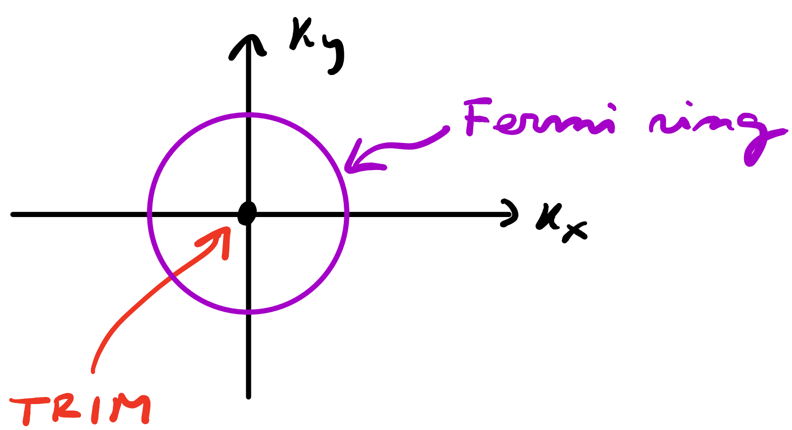
③ $\mathbb{Z}_2$ topological insulators in 3D

3.3 Surface states

Consider a strong top. ins.

with $(\nu_0;\ \nu_1, \nu_2, \nu_3) = (1;0,0,0)$

* Electrons on xy surface



2 fold degeneracy at $\vec{k} = 0$.

Degeneracy splits at $\vec{k} \neq 0$. How?

* Effective Hamiltonian:

$$h_{eff}(\vec{k}) = \epsilon(\vec{k}) + \vec{d}(\vec{k}) \cdot \vec{\sigma}$$

where σ^z labels the 2 states forming a Kramers pair at $\vec{k} = 0$.

$\vec{\sigma} \sim$ angular momentum.

* Symmetry constraints:

TRS $\Leftrightarrow$ $h_{eff}(\vec{k}) = \Theta \, h_{eff}(-\vec{k}) \, \Theta^{-1}$

where $\Theta = i \sigma^y \mathbb{K}$.

Then, TRS imposes

$$\epsilon(\vec{k}) = \epsilon(-\vec{k})$$
$$\vec{d}(\vec{k}) = -\vec{d}(-\vec{k})$$

* Small $\vec{k}$ expansion:

$$h_{eff}(\vec{k}) = \epsilon_0 + \alpha_{ij} k_i k_j$$

$$+ \beta_{i\alpha} k_i \sigma^{\alpha} + O(k^3)$$

sum over repeated indices

$i, j \in \{x, y\}$

$\alpha \in \{x, y, z\}$

* For small $\vec{k}$, we can assume rotational symmetry around $\hat{z}$ (normal to the surface).

Then,

$$h_{eff}(\vec{k}) = \epsilon_0 + \alpha \vec{k}^2 + \beta \vec{k} \cdot \vec{\sigma}$$

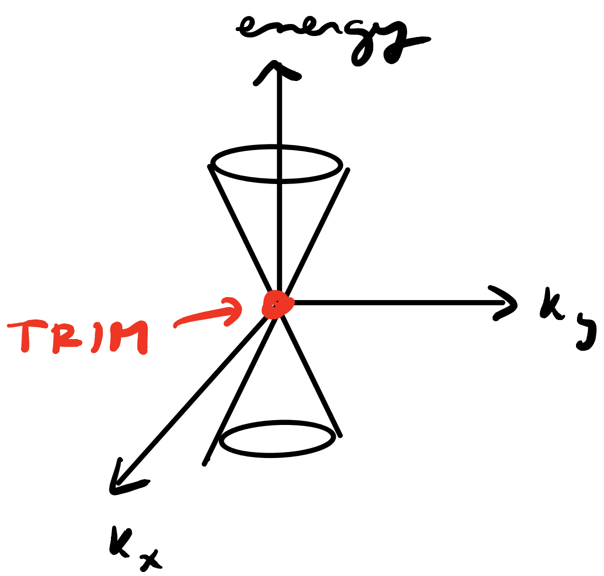
(scalar products are invariant under rotation)

we could also have

$$h_{eff}(\vec{k}) = \epsilon_0 + \alpha \vec{k}^2 + \beta (\vec{k} \times \hat{z}) \cdot \vec{\sigma}$$

The 2nd "choice" is more commonly realised in real materials.

* Energy spectrum :



2D massless Dirac fermions.

Difference with respect to graphene:

The Dirac cone here is non degenerate (one cone per surface, as opposed to four cones in graphene)

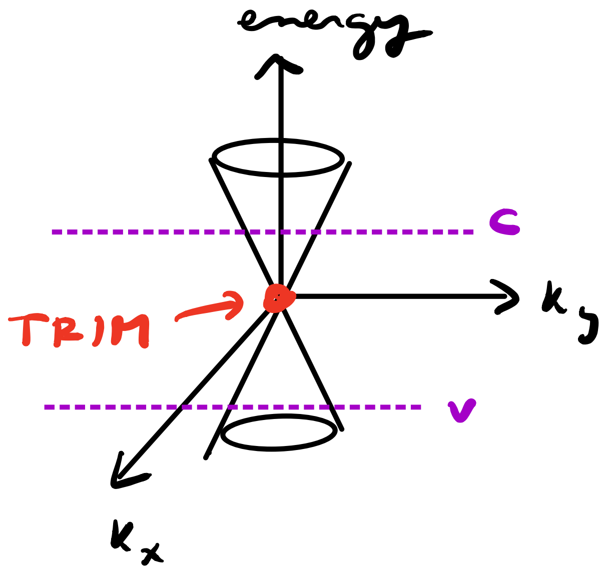
K / K'

spin ↑ / spin ↓

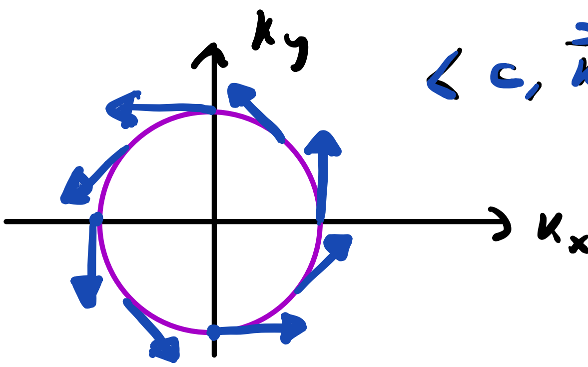
Another difference with respect to graphene: $\vec{\sigma}$ here is angular momentum (odd under TR), as opposed to A/B sublattice in graphene (even under TR).

* Spin-texture in momentum space:

$$h_{eff}(\vec{k}) = \epsilon_0(\vec{k}) + \underbrace{\beta(\hat{z}\times\vec{k})}_{\substack{\text{like an effective}\\ \text{B-field pointing}\\ \text{along } \hat{z}\times\vec{k}.}}\cdot\vec{\sigma}$$

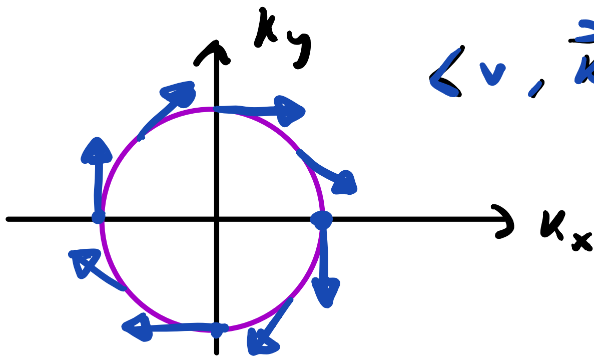


Constant energy surface c :



$\langle c, \vec{k} \,|\, \vec{\sigma} \,|\, c, \vec{k} \rangle$

Constant energy surface v:



$\langle v, \vec{k} | \vec{\sigma} | v, \vec{k} \rangle$

"Helical spin texture"

"Spin-momentum locking"

Berry phase of π for a constant energy contour

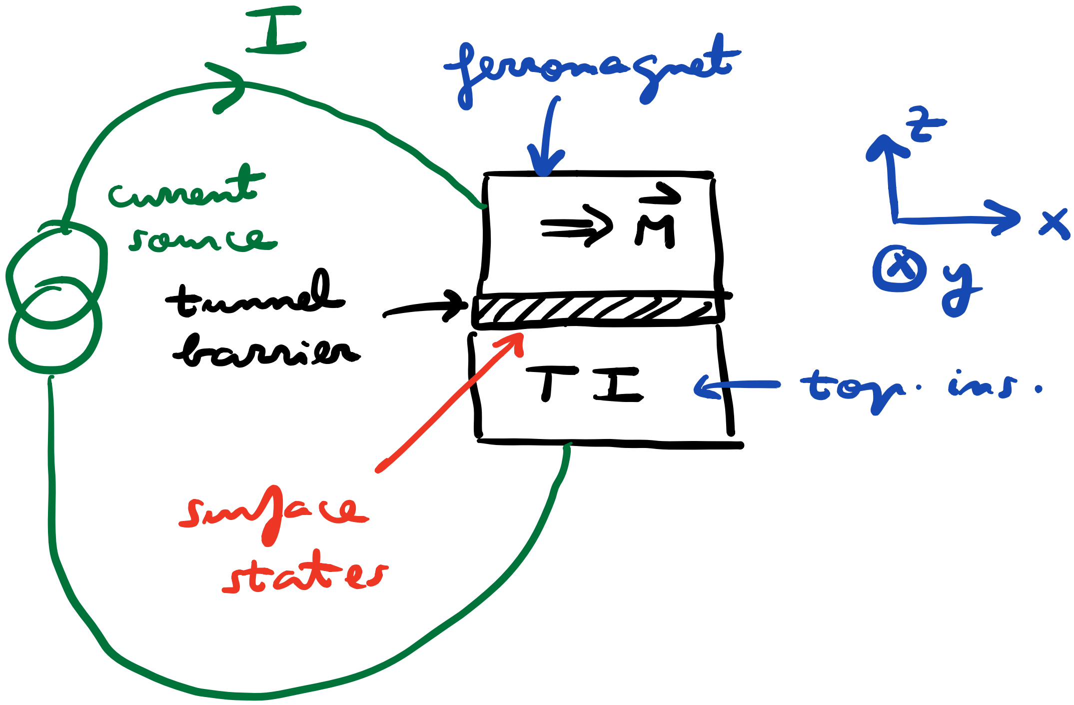
(lec. 5)

* Some physical consequences of spin-momentum locking:

(1) Enhanced surface conductivity:

180° scattering ("backscattering") is not possible for non-magnetic impurities because $\vec{k}$ and $-\vec{k}$ electronic states are TR-partners.

(2) Magnetoelectric (spintronics) effects:



Because of the magnetization $\vec{M}$, the electrons exiting the magnet have a net spin density along x.

Assume spin-conserving tunneling through barrier.

Because of spin-momentum locking, a current along y is generated on the surface of the TI.

If the circuit is "open" along y, then the accummulated charge will result in a voltage difference V along y.

(Hall voltage)

If $\vec{M} \to -\vec{M}$, $V \to -V$.

L. Liu et al., PRB 91, 235437 (2015)

Converse effect: if a current along y flows on the surface of the TI, then this current is spin-polarized along $-x$.

("Edelstein effect")

Such spin polarization can be used to manipulate (even reverse) the magnetization of a nearby magnet. "spin torque"

A. Mellnik et al., Nature 511, 449 (2014).