*NOISE IN PHYSICAL SYSTEMS AND 1/f NOISE*
*M. Savelli, G. Lecoy and J-P. Nougier (eds.)*



# FLUCTUATIONS IN DISSIPATIVE STEADY STATES OF THIN METALLIC FILMS

A.-M. S. Tremblay and François Vidal

Département de physique, Université de Sherbrooke,
Sherbrooke, Québec J1K 2R1
Canada

It is shown that high-frequency current fluctuations induced by the application of a steady electric field to a thin metallic film provide, in certain cases, a way to determine an inelastic-scattering time in a regime where it is otherwise hardly accessible because the usual transport coefficients and equilibrium fluctuations are mainly determined by elastic scattering.

Since the following work has already appeared in print[1], we restrict ourselves, for the benifit of this audience, to a short summary of our motivation and main results and we briefly discuss one related new development.

In our opinion, there are two reasons which make the study of fluctuations in nonequilibrium systems interesting.

a) Firstly, the fundamental aspect. The measurement of fluctuations probably constitutes the most stringent test of Statistical Mechanics available. Yet there are very few experimental tests of fluctuations in nonequilibrium systems, and very often these experiments are not very well understood. 1/f noise is in such a category. One can then certainly claim that Nonequilibrium Statistical Mechanics (N.E.S.M.) rests on much less solid grounds than its equilibrium counterpart. We thus think it is important to suggest experiments which could check certain aspects of the theory.

b) Secondly, we think that if one takes the opposite point of view and assumes that N.E.S.M. is correct, then one can use the predictions of the theory to learn about properties of materials. Indeed, there is no simple fluctuation-dissipation theorem for fluctuations about nonequilibrium states (even steady-states) and hence in general, measurements of fluctuations about nonequilibrium states give results which cannot be predicted from transport measurements and the like. Again, we can take 1/f noise as an example. 1/f noise may be telling us something about trap disbributions, atomic diffusion or similar processes which are not well known because they have very little bearing on traditional measurements.

The work of ref. 1 addressed a few examples of fluctuations in nonequilibrium steady states. We summarize here the results we obtained for the problem of high frequency current fluctuations in a resistor subjected to a constant electric field. We refer to frequencies lower than the usual high frequency cutoffs but higher than those at which other physical effects tend to give 1/f fluctuations.

Our calculation applies to a metallic resistor in the residual resistivity regime where the resistance is determined by elastic (defect) scattering processes. We take the phonons into account explicitly. They provide a sink for Joule heat. Our results are obtained from a Boltzmann-Langevin type theory[2], which can also be justified with Green's function techniques. The extra nonequilibrium current fluctuations are given by,

$$S_I(\omega) \equiv \int_{-\infty}^{\infty} dt\; e^{i\omega t} \langle I(t)\, I(0)\rangle_{ne} \tag{1}$$

$$= \frac{2}{R} k_B T \left[0.54\ \frac{eE\ell^*}{k_B T}\right]^2 \tag{2}$$

where $k_B$ is Boltzmann's constant, T the temperature, R the sample resistance, e the electron charge, E the applied electric field and

$$\ell^* \equiv v_F^2\, \tau_e\, \tau_o \tag{3}$$

is the inelastic mean free path. $v_F$ is the Fermi velocity, $\tau_e$ the elastic collision time and $\tau_o$ an inelastic collision time. (See Eq.(3.24) of Ref.1 for a more precise definition). We assume that $\tau_e \ll \tau_o$, i.e. that the resistance is, to an excellent approximation, determined by $\tau_e$ and hence temperature independent.

One can show explicitly that it is possible to find experimental conditions such that phonon heating is negligible. Hence the above results apply, for example, in films of the order of 1000 Å thick and for temperatures of the order of 1 K. With $\tau_e \sim 10^{-14}$s, one could obtain a 10% effect in a field of the order of $10^{-2}$ V/cm. Experimental conditions are discussed in detail in Eqs. (4.22) to (4.24) of ref. 1 .

Let us now discuss how the above results bear upon the two aspects of nonequilibrium fluctuations mentioned at the beginning.

a) The result (2), if measured experimentally, would be a check of the theory of nonequilibrium fluctuations only if the result could be obtained with great precision and if the parameters entering Eq. (2) were also known independently from other sources and with a comparable precision. Indeed, the order of magnitude of the result (2) can be obtained from the following simple argument. Consider the equilibrium Nyquist-Johnson result,

$$S_I(\omega) = \frac{2}{R} k_B T . \qquad (4)$$

In the nonequilibrium steady state, the electronic temperature will rise by an amount given roughly by the balance between the Joule heating rate and the rate at which electrons can loose their energy to the external world, this being limited by the electron-phonon inelastic collision time $\tau_o$ since the phonons are in very good contact with the thermal reservoir. More quantitatively we have

$$\frac{C_v \delta T}{\tau_o} = \frac{1}{\Omega} \frac{V^2}{R} \qquad (5)$$

where $C_v$ is the specific heat, $\delta T$ the electronic temperature change, $\Omega$ the volume and V the applied voltage. Since R is temperature independent, the most simple-minded ideas about nonequilibrium fluctuations would predict that the nonequilibrium current fluctuations are given by

$$S_I(\omega) = \frac{2}{R} k_B \, \delta T \qquad (6)$$

with $\delta T$ determined by Eq.(5). That result is a factor 5.33 smaller than the true result. If one becomes a bit more sophisticated and tries to evaluate the temperature rise from a more realistic formula than Eq.(5) one must first solve the Boltzmann equation appropriate to this problem. One then finds that the steady-state distribution cannot be described by a temperature. If one insists in calculating a temperature from the formula

$$\delta T = \frac{1}{C_v} \int \frac{d^3k}{(2\pi)^3} \varepsilon_k \, \delta f_k \qquad (7)$$

where $\varepsilon_k$ is the energy in state k and $\delta f_k$ the deviation from the equilibrium distribution function, one finds that the result Eq.(2) is a factor 1.27 larger than what may be obtained by substituting Eq.(7) in Eq.(6). It is in a sense this factor of 1.27 which is a real test of N.E.S.M. and, hence, one must measure the result Eq.(2) to at least 20% accuracy and know independently all the parameters entering that equation to claim that it is a check of N.E.S.M.

b) Assuming Statistical Mechanics to be correct and now considering Eq.(2) from the point of view of measuring material properties, we see that we have here a most interesting result. Indeed, Eq.(2) shows that measuring nonequilibrium current fluctuations gives a way of obtaining the inelastic relaxation time $\tau_o$ in a regime where it is not otherwise easily accessible because the usual transport measurements are sensitive to properties, when $\tau_e \ll \tau_o$, which are determined by elastic scattering. This inelastic time is relevant to at least the following three fields of Condensed Matter Physics:

i) In nonequilibrium superconductivity, the normal-state inelastic scattering time $\tau_o$ appears as a parameter in many of the results and hence it can be indirectly measured in the superconducting state[3] . Our calculation provides a way of obtaining this inelastic scattering time directly in the normal state.

ii) In the field of low-temperature refrigeration, it has been known for some time that in sufficiently small metallic systems, the thermal impedance between electrons and phonons becomes an important mechanism limiting the efficiency of heat transfer[4] . If we define the thermal resistance as $R_T = \delta T/\dot{Q}$, where $\dot{Q} = \sigma E^2 V$ ($\sigma$ is the electrical conductivity) one finds that nonequilibrium current fluctuations provide a way of measuring $\delta T$, and hence the thermal resistance between electrons and phonons. Given the widespread use of metal sinters in the refrigeration process, this electron-phonon thermal resistance is very relevant for low temperature experimentation. The thermal resistance can also be measured by different techniques[4].

iii) Finally, it is interesting to speculate that if results analogous to those we have derived hold in the localized regime of two-dimensional metallic films, high-frequency nonequilibrium current fluctuations may provide a way to independently determine the inelastic scattering rate which arises as a parameter determining the logarithmic temperature dependence of the resistance in the theory of localization in two dimensions[5]. In view of the existence of a competing theory[6] for this logarithmic temperature dependence of resistance, it may be important to have this additional way to estimate the inelastic relaxation rate to remove one adjustable parameter in the theory.

To conclude, we should like to point out that the calculation we described has recently been extended non-perturbatively to the high-field limit ($eE\ell^*/k_B T \gg 1$) by M. Arai[7]. He finds that $\delta T$ in Eq.(6) is proportional to $E^{2/5}$, a highly non-ohmic effect which would be most interesting to measure. Note that in the limit $T \to 0$ one is always in the high field limit and hence, as Arai points out, his result gives a fundamental noise limit to many electrical measurements.

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