Interacting Topological Materials

Joseph Maciejko
University of Alberta

CIFAR Quantum Materials Summer School 2018
May 30, 2018

UNIVERSITY OF ALBERTA

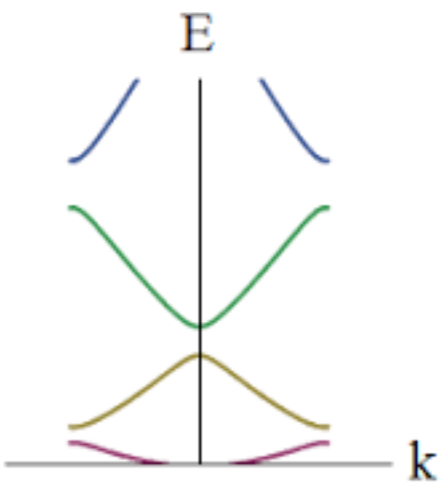
CIFAR | ICRA
CANADIAN INSTITUTE FOR ADVANCED RESEARCH | INSTITUT CANADIEN DE RECHERCHES AVANCÉES

Canada Research Chairs
www.chairs-chaires.gc.ca

NSERC
CRSNG

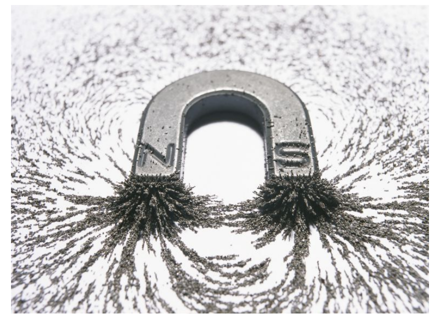
Why topology + interactions?

- New paradigm in many-body physics

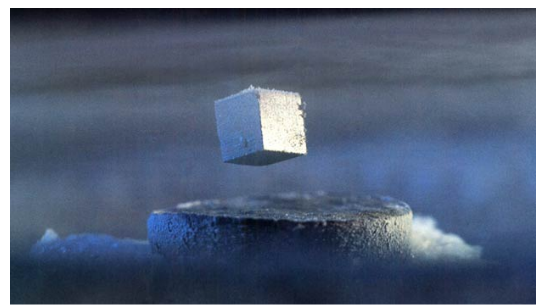


conventional
bandstructure

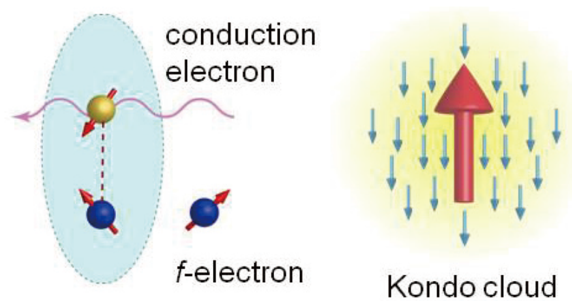
interactions



magnetism



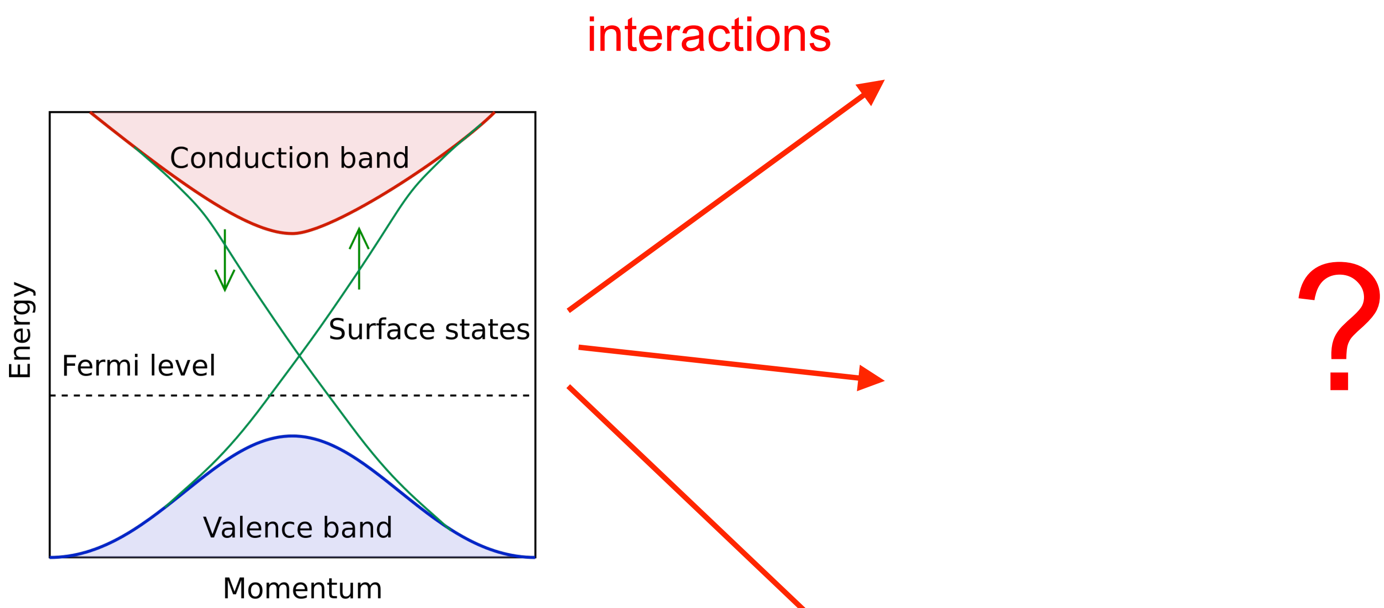
superconductivity



Kondo effect

Why topology + interactions?

- New paradigm in many-body physics

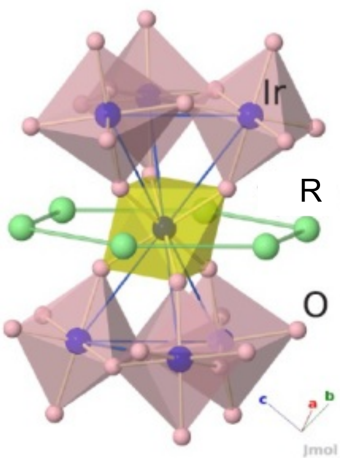


topological
bandstructure

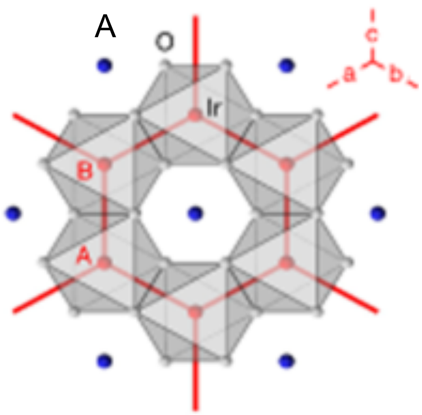
Why topology + interactions?

- Possibly relevant to many quantum materials of current interest with strong SOC + strong interactions – at least to understand much of the theoretical literature on these materials

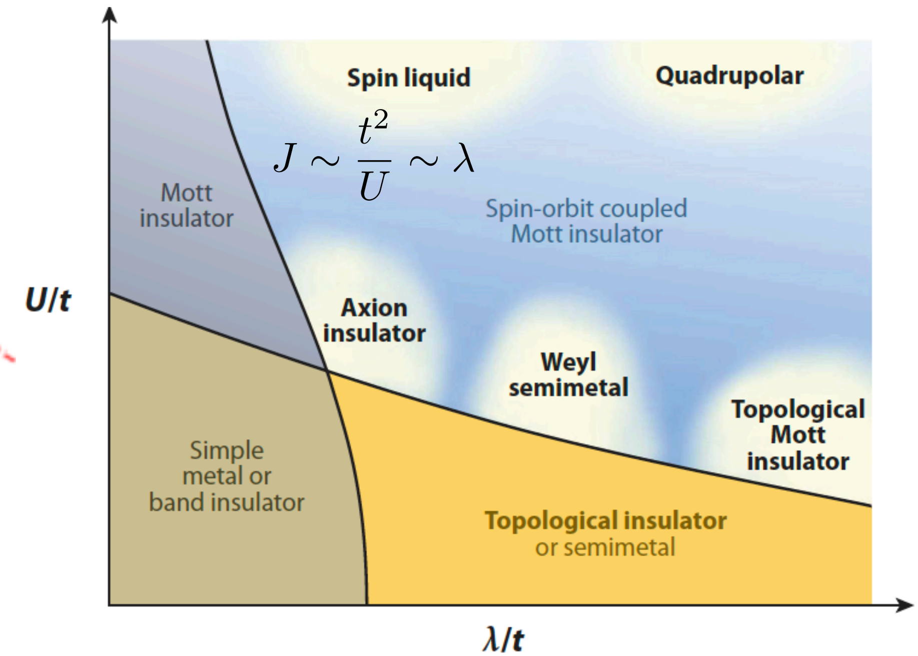
$$H = \sum_{i,j;\alpha\beta} t_{ij,\alpha\beta}\, c^\dagger_{i\alpha} c_{j\beta} + \text{h.c.} + \lambda \sum_i \boldsymbol{L}_i \cdot \boldsymbol{S}_i + U \sum_{i,\alpha} n_{i\alpha}(n_{i\alpha} - 1)$$



$R_2Ir_2O_7$



A_2IrO_3



Witczak-Krempa, Chen, Kim, Balents, Annu. Rev. CMP '14

Outline

I. Stability of free-fermion topological phases to interactions

- Perturbative stability
- Spontaneous symmetry breaking
- Reduction of the free-fermion classification

II. Interaction-induced topological phases

- Topological Mott insulators
- Topological Kondo insulators

III. Strongly correlated topological phases

- Symmetry-protected topological phases
- Fractionalized topological phases

Outline

I. Stability of free-fermion topological phases to interactions

- Perturbative stability
- Spontaneous symmetry breaking
- Reduction of the free-fermion classification

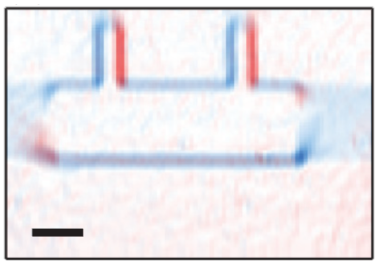
II. Interaction-induced topological phases

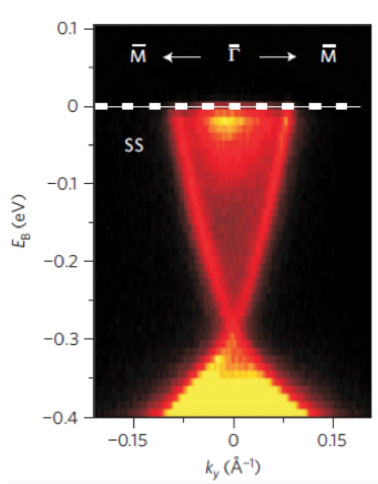
- Topological Mott insulators
- Topological Kondo insulators

III. Strongly correlated topological phases

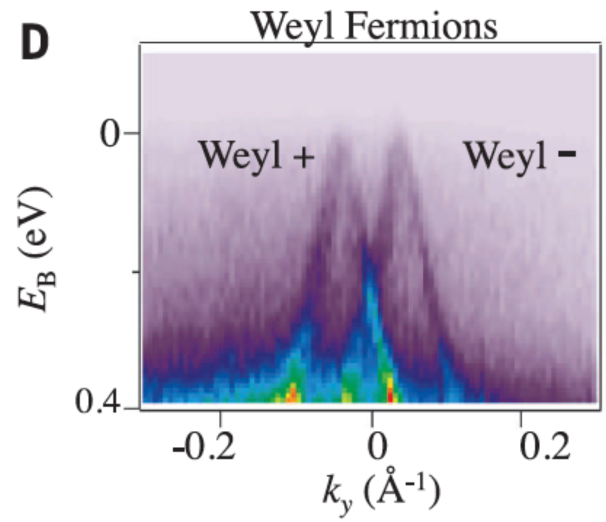
- Symmetry-protected topological phases
- Fractionalized topological phases

Free-fermion topological phases





topological insulators



Weyl semimetals

- Cannot be smoothly connected to atomic insulator (product state) unless some **discrete protecting symmetry** is broken
- TIs: time reversal; WSMs: lattice translation (+ plethora of phases protected by point/space group symmetries: topological crystalline insulators, Dirac/ nodal line semimetals, higher-order topological insulators…)
- Smoothly = without a phase transition: topological phases should be **stable** against small symmetry-preserving perturbations, including **interactions**

Interacting topological insulators

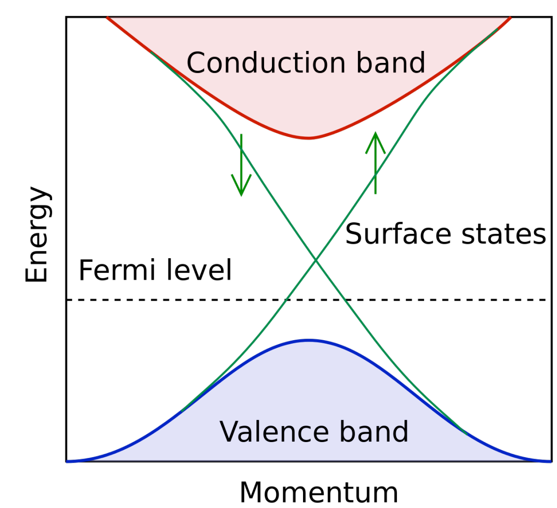
- Topological insulators: bulk is gapped, perturbation theory converges

$$\Delta E_0 = \sum_{n \neq 0} \underbrace{\frac{|\langle n|V|0\rangle|^2}{E_0 - E_n}}_{\text{gap}}$$

- Noninteracting edge/surface is gapless, naive perturbation theory diverges!

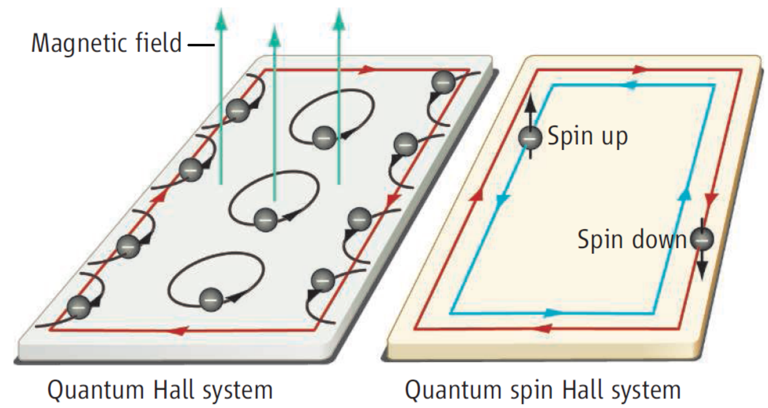
- Stability of interacting edge/surface metals: highly dependent on **dimensionality**

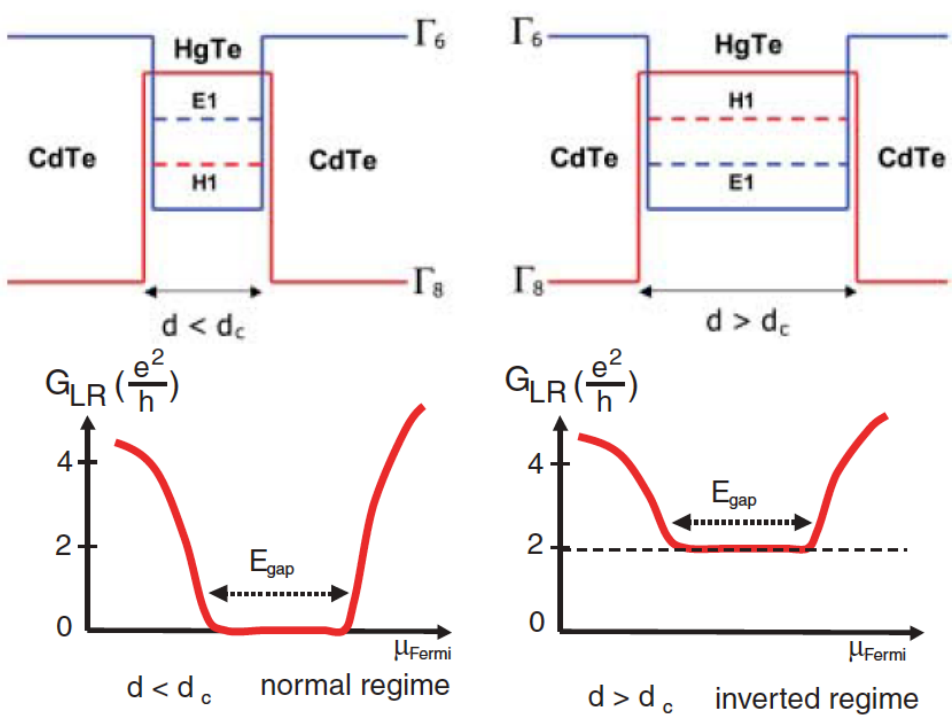
- Quantum spin Hall effect: d=1 edge,
  3D topological insulator: d=2 surface



Quantum spin Hall effect

- T-invariant generalization of the QHE (Kane & Mele, PRL '05)
- 2 counterpropagating edge modes with opposite spin



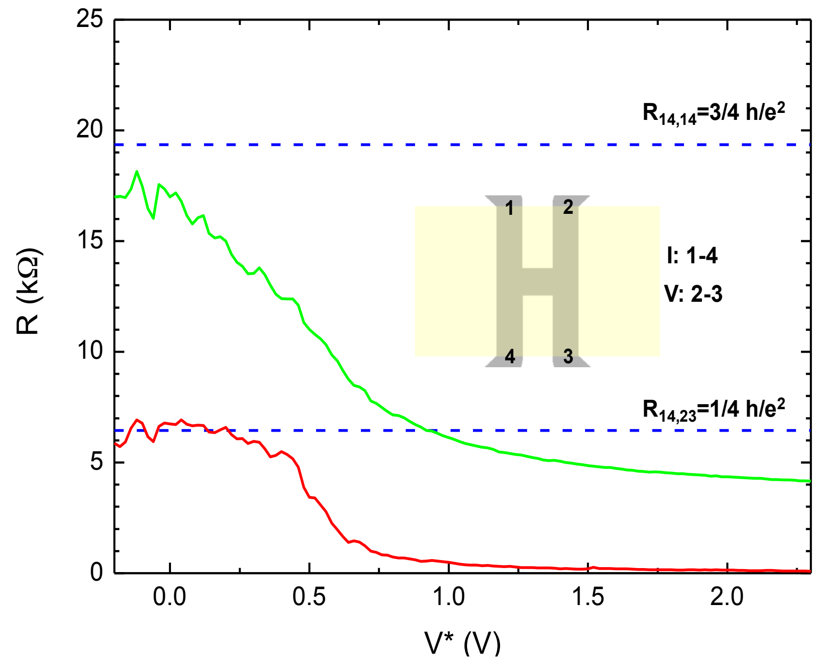


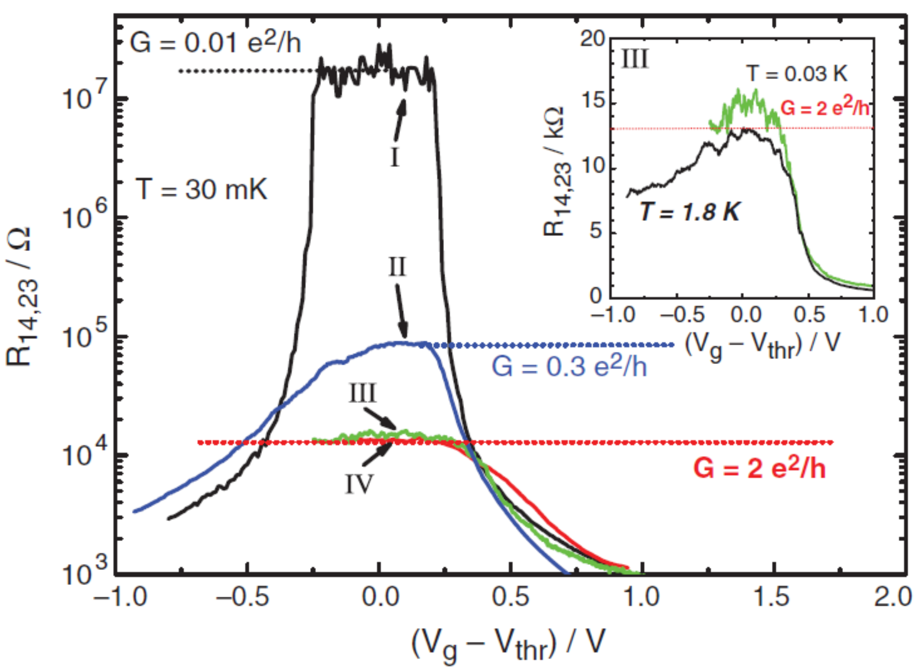
- Prediction in HgTe/CdTe quantum wells (Bernevig, Hughes, Zhang, Science '06)

QSHE in HgTe

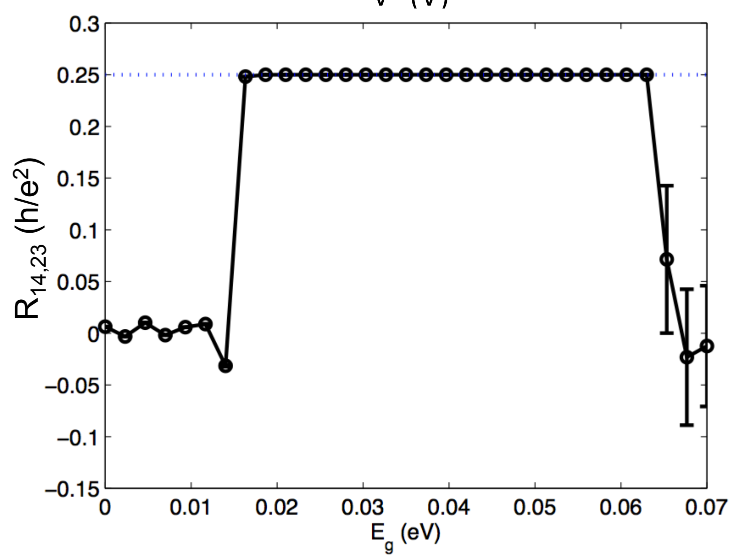
Roth, Brüne, Buhmann, Molenkamp, JM, Qi, Zhang, Science '09

- 2-terminal conductance quantized to $2e^2/h$
- Quantized nonlocal edge transport without magnetic fields



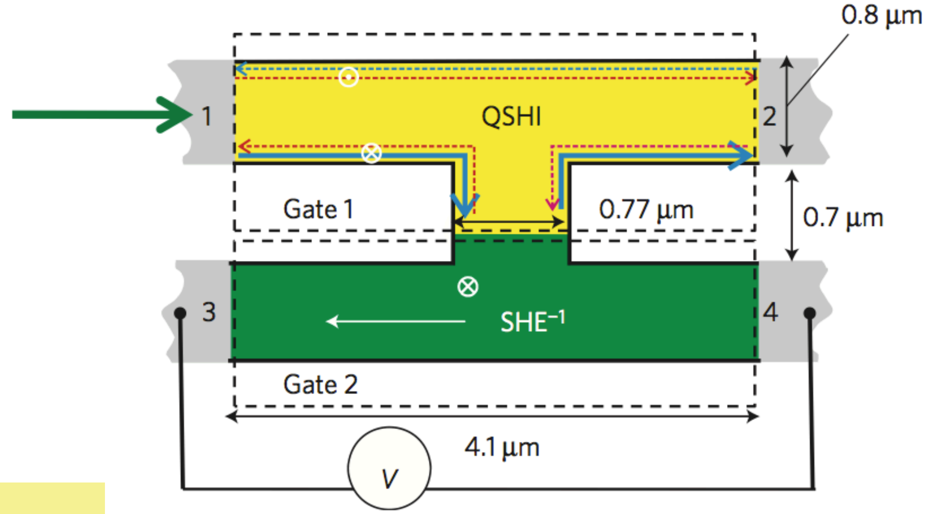


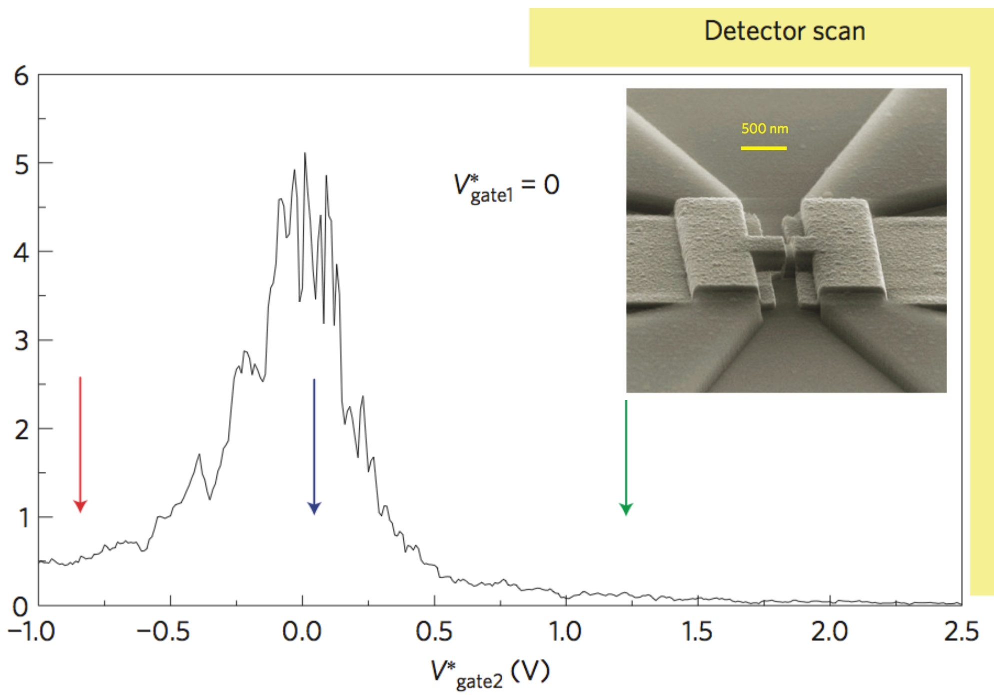
König et al., Science '07

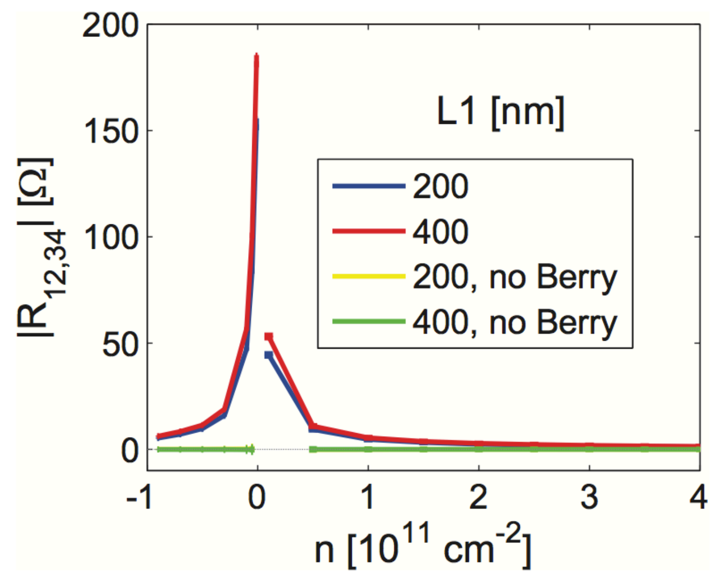


Helical edge states

- "Helical" nature of edge states detected via inverse SHE

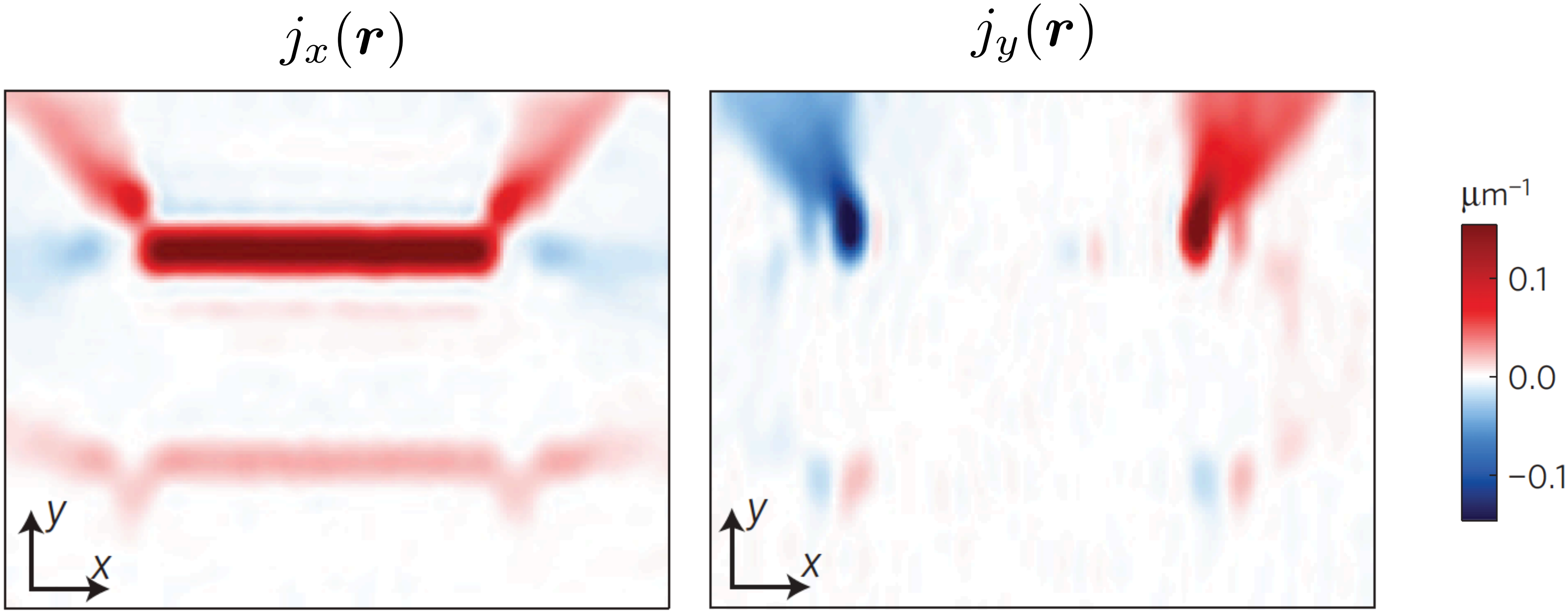


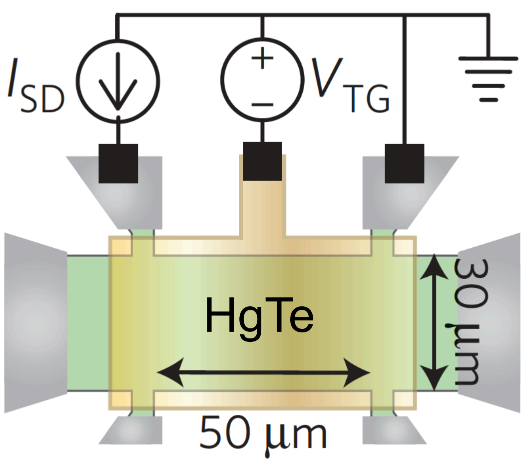


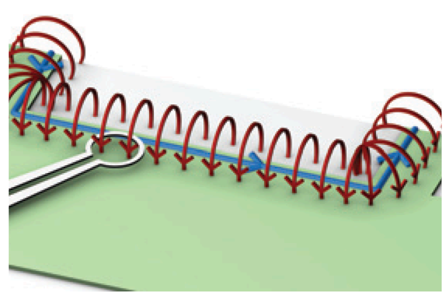


Brüne, Roth, Buhmann, Hankiewicz, Molenkamp, JM, Qi, Zhang, Nat. Phys. '12

Seeing the edge states

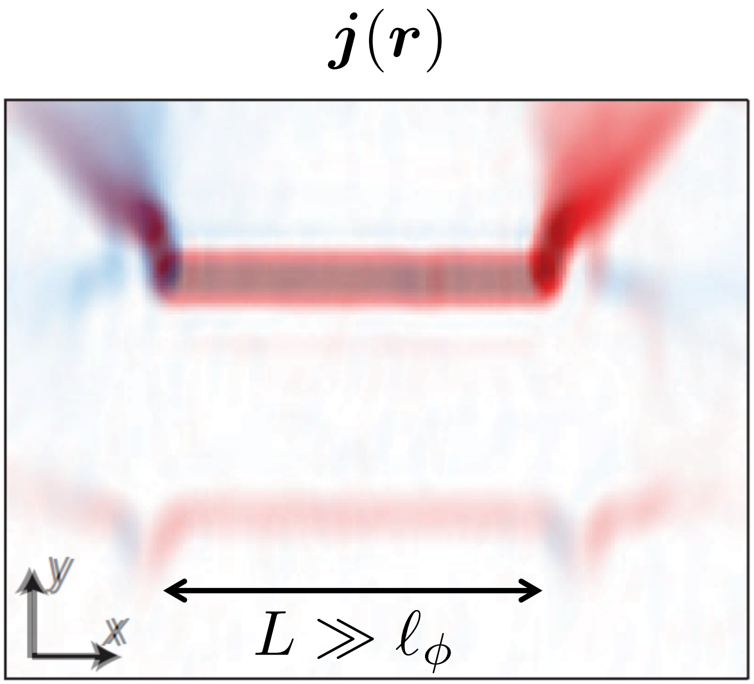


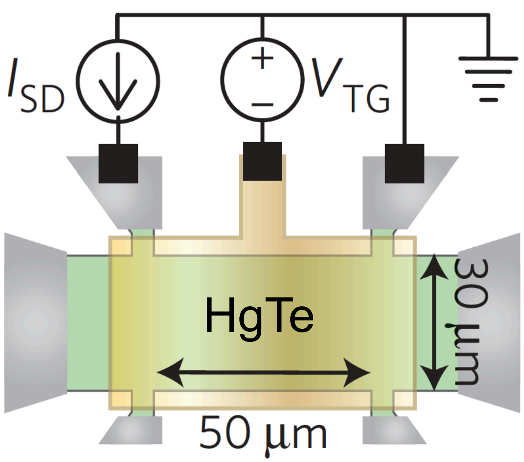


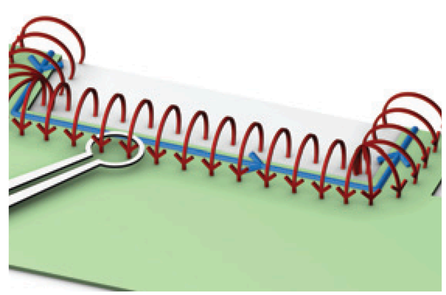


Nowack et al., Nat. Mater. '13

Seeing the edge states







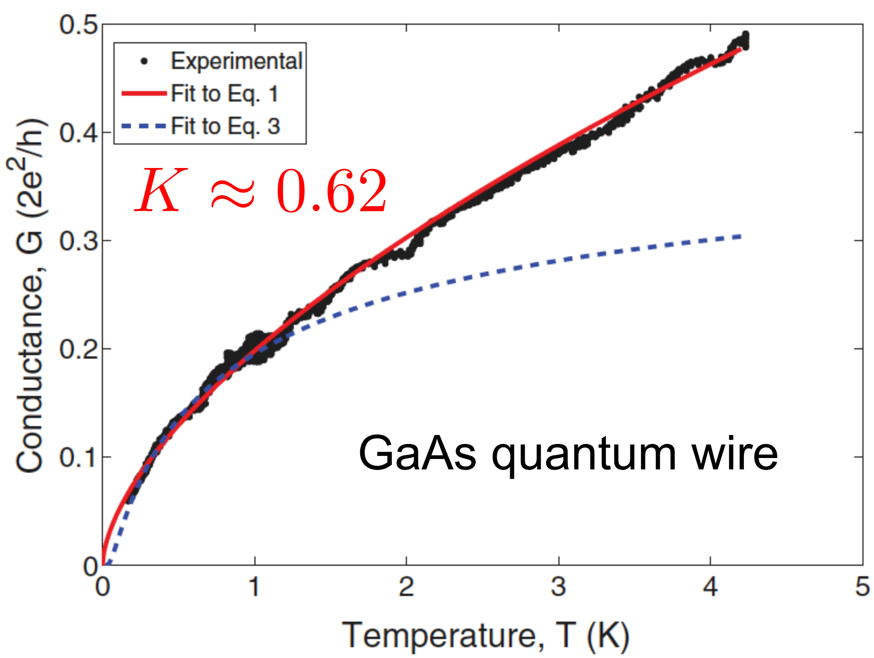
Nowack et al., Nat. Mater. '13

Edge metal-insulator transition

- With interactions, edge is a **helical Luttinger liquid (LL)** ~ "spinless" LL (Wu, Bernevig, Zhang, PRL' 06; Xu, Moore, PRB '06): interaction strength described by the Luttinger parameter K

- Spinless LL: single impurity can induce a metal-insulator transition at K=1 (Kane & Fisher, PRB '92)

$$G(T) \propto \left(\frac{T}{T^*}\right)^{1/K-1}$$



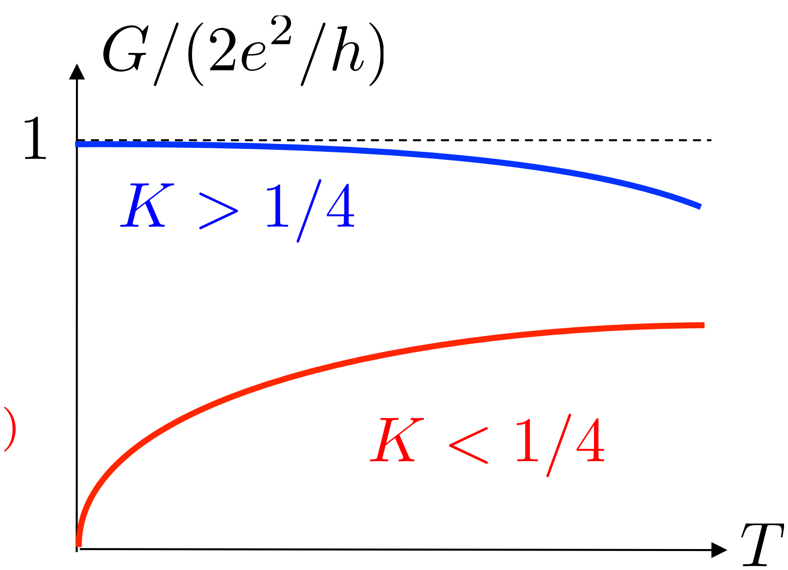
Levy et al., PRB '12

Edge metal-insulator transition

- With interactions, edge is a **helical Luttinger liquid (LL)** ~ "spinless" LL (Wu, Bernevig, Zhang, PRL' 06; Xu, Moore, PRB '06): interaction strength described by the Luttinger parameter K

- Helical LL: metal-insulator transition is at K=1/4 (Wu, Bernevig, Zhang, PRL '06; JM et al., PRL '09), **spontaneous T breaking** for K < 1/4

$$G(T) \propto \left(\frac{T}{T^*}\right)^{2(1/4K-1)}$$



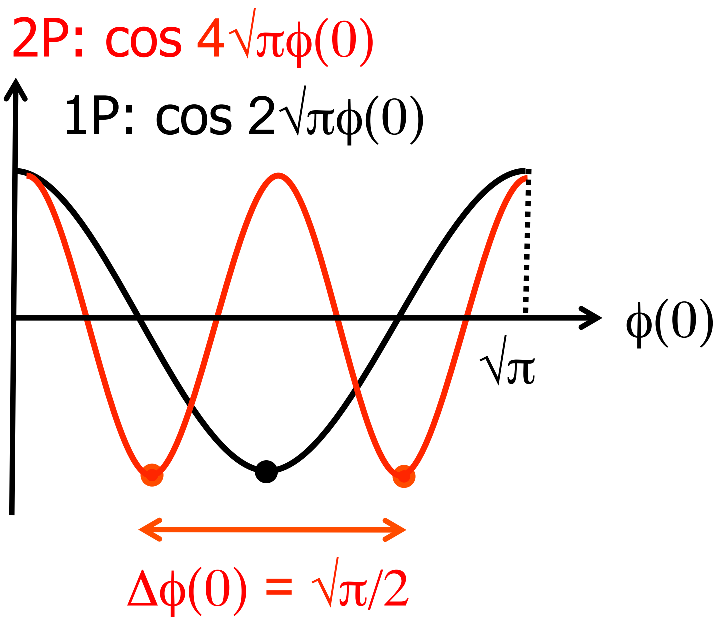
1- vs 2-particle backscattering

- 1-particle backscattering forbidden by T symmetry

$$\int dx\, \delta(x) \psi^{\dagger}_{R\uparrow} \psi_{L\downarrow} + \text{h.c.} \sim \cos 2\sqrt{\pi}\phi(0)$$

- 2-particle backscattering allowed by T symmetry

$$\int dx\, \delta(x) \psi^{\dagger}_{R\uparrow} \partial_x \psi^{\dagger}_{R\uparrow} \psi_{L\downarrow} \partial_x \psi_{L\downarrow} + \text{h.c.} \sim \cos 4\sqrt{\pi}\phi(0)$$

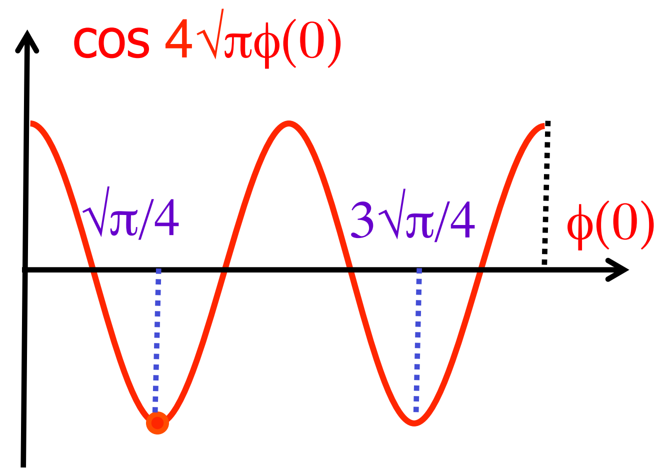


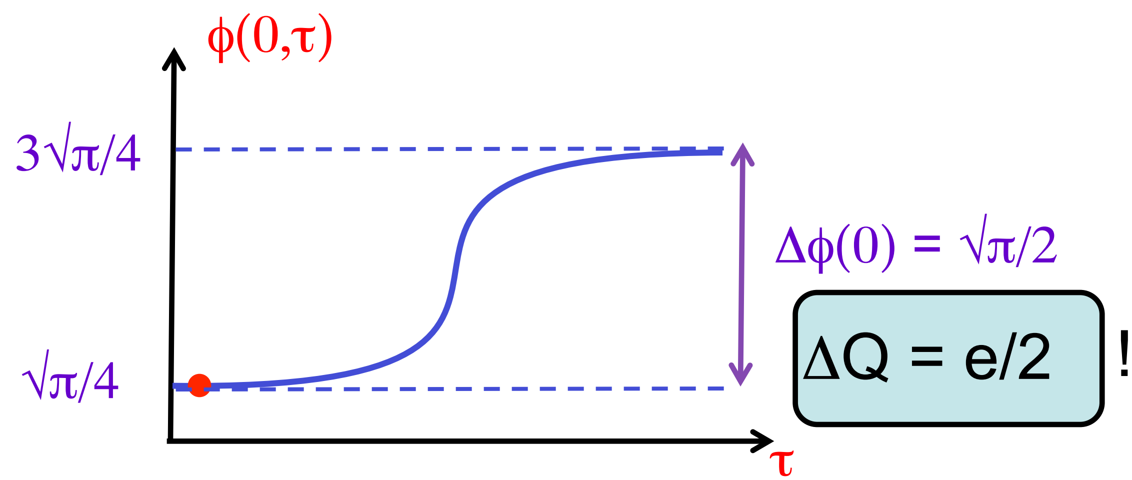
ground state nondegenerate
(potential scattering: Kane & Fisher, PRB '92)

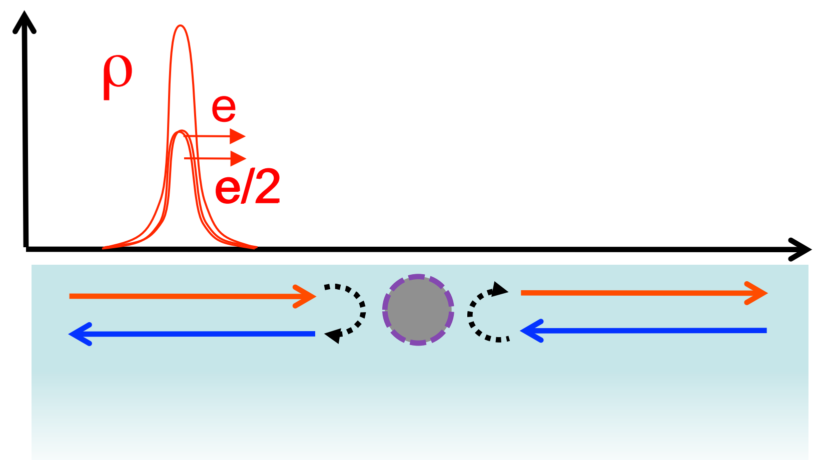
ground state 2-fold degenerate
(related by T)

Charge fractionalization

- Expect fractionalization from ground state degeneracy (cf. Su-Schrieffer-Heeger model of polyacetylene)
- Instanton (temporal soliton) pumps charge e/2: Fano factor = 1/2







$$j = -\partial_t \phi / \sqrt{\pi}$$
$$\rho = \partial_x \phi / \sqrt{\pi}$$

- Time counterpart of domain wall (spatial soliton) carrying charge e/2 (Qi, Hughes, Zhang, Nat. Phys. '08)

A helical Luttinger liquid?

- Effects of interactions in 1D edge channels? HgTe is weakly interacting:

$$K \sim 0.8$$

Teo & Kane, PRB '09

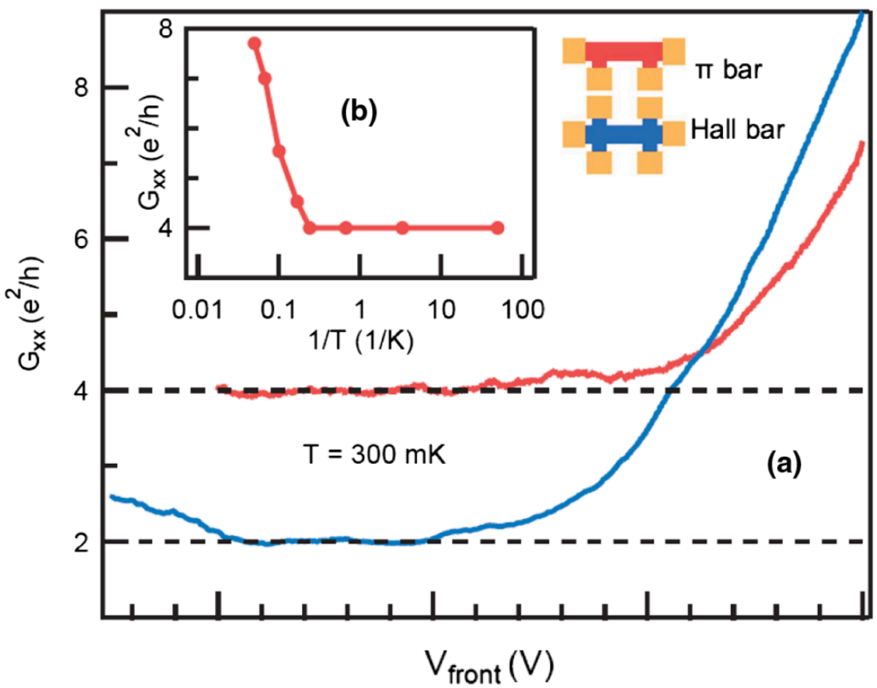
- Possibility of stronger interaction effects in InAs/GaSb QW:

$$K \sim 0.2$$

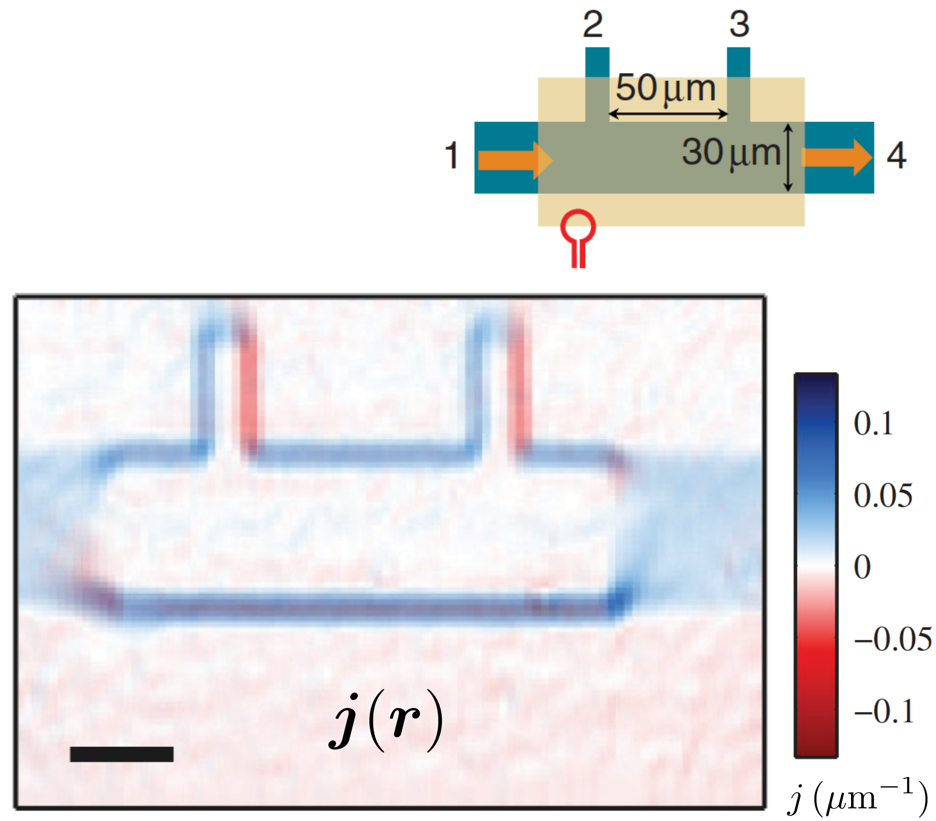
JM et al., PRL '09

- K < 1/4 in InAs/GaSb?

QSHE in InAs/GaSb



Du et al., PRL ‘15



Spanton et al., PRL ‘14

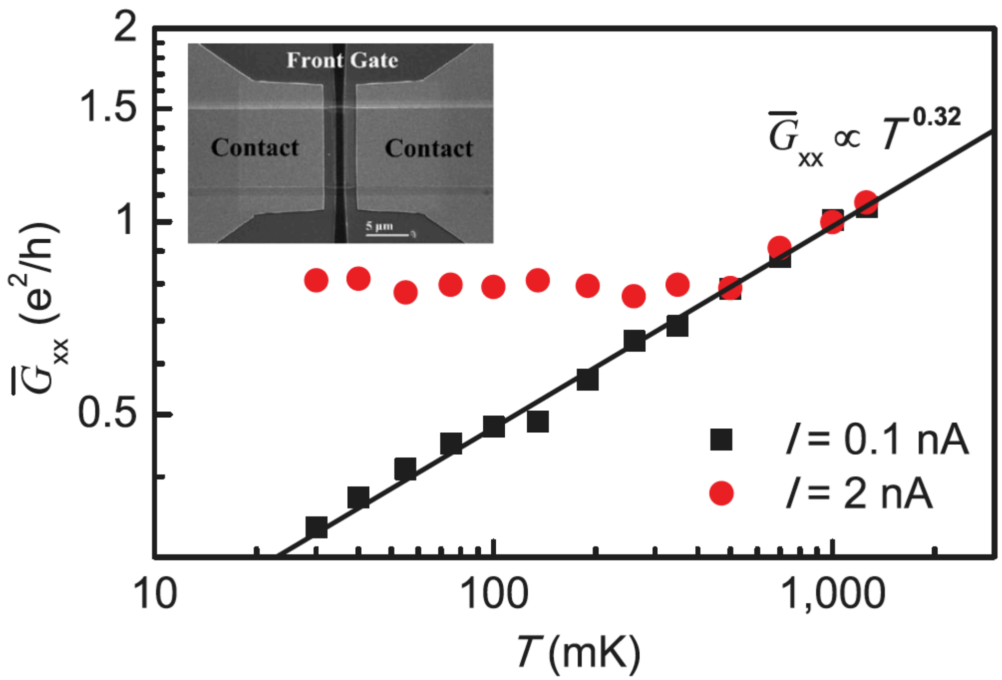
- G quantized to better than 1% (better than HgTe)

Helical LL in InAs/GaSb?

PRL **115,** 136804 (2015) PHYSICAL REVIEW LETTERS week ending 25 SEPTEMBER 2015

Observation of a Helical Luttinger Liquid in InAs/GaSb Quantum Spin Hall Edges

Tingxin Li,[1,4] Pengjie Wang,[1,4] Hailong Fu,[1,4] Lingjie Du,[2] Kate A. Schreiber,[3] Xiaoyang Mu,[1,4] Xiaoxue Liu,[1,4] Gerard Sullivan,[5] Gábor A. Csáthy,[3] Xi Lin,[1,4] and Rui-Rui Du[1,2,4,*]



- Insulating behavior
- Power-law T dependence, saturates below $T_{\text{cutoff}} \sim eV/k_B$

$$G \propto T^{2(1/4K-1)}$$

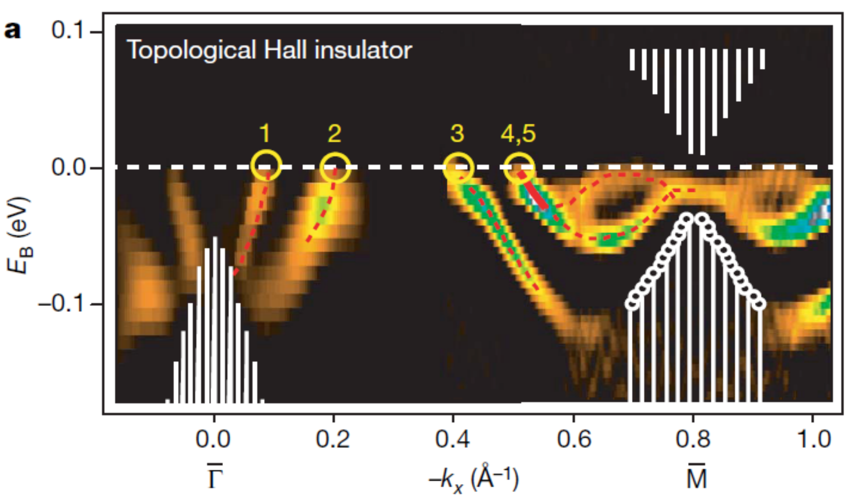
$$K \approx 0.21$$

- Fano factor = 1/2?
- K in principle tunable by bandgap engineering: observe M-I transition?

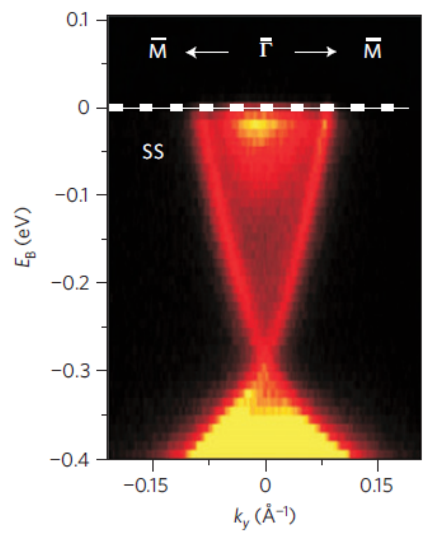
3D topological insulators

- 3D generalization of the QSHE (Fu, Kane, Mele, PRL ’07); predicted in $Bi_{1-x}Sb_x$ (Fu, Kane, PRB ‘07)
- Odd # of 2D massless Dirac fermions on the surface: Z_2 invariant, like QSHE (Moore & Balents, PRB ‘07; Roy, PRB ‘09)
- Single Dirac cone: prediction in Bi_2Se_3, Bi_2Te_3, Sb_2Te_3 (Zhang et al., Nat. Phys. ‘09)
- Observed with ARPES (M. Z. Hasan, Z. X. Shen)

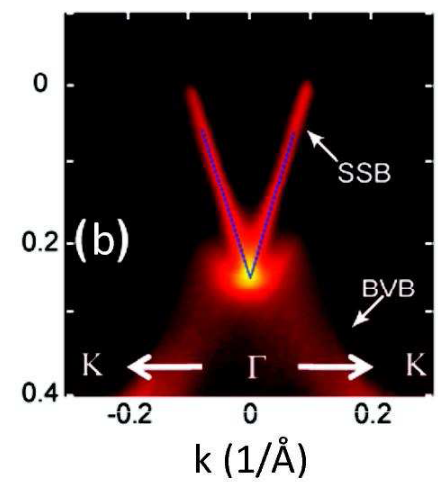
Bi_xSb_{1-x} (Hsieh et al., Nature ‘08)



Bi_2Se_3 (Xia et al., Nat. Phys. ‘09)

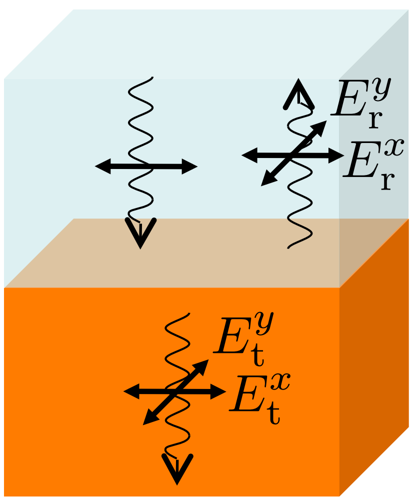


Bi_2Te_3 (Chen et al., Science ‘09)



Axion electrodynamics

- E&M response contains magnetoelectric ~θ**E**·**B** coupling with quantized θ=π (Qi, Hughes, Zhang, PRB '08)
- θ angle can be measured via Kerr/Faraday effect



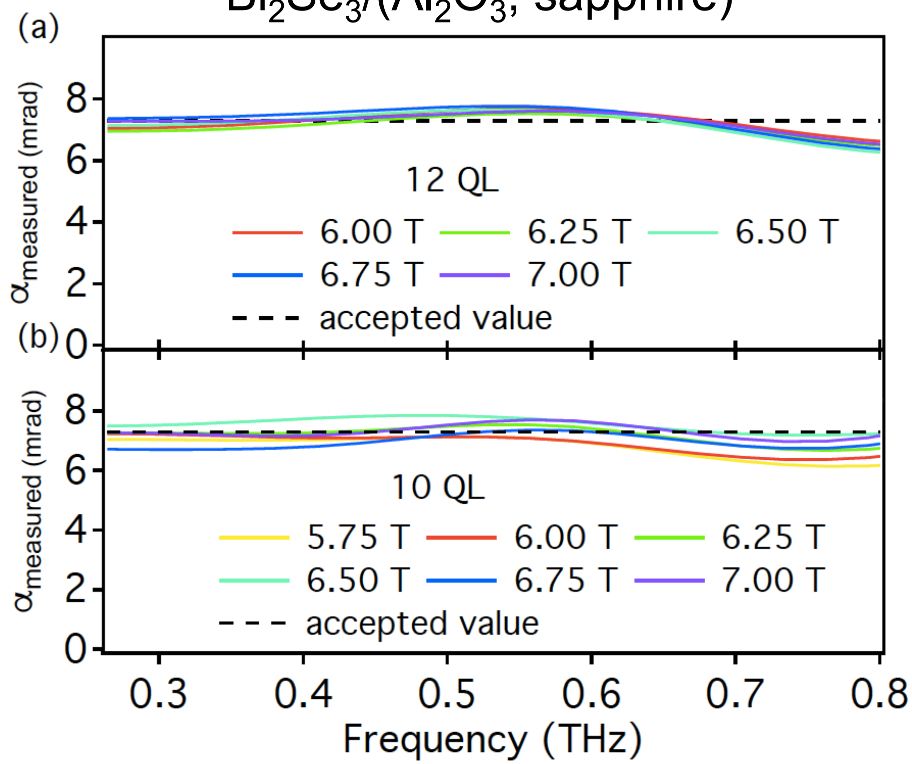
$$\tan\theta_K = \frac{E^y_{\mathrm{r}}}{E^x_{\mathrm{r}}}$$

$$\tan\theta_F = \frac{E^y_{\mathrm{t}}}{E^x_{\mathrm{t}}}$$

$$\frac{\cot\theta_F + \cot\theta_K}{1+\cot^2\theta_F} = \alpha$$

JM et al., PRL '10

$Bi_2Se_3/(Al_2O_3$, sapphire)



Wu et al., Science '16

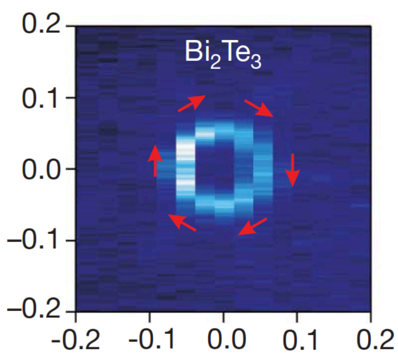
Spin-momentum locking

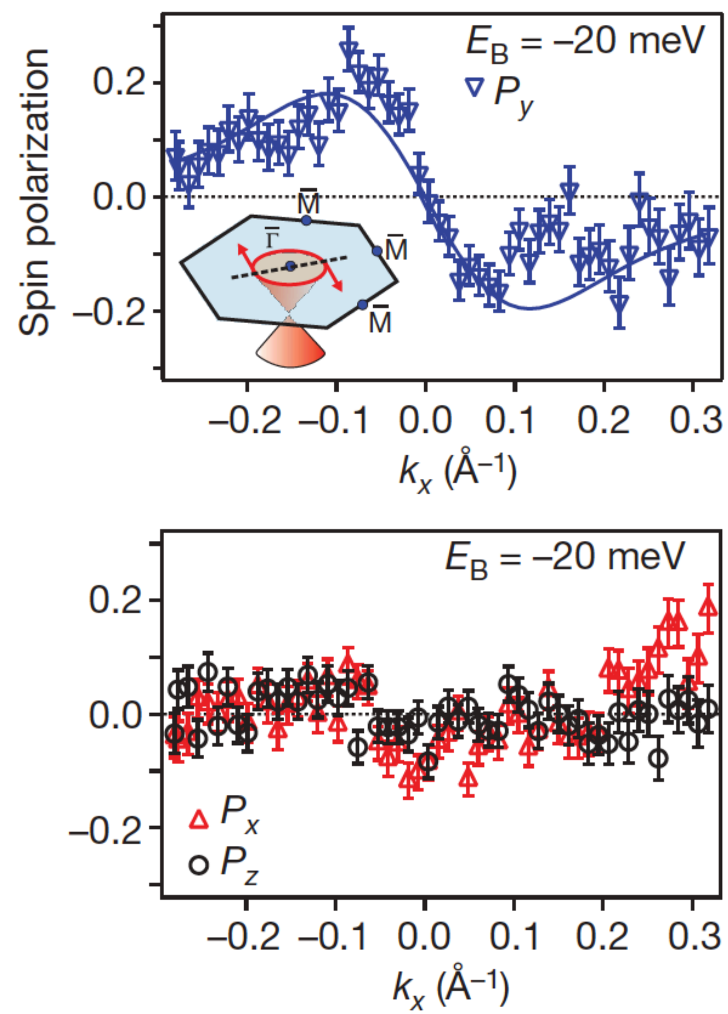
Hsieh et al., Nature '09

- Single nondegenerate Fermi surface with spin-momentum locking: spin-resolved ARPES

$$H_{\text{surface}} = \hbar v_F \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{k})$$

real spin, not pseudospin!



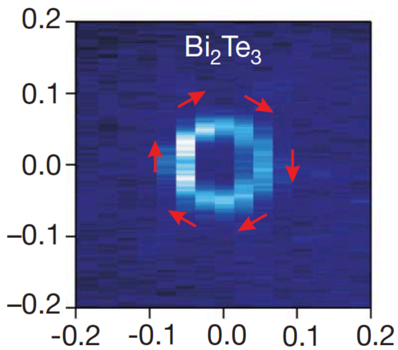


Spin-momentum locking

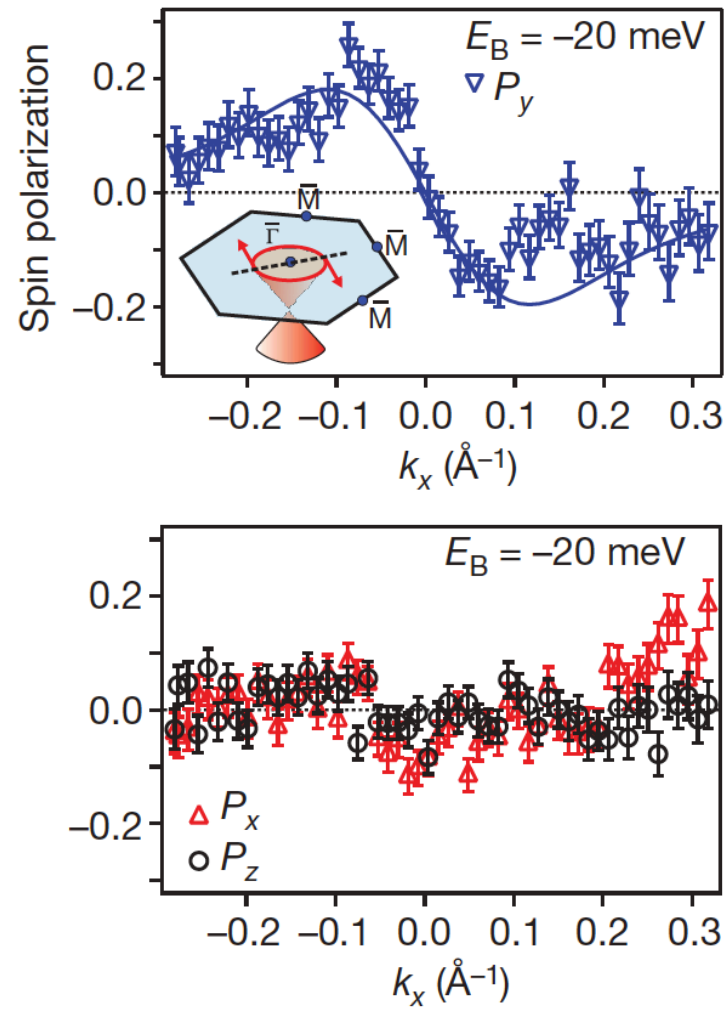
Hsieh et al., Nature '09

- Single nondegenerate Fermi surface with spin-momentum locking: spin-resolved ARPES

$$H_{\text{surface}} = \hbar v_F \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{k})$$



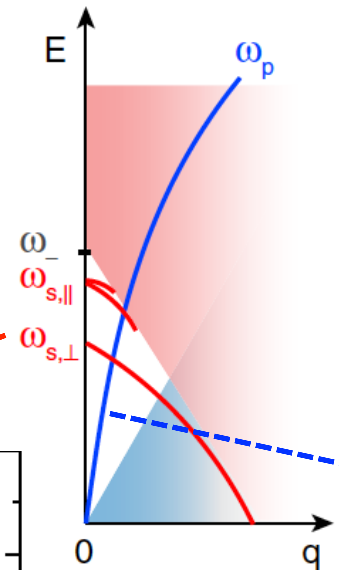
real spin, not pseudospin!

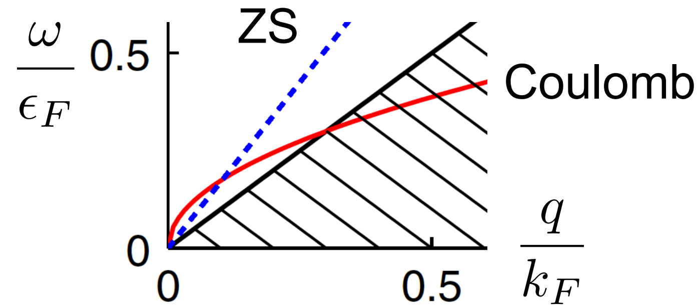


- For rotationally invariant FS, interactions described by **helical** Landau Fermi liquid theory: 10 Landau parameters F_ℓ^α, $\alpha = 1, \ldots, 10$ (Lundgren & JM, PRL '15)

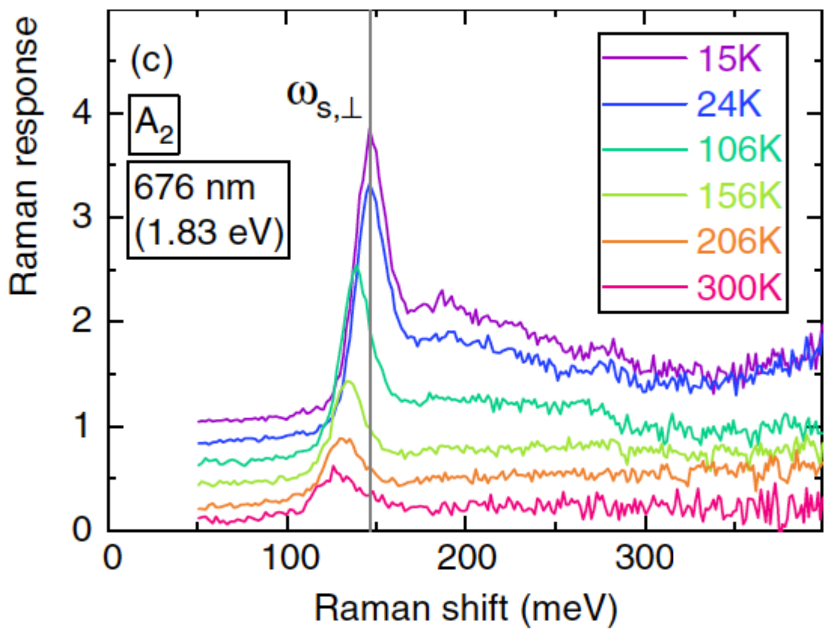
Surface collective modes

(Bi_2Se_3)



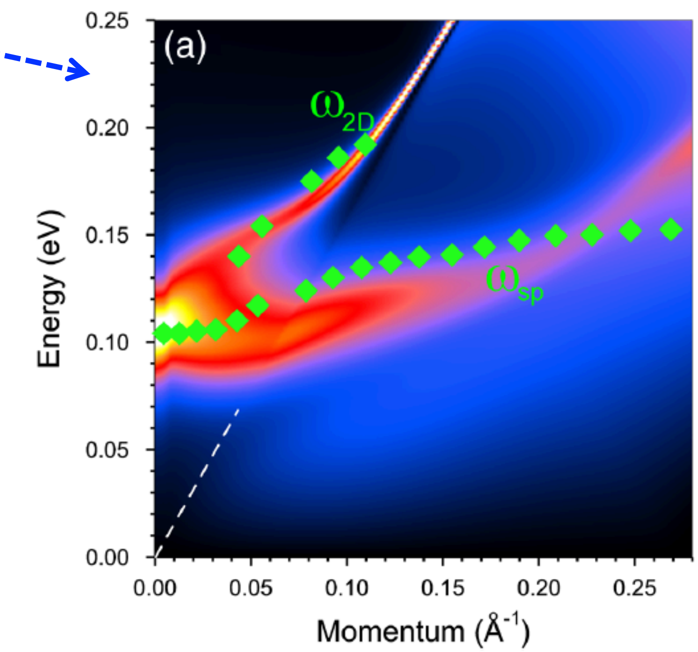


chiral spin mode (Ashrafi & Maslov, PRL ‘12)



Kung et al., PRL ‘17

Dirac spin plasmon (Raghu et al., PRL ‘10)



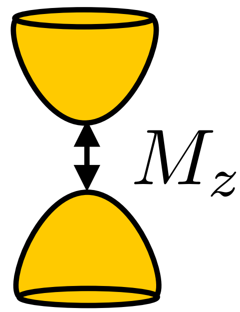
Politano et al., PRL ’15

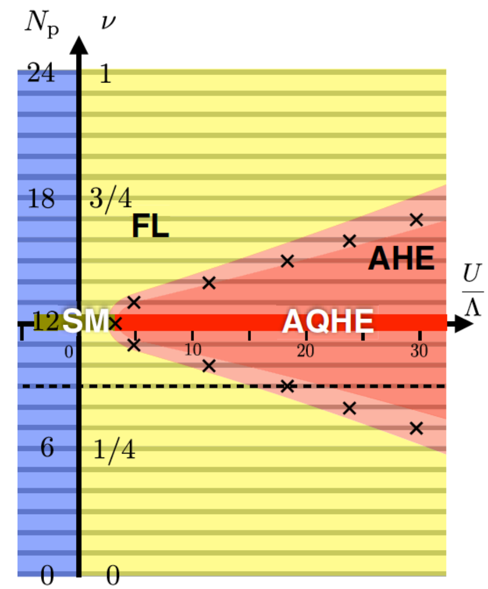
(also Di Pietro et al., Nat. Nano. ‘13)

Particle-hole instabilities

- As on QSH edge, strong repulsive interactions can lead to spontaneous symmetry breaking (SSB) on the TI surface…

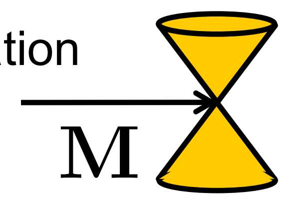
  - Ising ferromagnetic order: breaks T
    (Xu, PRB '10), $\sigma_{xy} = \pm e^2/2h$



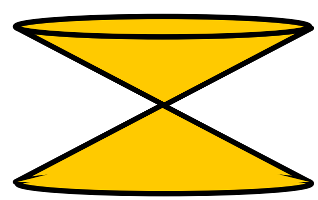


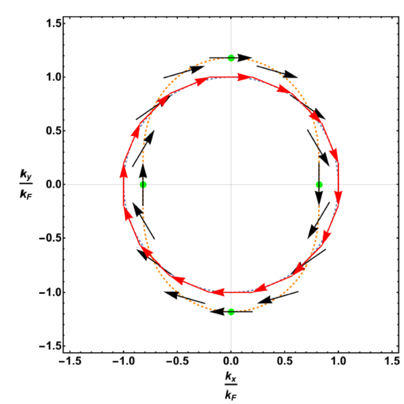
Neupert et al., PRL '15

  - XY ferromagnetic order: breaks T + rotation
    (Xu, PRB '10)



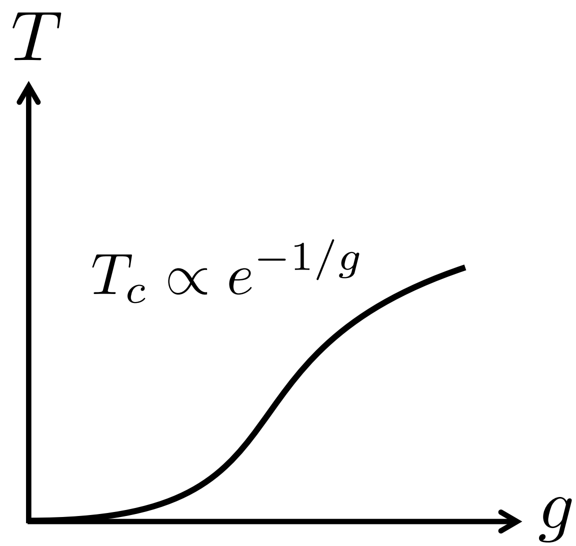
  - Nematic order: breaks rotation but not T
    (Lundgren, Yerzhakov, JM, PRB '17)

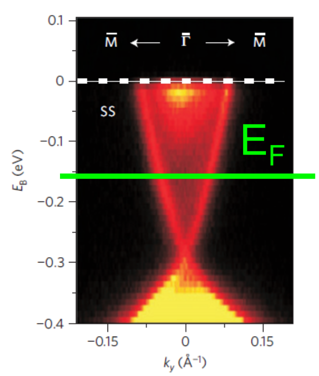


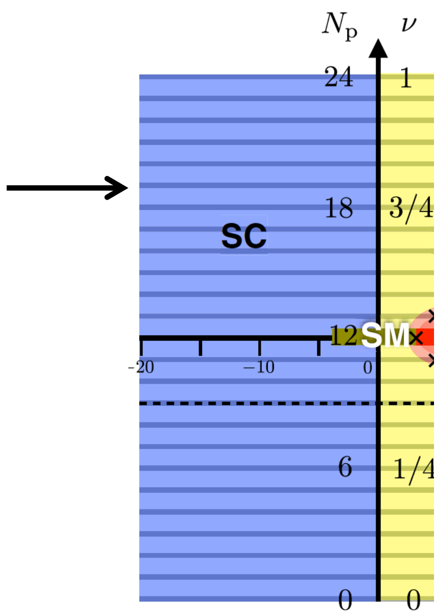


Superconducting instabilities

- … while attractive interactions generate (topological) superconductivity



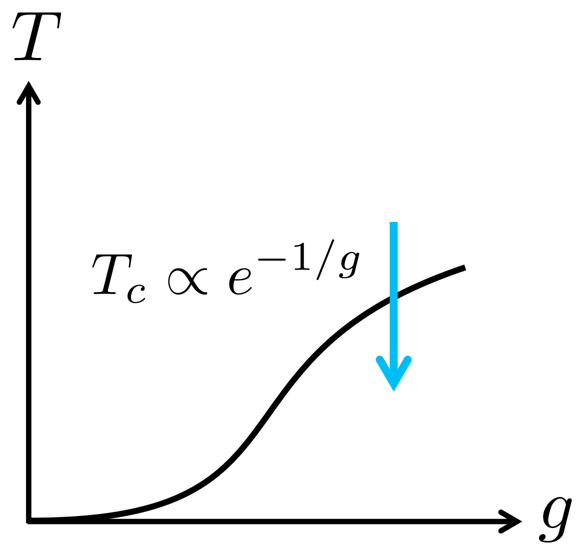


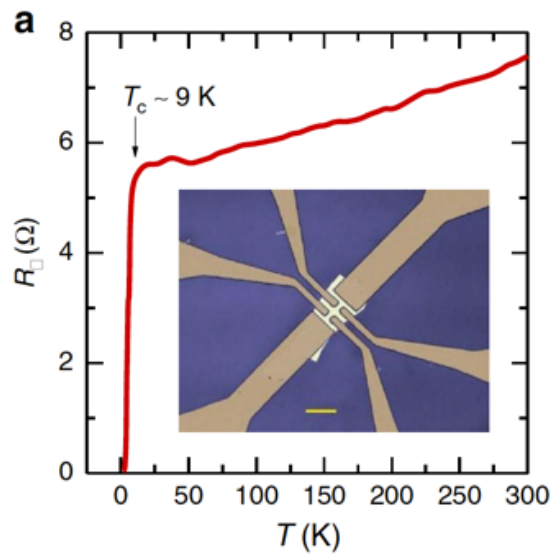


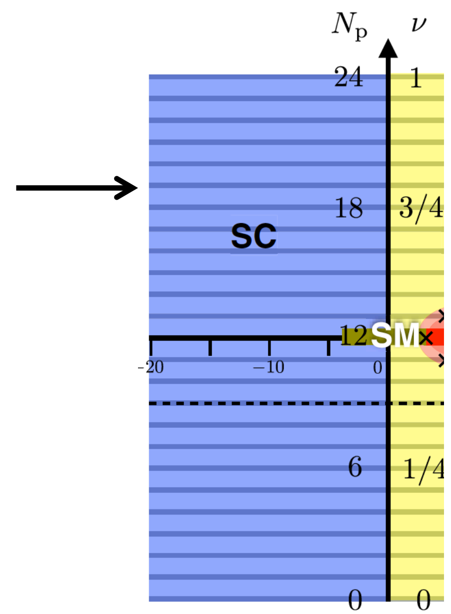
Neupert et al., PRL ‘15

Superconducting instabilities

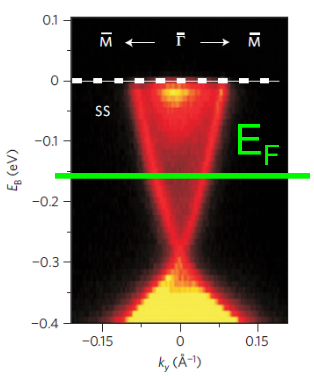
- … while attractive interactions generate (topological) superconductivity







Neupert et al., PRL '15



ARTICLE

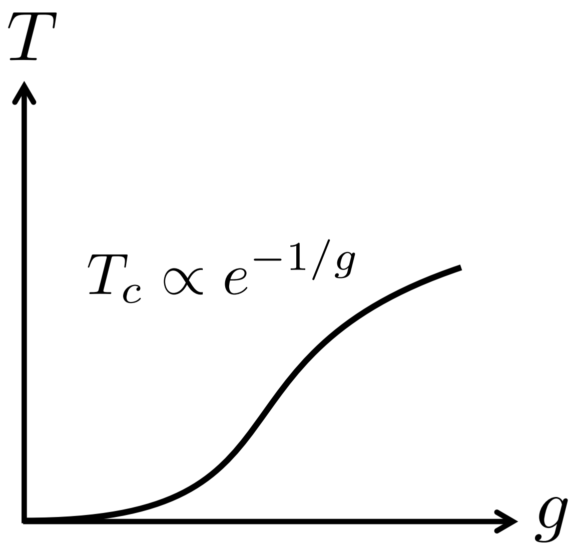
Received 31 Aug 2014 | Accepted 6 Aug 2015 | Published 11 Sep 2015
DOI: 10.1038/ncomms9279

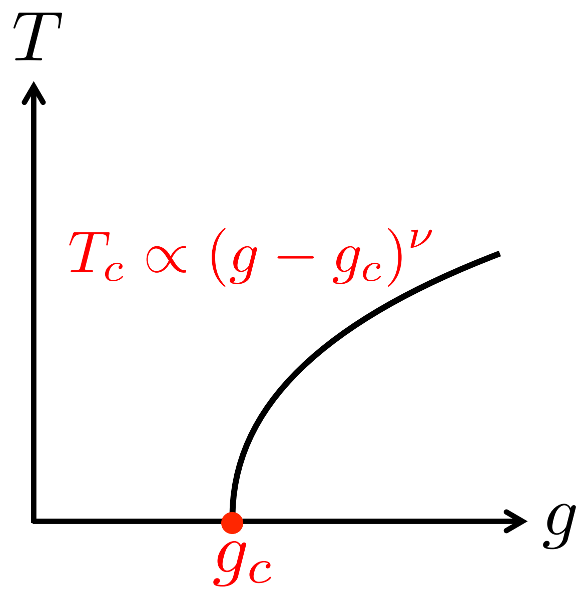
Emergent surface superconductivity in the topological insulator Sb_2Te_3

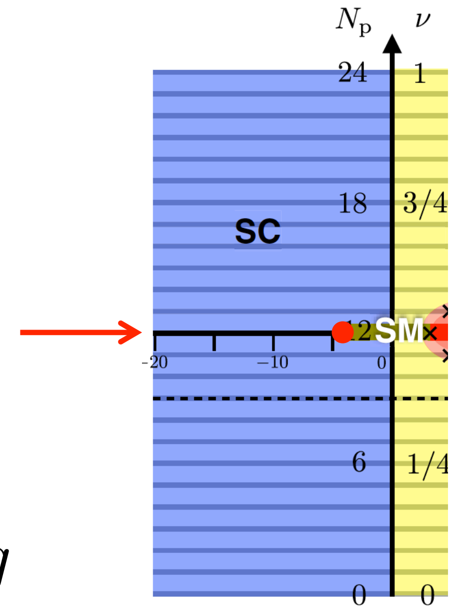
Lukas Zhao[1], Haiming Deng[1], Inna Korzhovska[1], Milan Begliarbekov[1], Zhiyi Chen[1], Erick Andrade[2], Ethan Rosenthal[2], Abhay Pasupathy[2], Vadim Oganesyan[3,4] & Lia Krusin-Elbaum[1,4]

Superconducting instabilities

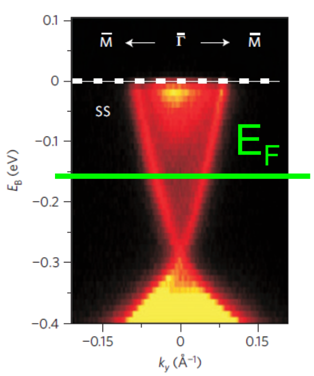
- … while attractive interactions generate (topological) superconductivity

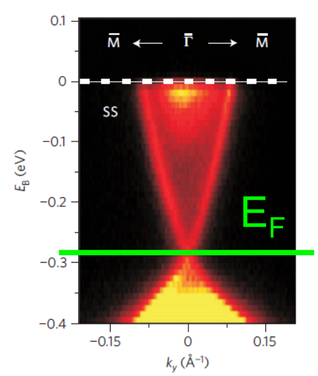






Neupert et al., PRL ‘15

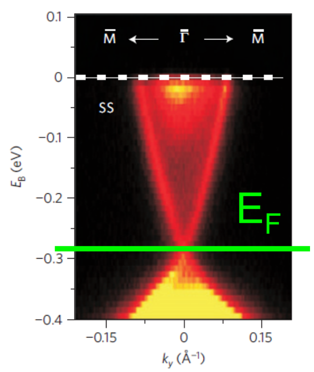


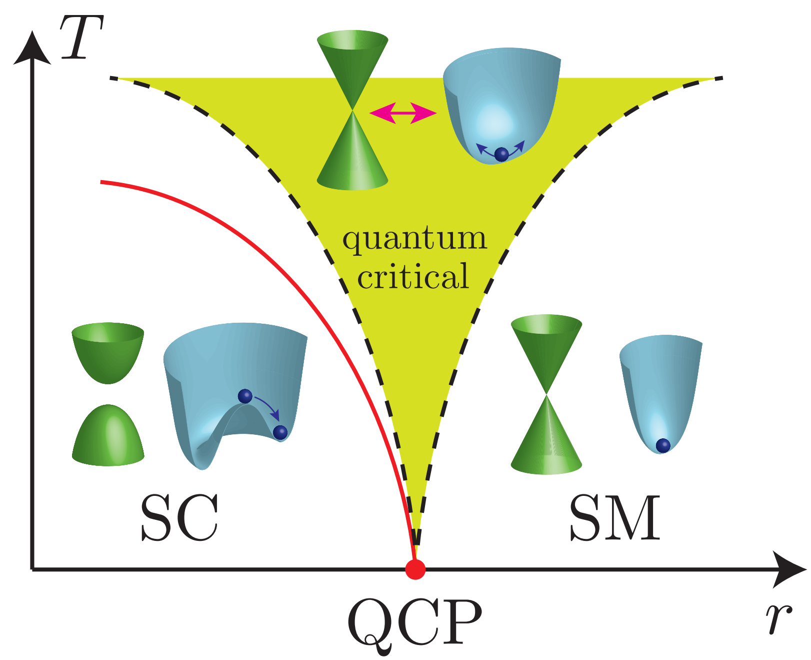


- Additional quantum critical point (Roy, Juričić, Herbut, PRB ‘13; Nandkishore, JM, Huse, Sondhi, PRB ‘13)

Semimetal-superconductor QCP

- QCP has an emergent (2+1)D **supersymmetry**: N=2 Wess-Zumino model (Grover, Sheng, Vishwanath, Science ‘14; Ponte, Lee, NJP ‘14)





$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + |\partial_\mu\phi|^2 + r|\phi|^2 + h^2|\phi|^4 + h(\phi^*\psi^T i\sigma^y\psi + \text{h.c.})$$

SUSY QCP: critical exponents

- Strongly coupled QCP: anomalous dimensions exactly known from SUSY (Aharony et al., NPB ‘97)

$$\eta_\phi = \eta_\psi = \frac{1}{3}$$

- Correlation length exponent: $\xi \sim (g - g_c)^{-\nu}$

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \mathcal{O}(\epsilon^2) \approx 0.75$$

1-loop RG (Thomas, ‘05; Lee, PRB ’07)

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \frac{\epsilon^2}{24} + \left(\frac{\zeta(3)}{6} - \frac{1}{144}\right)\epsilon^3 + \mathcal{O}(\epsilon^4) \approx 0.985$$

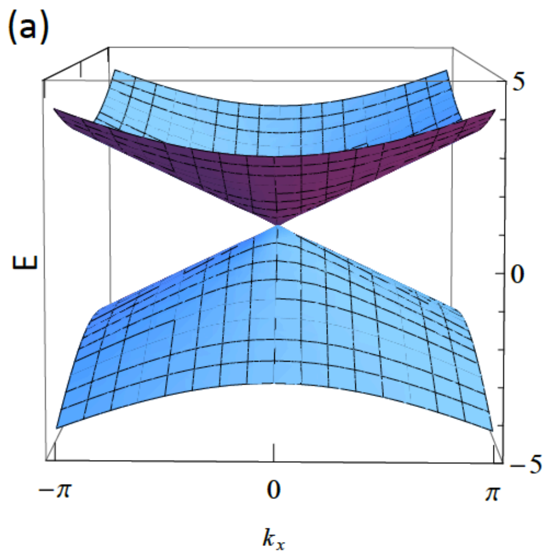
3-loop RG (Zerf, Lin, JM, PRB ’16)

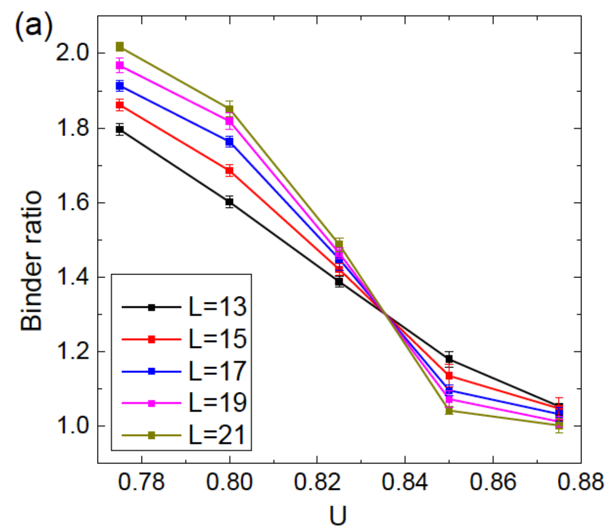
$\nu \approx 0.9174$ Padé extrapolation of 3-loop result (Fei et al., PTEP ‘16)

$\nu \approx 0.9173$ conformal bootstrap (Bobev et al., PRL ‘15)

SUSY QCP: QMC study

- Single Dirac cone + attractive Hubbard U can be simulated on a 2D lattice with long-range hopping (Li et al., arXiv '17)

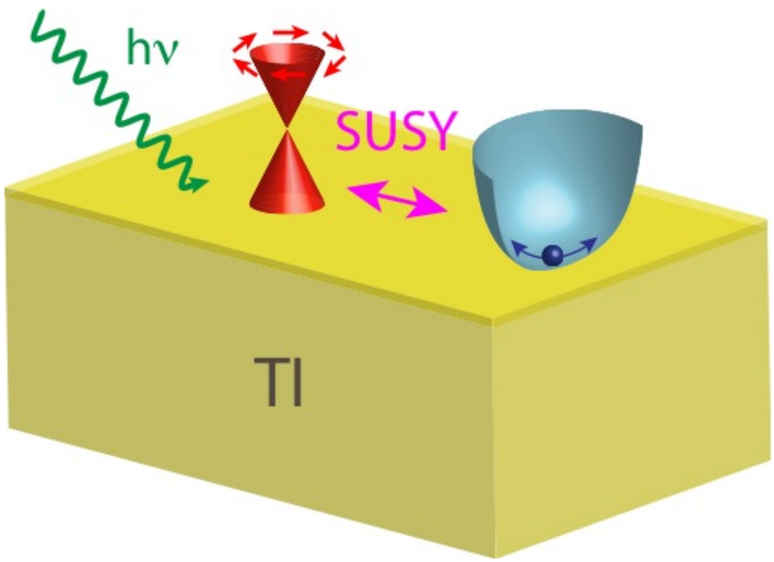




- QCP found in sign-problem-free QMC at $U_c/t \approx 0.83$
- Critical exponents: $\nu = 0.87 \pm 0.05$, $\eta_\phi = 0.32 \pm 0.02$, $\eta_\psi = 0.34 \pm 0.05$
- Consistent with SUSY! ($\nu \approx 0.917$, $\eta_\phi = \eta_\psi = 1/3$)

SUSY QCP: optical conductivity

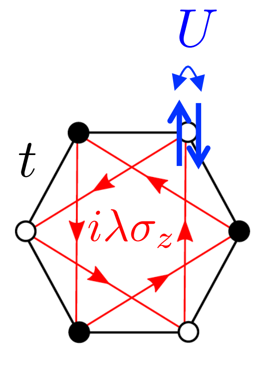
- Optical conductivity at a 2D QCP (T=0) should be spectrally flat, given by a universal constant (Damle & Sachdev, PRB '97)



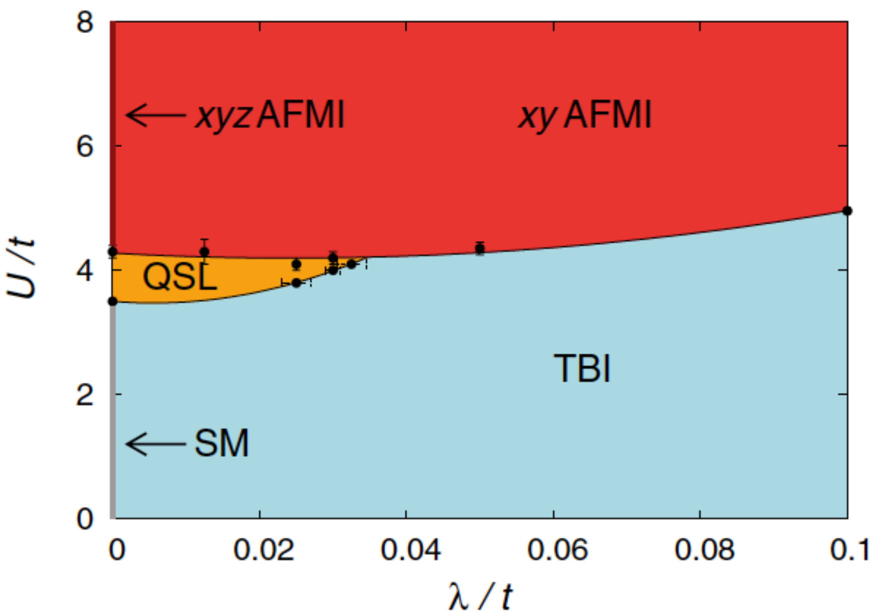
- Can be calculated exactly using SUSY at the strongly correlated Dirac SM-SC QCP (Witczak-Krempa and JM, PRL '16)
- Benchmark for QMC study

$$\sigma(\omega, 0) = \frac{5(16\pi - 9\sqrt{3})}{243\pi} \frac{e^2}{\hbar} \approx 0.2271 \frac{e^2}{\hbar}$$

Bulk symmetry breaking

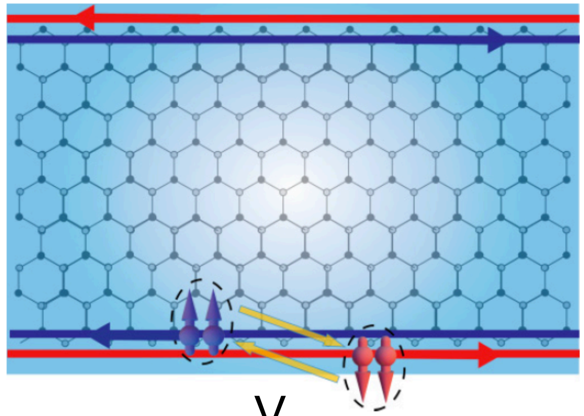


- When interaction strength ~ bulk gap, bulk SSB is possible: destroys bulk topology
- Model for interacting QSHE: Kane-Mele-Hubbard (KMH) model, yields in-plane (XY) antiferromagnetism

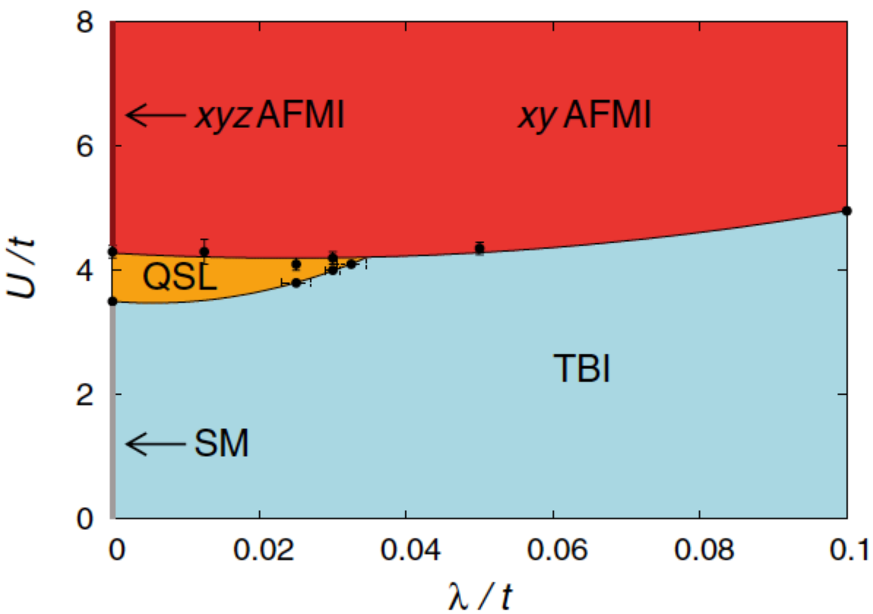


KMH model
(Hohenadler et al., PRB '12)

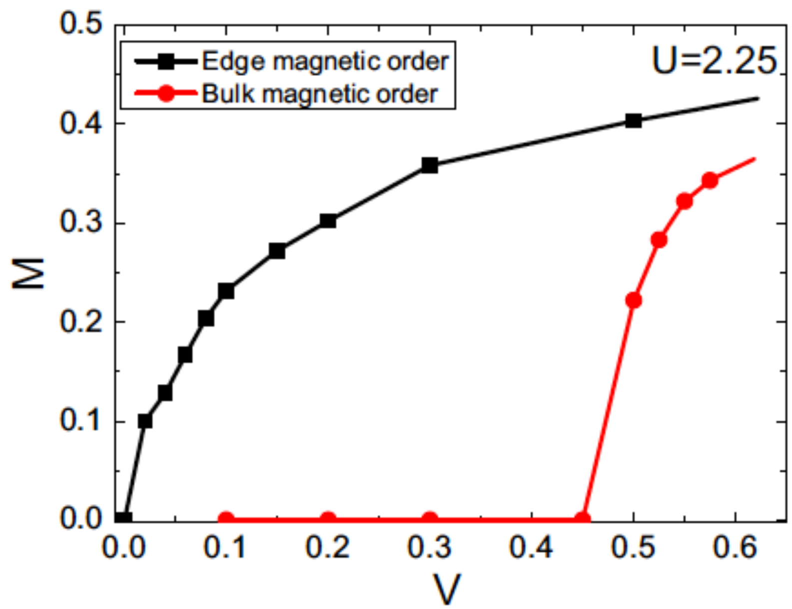
Bulk symmetry breaking



- When interaction strength ~ bulk gap, bulk SSB is possible: destroys bulk topology
- Model for interacting QSHE: Kane-Mele-Hubbard (KMH) model, yields in-plane (XY) antiferromagnetism



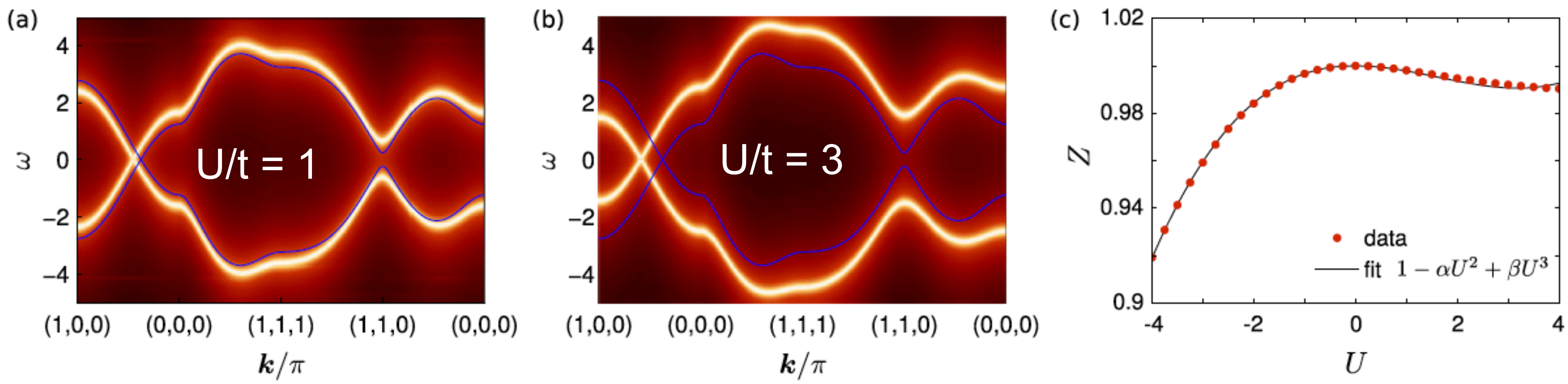
KMH model
(Hohenadler et al., PRB ’12)



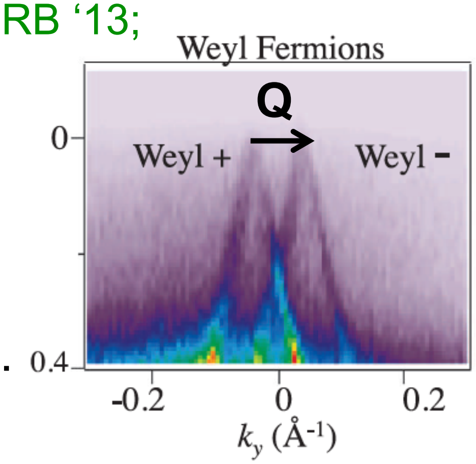
KMH + Rashba: 2-particle backscattering
(Li & Yao, PRB ‘17)

Interacting Weyl semimetals

- Bulk is gapless, but perturbatively stable: interactions renormalize quasiparticle residue (Witczak-Krempa, Knap, Abanin, PRL ‘14)



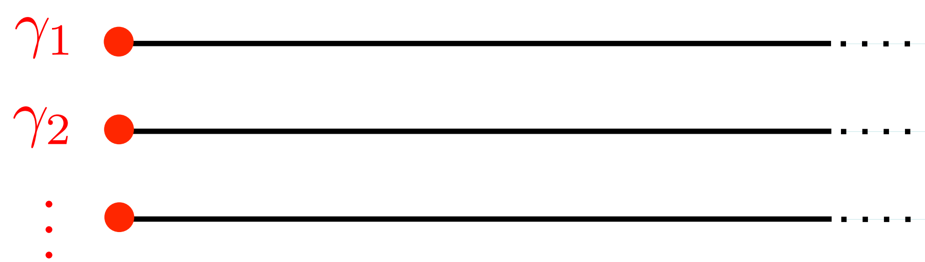
- For strong enough interactions, protecting lattice translation symmetry can be broken spontaneously: CDW / SDW order (Wang, Zhang, PRB ‘13; JM, Nandkishore, PRB ’14)
- Topological defects = “axion strings”, trap chiral fermion zero modes (Callan, Harvey, NPB ’85)
- Sliding mode = “dynamical axion field”, couples to $\mathbf{E} \cdot \mathbf{B}$
- Search for interacting Weyl materials: $CeRu_4Sn_6$, CeSb, …



Reduction of the classification

- Interactions can “reduce the classification”: adiabatically connect two phases thought to be distinct without interactions (Fidkowski & Kitaev, PRB ’10)
- Example: BDI class in d=1, Majorana zero modes (MZM) protected by “spinless TRS”

$$T\gamma_j T^{-1} = \gamma_j ,$$
$$TiT^{1} = -i$$

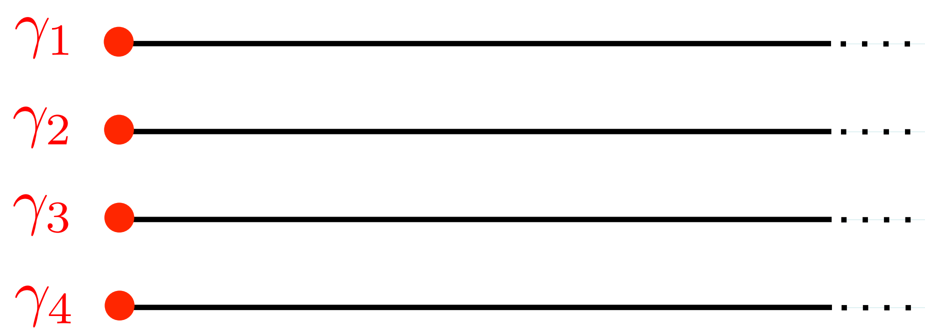


- Noninteracting (quadratic) terms that can gap out the MZM are of the form $i\gamma_j\gamma_k$, forbidden by TRS: Z invariant ν, counts number of MZM

Reduction of the classification

- Interactions can "reduce the classification": adiabatically connect two phases thought to be distinct without interactions (Fidkowski & Kitaev, PRB '10)
- Example: BDI class in d=1, Majorana zero modes (MZM) protected by "spinless TRS"

$$c_1 = (\gamma_1 + i\gamma_2)/2,$$
$$c_2 = (\gamma_3 + i\gamma_4)/2$$



- TRS-preserving interaction with ν = 4:

$$\frac{U}{4}\gamma_1\gamma_2\gamma_3\gamma_4 = -U(n_1 - \tfrac{1}{2})(n_2 - \tfrac{1}{2})$$

- Ground state has energy –U/4, but 2-fold degenerate ($|00\rangle$ and $|11\rangle$): still distinct from trivial state ν = 0 (no MZM, unique ground state)

Reduction of the classification: Z to Z_8

- Interactions can "reduce the classification": adiabatically connect two phases thought to be distinct without interactions (Fidkowski & Kitaev, PRB '10)
- Example: BDI class in d=1, Majorana zero modes (MZM) protected by "spinless TRS"
- TRS-preserving interaction with ν = 8:

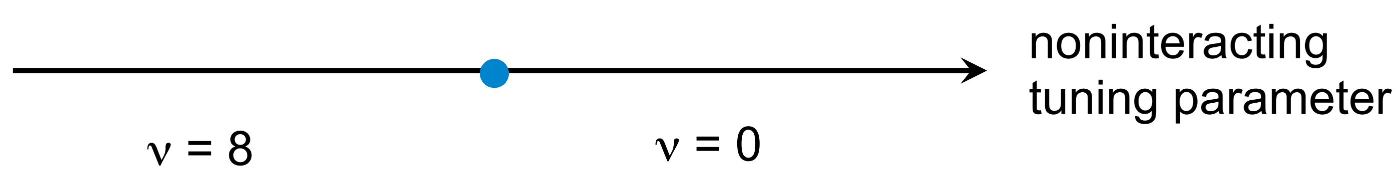
$$\mathbf{S}_1 \equiv \begin{pmatrix} c_1^\dagger & c_2^\dagger \end{pmatrix} \boldsymbol{\sigma} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\mathbf{S}_2 \equiv \begin{pmatrix} c_3^\dagger & c_4^\dagger \end{pmatrix} \boldsymbol{\sigma} \begin{pmatrix} c_3 \\ c_4 \end{pmatrix}$$

$J\mathbf{S}_1 \cdot \mathbf{S}_2$: for J > 0, unique (singlet) gapped ground state, adiabatically connected to trivial phase ν = 0

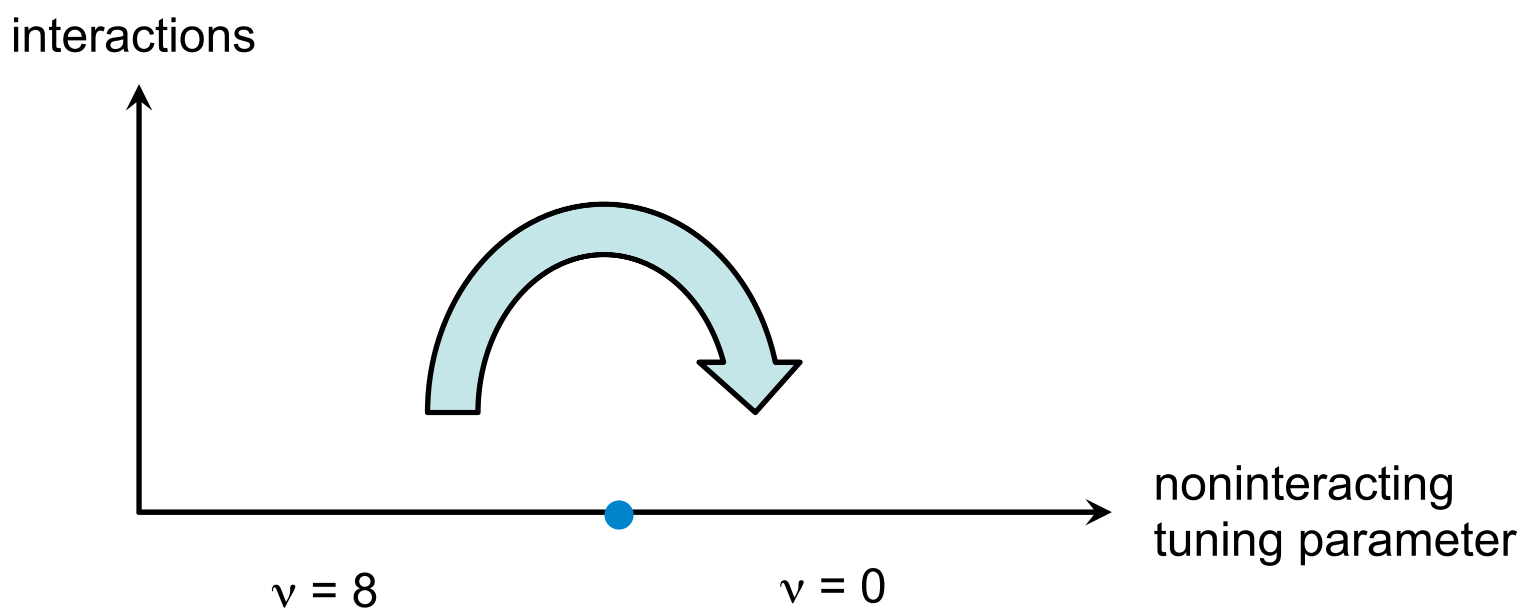
Reduction of the classification: Z to Z_8

- Interactions can "reduce the classification": adiabatically connect two phases thought to be distinct without interactions (Fidkowski & Kitaev, PRB '10)
- Example: BDI class in d=1, Majorana zero modes (MZM) protected by "spinless TRS"



Reduction of the classification: Z to Z_8

- Interactions can “reduce the classification”: adiabatically connect two phases thought to be distinct without interactions (Fidkowski & Kitaev, PRB ’10)
- Example: BDI class in d=1, Majorana zero modes (MZM) protected by “spinless TRS”



Outline

I. Stability of free-fermion topological phases to interactions

- Perturbative stability
- Spontaneous symmetry breaking
- Reduction of the free-fermion classification

II. Interaction-induced topological phases

- Topological Mott insulators
- Topological Kondo insulators

III. Strongly correlated topological phases

- Symmetry-protected topological phases
- Fractionalized topological phases

Topology from interactions

- Can a topological bandstructure emerge spontaneously (in a mean-field sense) from interactions?

Topological superconductivity (-fluidity)

- Can a topological bandstructure emerge spontaneously (in a mean-field sense) from interactions?

- First example: topological superconductors/superfluids! Dynamical generation of topological pairing gap from interactions; support edge/surface Majorana fermions (Read & Green, PRB ‘00; Roy, PRB ‘09; Volovik, PRB ’09; Qi et al., PRL ‘09)

- Materials: Sr_2RuO_4 (?), B phase of superfluid ^3He

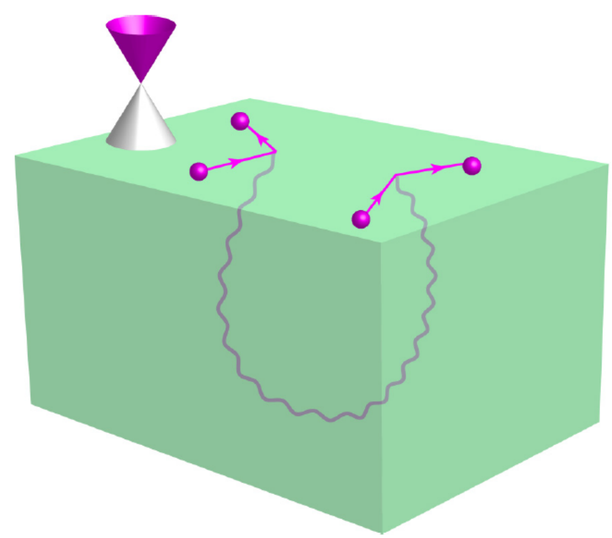
Topological superconductivity (-fluidity)

- Can a topological bandstructure emerge spontaneously (in a mean-field sense) from interactions?

- First example: topological superconductors/superfluids! Dynamical generation of topological pairing gap from interactions; support edge/surface Majorana fermions (Read & Green, PRB ‘00; Roy, PRB ‘09; Volovik, PRB ’09; Qi et al., PRL ‘09)

- Materials: Sr_2RuO_4 (?), B phase of superfluid ^3He

- Edge/surface Majorana fermions can interact by exchanging SC/SF order parameter fluctuations (Park, Chung, JM, PRB ‘15)
- Can induce spontaneous T breaking on the surface: possibility of QCPs with emergent N=1 SUSY (Grover, Sheng, Vishwanath, Science ’14)



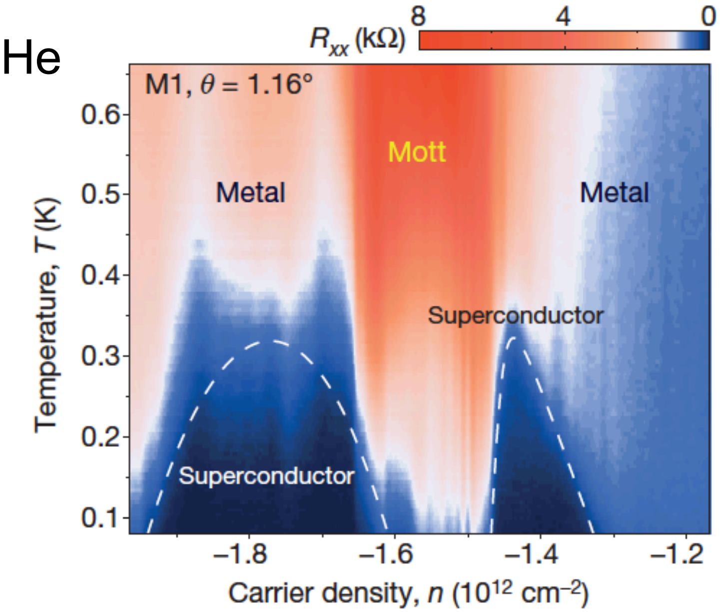
Topological superconductivity (-fluidity)

- Can a topological bandstructure emerge spontaneously (in a mean-field sense) from interactions?

- First example: topological superconductors/superfluids! Dynamical generation of topological pairing gap from interactions; support edge/surface Majorana fermions (Read & Green, PRB ‘00; Roy, PRB ‘09; Volovik, PRB ’09; Qi et al., PRL ‘09)

- Materials: Sr_2RuO_4 (?), B phase of superfluid ^3He

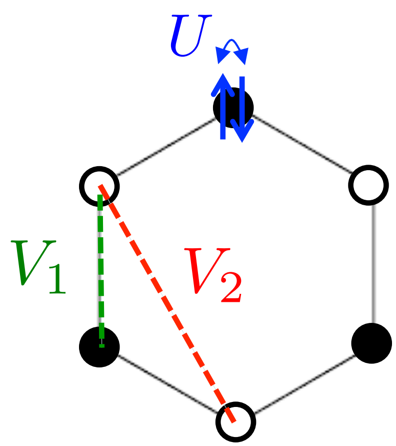
- Superconductivity in graphene Moire superlattices: possible topological spin-triplet $d_{x2-y2}+id_{xy}$ pairing? (Xu & Balents, arXiv ‘18)

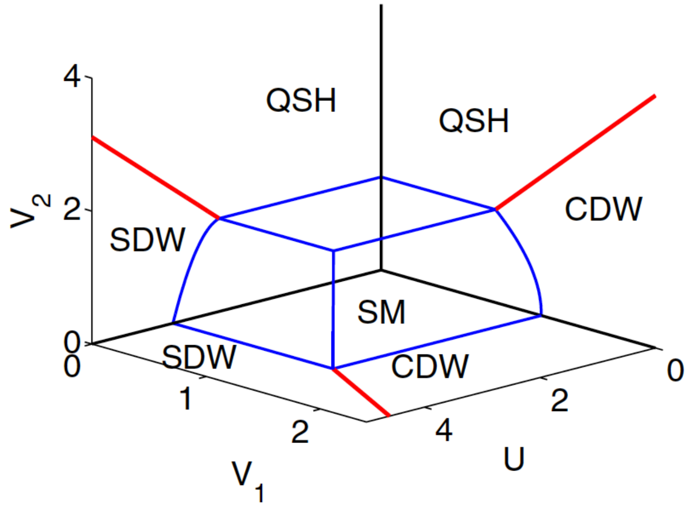


Cao et al., Nature ’18

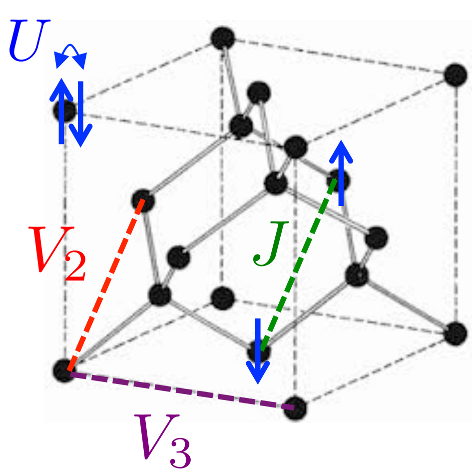
Topological Mott insulators

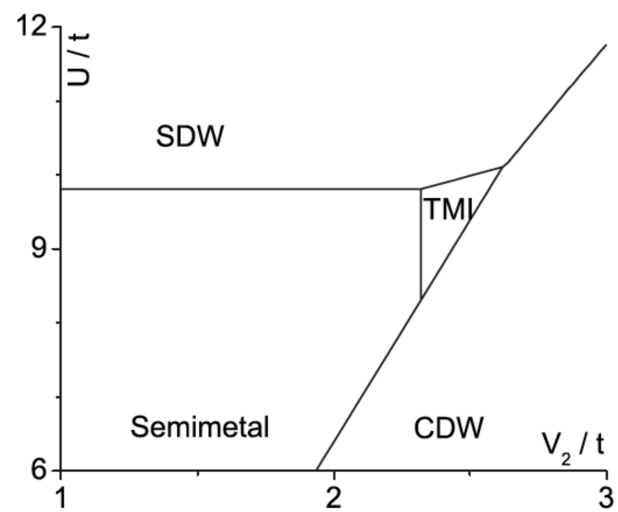
- Topologically **insulating** gap can also be dynamically generated from (strong) interactions: spontaneous generation of spin-orbit coupling





2D topological Mott insulator
(Raghu et al., PRL '08)





3D topological Mott insulator
(Zhang, Ran, Vishwanath, PRB '09)

Topological Mott insulators

- Topologically **insulating** gap can also be dynamically generated from (strong) interactions: spontaneous generation of spin-orbit coupling

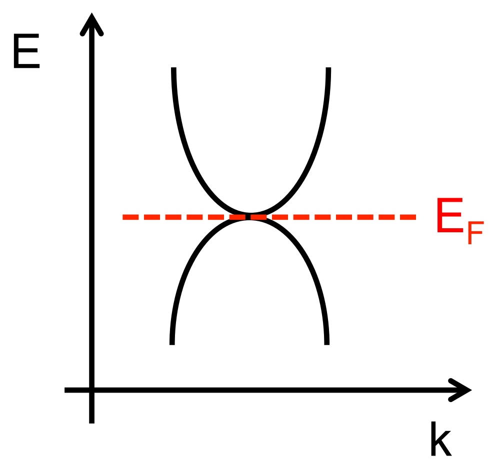
- Contrasts with weak-coupling BCS instability towards paired states. Can topological Mott insulators be stabilized for weak interactions?

Quadratic band crossings

- Topologically **insulating** gap can also be dynamically generated from (strong) interactions: spontaneous generation of spin-orbit coupling

- Contrasts with weak-coupling BCS instability towards paired states. Can topological Mott insulators be stabilized for weak interactions?

- **Quadratic band crossings in 2D**: finite DOS at Fermi level enhances particle-hole instabilities → topological Mott insulators (e.g. QSH) stabilized for infinitesimal interactions (Sun et al., PRL '09)

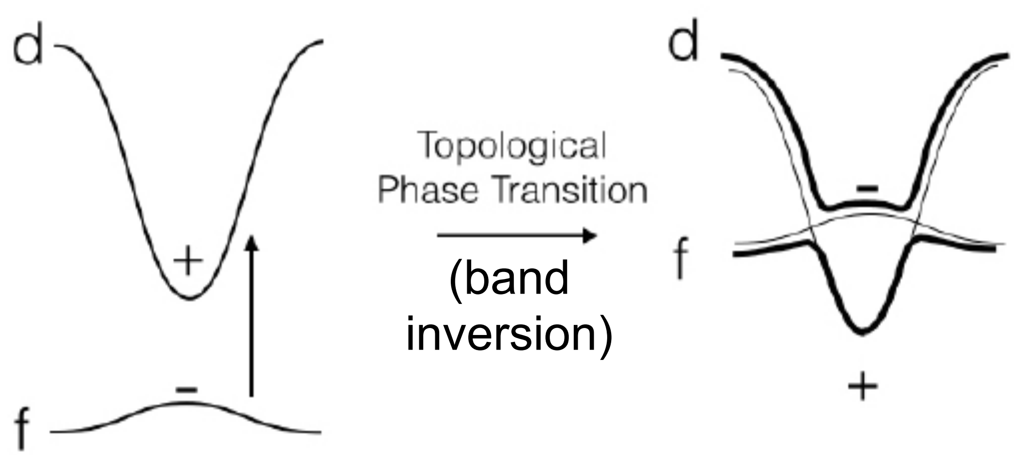


- Examples: checkerboard lattice (QBC protected by TRS and C_4 symmetry), kagome lattice (TRS and C_6 symmetry)

- In 3D: "gapless semiconductors" (HgTe, α-Sn), may realize a 3D topological Mott insulator at low T (Herbut, Janssen, PRL '14)

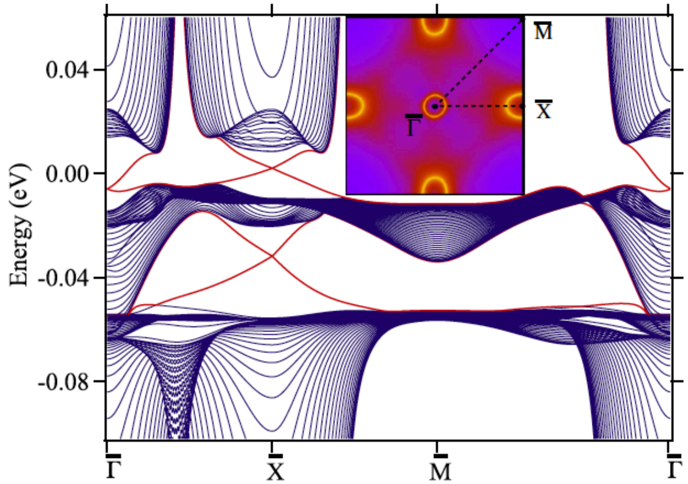
Topological Kondo insulators

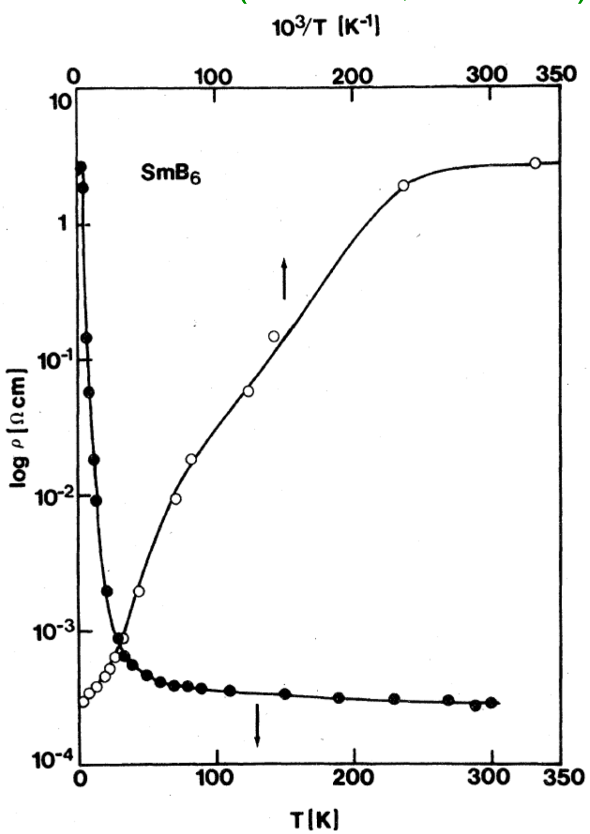
- Two main mechanisms to open a topological gap from interactions:
  - Spontaneous generation of spin-orbit coupling (topological Mott insulators): spontaneous breaking of SU(2) spin rotation symmetry, sharp phase transition below critical temperature T_c (in 3D)
  - Kondo hybridization between strongly spin-orbit coupled, localized *f* electrons, and extended *d* electrons: crossover from metallic behavior to **topological Kondo insulator** below coherence temperature T*
    (Dzero et al., PRL '10)



SmB_6, a topological Kondo insulator?

- First-principles studies predict that SmB_6 is a topological Kondo insulator with Dirac surface states at the X points on the (001) surface (Lu et al., PRL '13)

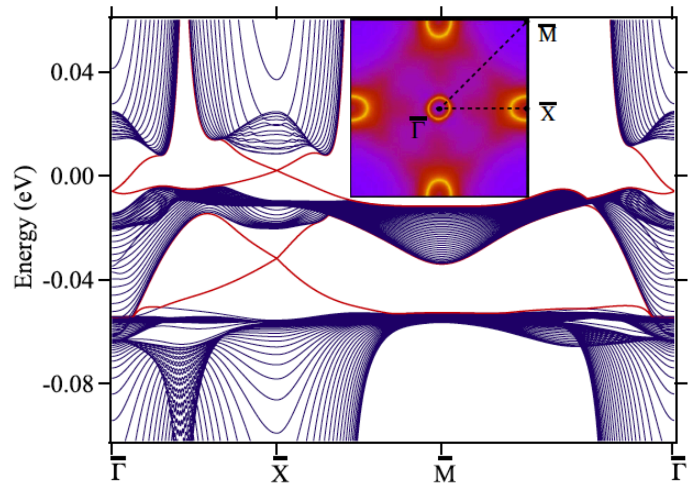


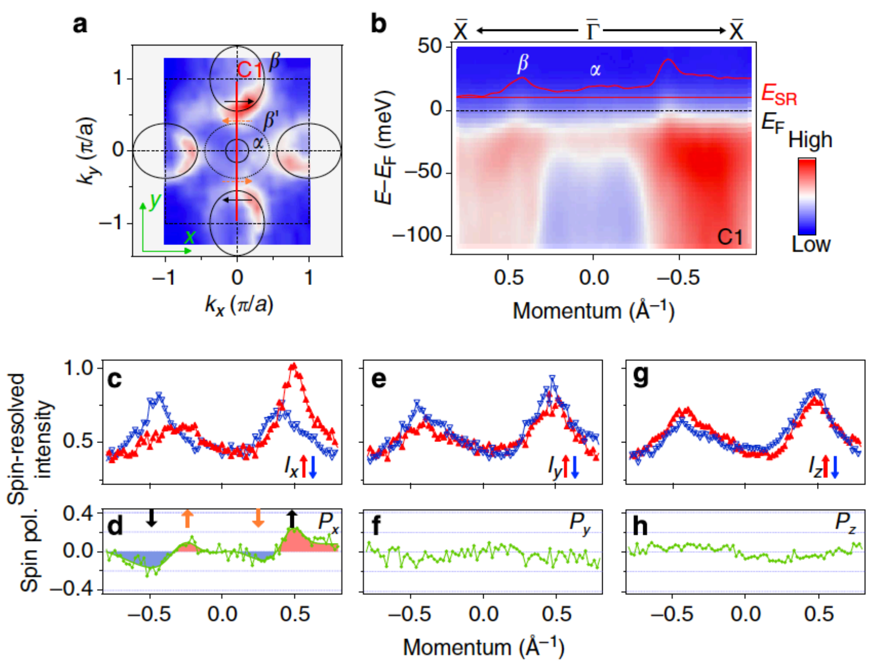


- Topological surface states may explain long-standing puzzle of residual low-T resistivity in SmB_6 (Allen, Batlogg, Wachter, PRB '79)

SmB_6, a topological Kondo insulator?

- First-principles studies predict that SmB_6 is a topological Kondo insulator with Dirac surface states at the X points on the (001) surface (Lu et al., PRL '13)





Xu et al., Nat. Comm. '14

- Spin-resolved ARPES consistent with helical surface states, but still controversial

Outline

I. Stability of free-fermion topological phases to interactions

- Perturbative stability
- Spontaneous symmetry breaking
- Reduction of the free-fermion classification

II. Interaction-induced topological phases

- Topological Mott insulators
- Topological Kondo insulators

III. Strongly correlated topological phases

- Symmetry-protected topological phases
- Fractionalized topological phases

Symmetry-protected topological phases

- Free-fermion topological insulators, topological superconductors, topological Mott/Kondo insulators: adiabatically connected to free-fermion topological insulators (ignoring Goldstone/collective modes)
- Simplest examples of SPT phases (Chen, Gu, Wen, PRB '10): cannot be adiabatically connected to a trivial product state if symmetry is preserved
- Generally interacting, but assume not fractionalized: unique ground state, no deconfined bulk excitations with fractional quantum numbers/statistics
- Are there SPT phases that are **not** adiabatically connected to free-fermion topological phases?

Surface terminations

- First, back to ordinary 3D topological insulator. Protected by T and U(1) symmetries. Possible surface “terminations” with interactions?
    - Helical Fermi liquid: gapless, T and U(1) symmetric
    - Ising FM order: gapped, U(1) symmetric, breaks T
    - s-wave SC order: gapped, T symmetric, breaks U(1)
- Can we gap out the surface without breaking symmetries?

Surface topological order

- Yes! But at the expense of developing **surface topological order** = 2D surface must support anyonic excitations (Bonderson, Nayak, Qi, J. Stat. Mech. ’13; Wang, Potter, Senthil, PRB ’13; Chen, Fidkowski, Vishwanath, PRB ’14; Metlitski, Kane, Fisher, PRB ’15)

- Basic idea: start from U(1)-breaking SC surface; attempt to trigger 2D superconductor-insulator transition by condensing vortices to restore U(1) symmetry (Fisher, Weichman, Grinstein, Fisher, PRB ‘89)

- Unlike ordinary SC, vortices on SC surface host Majorana fermions (Fu, Kane, PRL ‘08): not bosonic (non-Abelian!), single hc/2e vortex cannot condense

- Simplest bosonic object that can condense while preserving T is 4-fold (2hc/e) vortex; destroys SC order and gaps surface but introduces non-Abelian topological order

Interacting 3D topological insulators

- Classifying all possible types of topological order that can only exist on 2D boundary of 3D system ("anomalous"), consistent with T and U(1) symmetries, enables one to classify all possible 3D interacting topological insulators = 3D fermionic SPT phases protected by T and U(1)

- In total: 8, of which 6 not adiabatically connected to free-fermion insulator (Wang, Potter, Senthil, Science '14)

- 6 nontrivial SPT phases = electronic Mott insulator where spins form a "bosonic SPT" phase protected by T

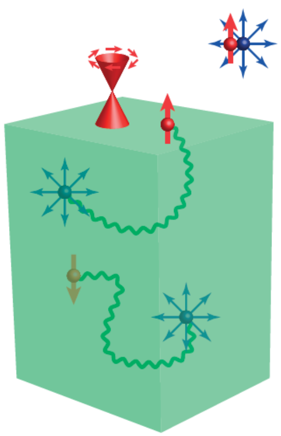
Fractionalized topological insulators

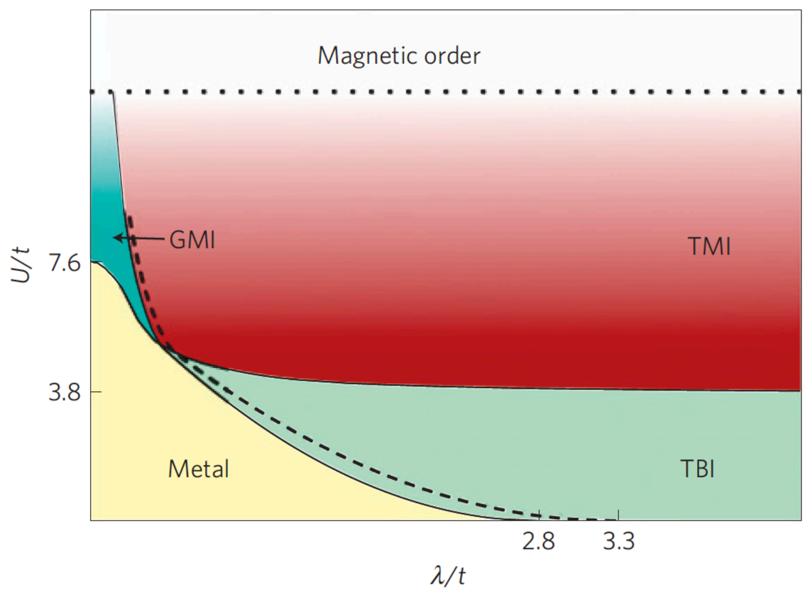
- SPT phase = simplest kind of interacting topological insulator: no fractionalization in the bulk (but allowed on the surface)
- We know strong correlations can induce fractionalization: spin-charge separation (quantum spin liquids), fractionalization of charge (FQHE)
- Novel types of correlated topological insulators if fractionalization is allowed in the bulk?
- Basic idea: assume fractionalized excitations (e.g. spinons, fractionally charged quasiparticles) occupy a topological bandstructrure

Fractionalized topological insulators

- Strong repulsive interactions + strong spin-orbit coupling: assume fermionic spinons (charge 0, spin 1/2) occupy a topological bandstructure

- In 2D, fractionalized QSHE (Young, Lee, Kallin, PRB '08; Rachel & Le Hur, PRB '10): unstable against gauge fluctuations (instanton proliferation)





- In 3D, fractionalized topological insulator (Pesin & Balents, Nat. Phys. '10): stable against gauge fluctuations = 3D U(1) spin liquid with emergent gapless "photon" and helical spinon surface states

- Potentially relevant to pyrochlore iridates

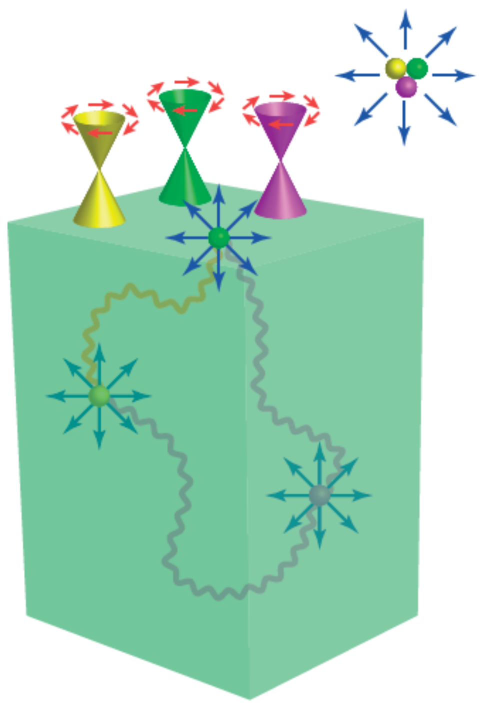
Fractional topological insulators

- Another possibility: electron (charge e) fractionalizes into quasiparticles of charge e/N

- 2D: fractional quantum spin Hall effect (Bernevig & Zhang, PRL '06; Levin & Stern, PRL '09) ≈ two copies of FQHE with opposite chirality for opposite spins

- 3D: fractional topological insulator (JM et al., PRL '10; Swingle et al., PRB '11), exhibits quantized but fractional magnetoelectric effect ($\theta=\pi/N$) while preserving T symmetry

- Surface states gapless = could be seen in ARPES, but electron spectral function is power-law due to fractionalization (Swingle, PRB '12)



Outlook

- Topics not covered: effect of interactions in…
  - Chern insulators (including fractional Chern insulators)
  - Novel topological semimetals (nodal line, type-II Weyl, “new fermions”/ higher-order band crossings…)
  - Beyond electrons: topological phases with ultracold atoms, topological magnon/phonon bandstructures, topoelectrical circuits…
  - Nonequilibrium topological phases (Floquet)
- Many concepts overlap with topics in frustrated magnetism (spin liquids) and FQHE physics
- Ongoing search for experimental candidates, in particular materials with strong spin-orbit coupling + correlations