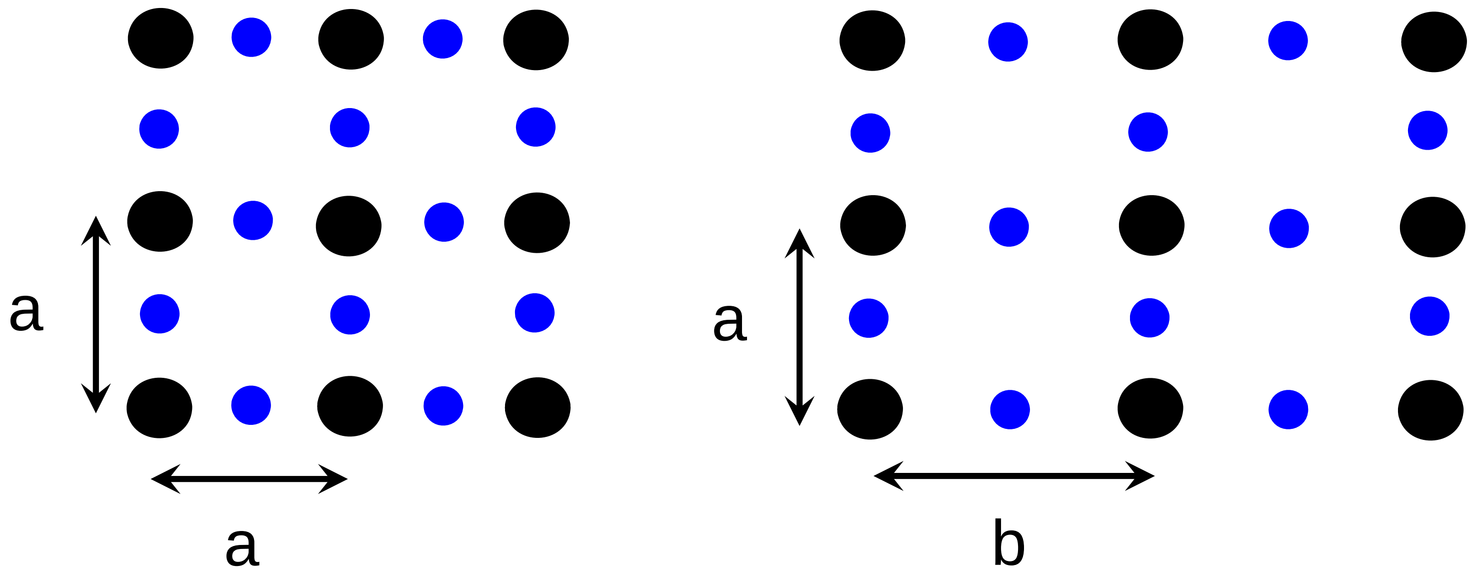
Spontaneously generated flux with half the conventional flux quantum

*C.C. Tsuei et al. PRL **73**, 593 (1994);*
*J.R. Kirtley et al., Nature **373**, 225 (1995).*

⇨ **d-wave!**

Adapted from I. Vekhter

Evidence for an s-wave component?



Tetragonal: Tl-2201

$\Longrightarrow d_{x^2-y^2}$

Orthorhombic: YBCO

Can have admixture of d and s

$\Longrightarrow d_{x^2-y^2} + s$

Angle-resolved phase-sensitive determination of the in-plane gap symmetry in $YBa_2Cu_3O_{7-\delta}$

Nature Physics **2**, 190 (2006).

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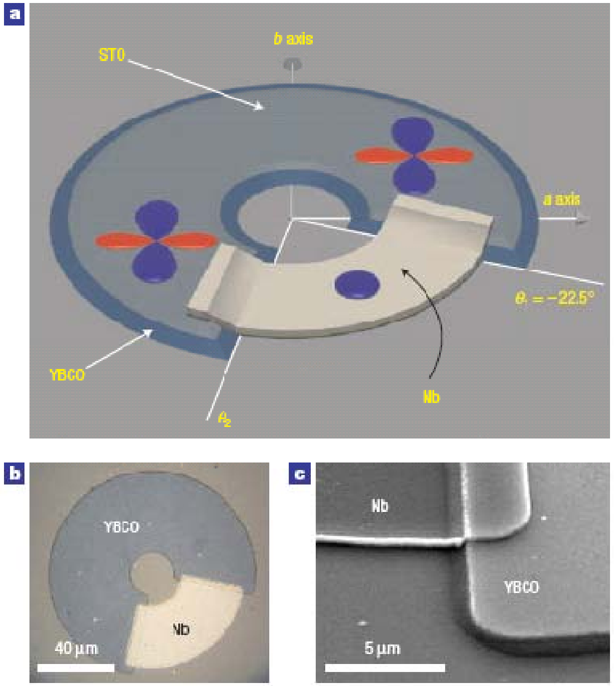
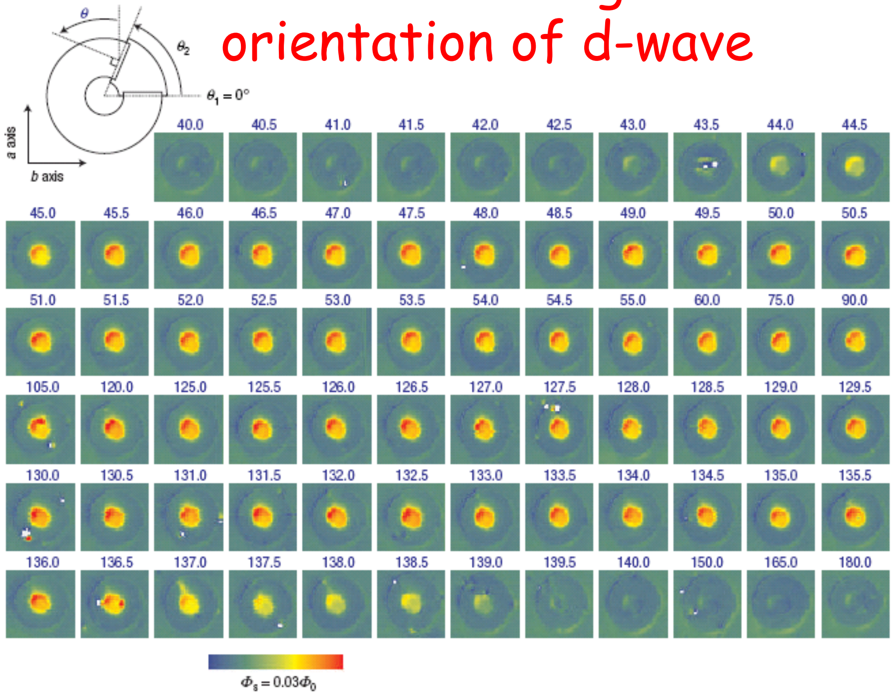
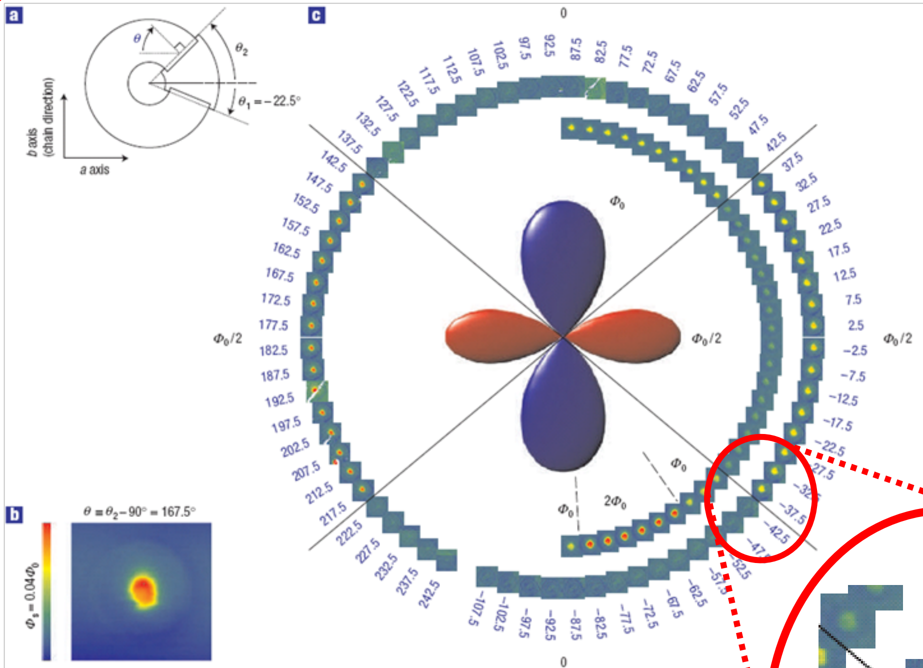
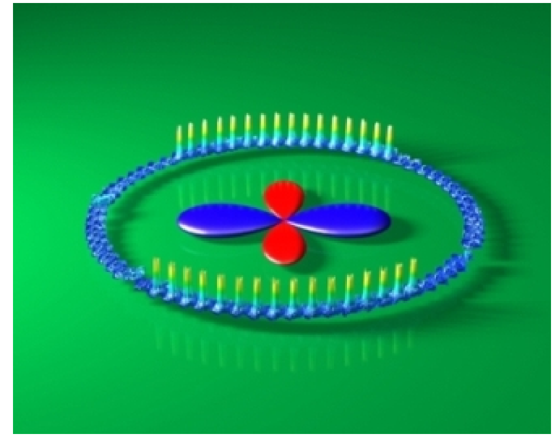
[2]Faculty of Science and Technology and MESA+ Institute for Nanotechnology, University of Twente, PO Box 217, 7500 AE Enschede, The Netherlands
*e-mail: kirtley@us.ibm.com

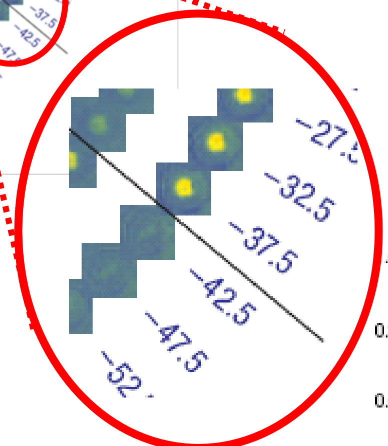
Make YBCO-Nb rings with variable angle which connects to different angular orientation of d-wave



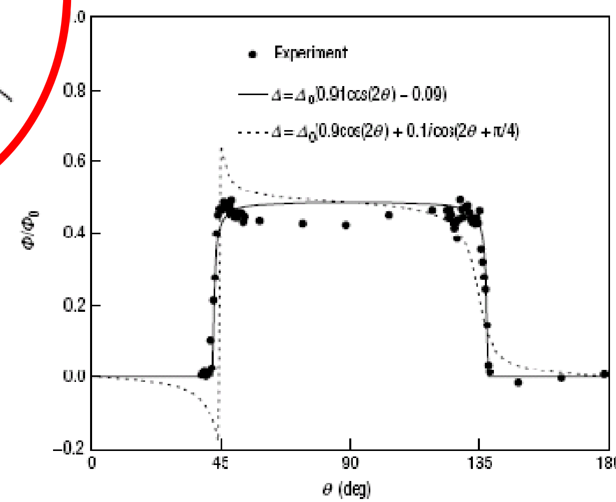








10% s-wave component inferred



When angle such that + lobe connects to - lobe, frustrated ring will spontaneously generate magnetic flux to heal the frustration

Special thanks to my great guys!

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Ilya Vekhter,
LSU