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MULTIFRACTALS AND NOISE IN METAL–INSULATOR MIXTURES

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Invited paper

A short review of the critical properties of $1/f$ noise in percolating mixtures is presented, including continuum corrections and comparisons with experiments. The relation to multifractals and analogies with ordinary critical phenomena are recalled. New results on the effect of noisy insulators are also presented.

1. Introduction

The problem of electrical noise in metal–insulator mixtures seems at first sight of limited interest. It however turns out to be a problem with far reaching experimental and theoretical consequences. In this short review paper, which expands on ref. [1], we recall that even exponents to compare with experiment are difficult to obtain because of continuum corrections and other complications of actual materials. We also present new results for one such complication, namely that which can come from noisy insulators. In the last part, we discuss how the problem of noise has lead to multifractals in the context of percolation and we briefly summarize our work in this area, which has been mainly concerned with connections between standard critical phenomena and multifractals.

2. Noise in metal–insulator mixtures

We are interested in the $1/f$ noise of metal–insulator mixtures. This should be distinguished from ordinary Nyquist–Johnson (thermal) noise which, by the fluctuation–dissipation theorem, is simply a measure of the resistance. The problem of the origin of $1/f$ noise is in itself rather subtle and only now is starting to be understood [2]. We will assume that $1/f$ noise simply comes from resistance fluctuations [3]. There are then two cases of interest: a) If $1/f$ fluctuations are uncorrelated for distances larger than the lattice spacing, we can assume that every resistor is fluctuating independently. The overall spectrum of fluctuations is the same as that of the individual resistors and the only question left is how does the *magnitude* of $1/f$ noise diverge close to the percolation threshold. b) If $1/f$ noise is correlated over distances larger than the lattice spacing, an additional question arises: how is the spectrum of fluctuations modified by the fractal structure of the lattice? The answer to the latter question clearly depends on the noise mechanism. We will not address this question here. The case of diffusion noise is treated in refs. [4, 1, 5] and references therein.

2.1. Theory for lattice and continuum cases

Let us assume then that $1/f$ noise is correlated over distances much shorter than the lattice spacing. Experimentally, this should be the case in many situations of interest since the correlation length of $1/f$ noise is usually extremely short [6]. Consider then a percolating lattice model where each resistance fluctuates in time around an average value r, independently from the other resistances. In other words, if α and β are two of the resistors of the system,

$$r_\alpha = r + \delta r_\alpha \,, \tag{1a}$$

$$\langle \delta r_\alpha \rangle = 0 \,, \tag{1b}$$

$$\text{F.T.}\langle \delta r_\alpha \, \delta r_\beta \rangle = \rho(\omega)\delta_{\alpha,\beta} \,, \tag{1c}$$

where F.T. stands for Fourier transform, and the brackets stand for ensemble average (or in practice time average). The last δ symbol is Kronecker's, i.e. is equal to 1 when α and β are identical and 0 otherwise. Note that we assume that each of the elementary fluctuating resistance has an indentical spectrum $\rho(\omega)$, but the exact frequency ω dependence of this spectrum is not important for the following.

To compute the magnitude of the resistance noise, recall first that the total resistance may be calculated from

$$RI^2 = \sum_\alpha r_\alpha i_\alpha^2 = r \sum_\alpha i_\alpha^2 \,, \tag{2}$$

where I is the total input current and i_α is the current in branch α of the network. To compute the overall resistance fluctuations of the circuit, we resort to theorems used before in the context of $1/f$ noise by the Dutch school, among others. Cohn's theorem, which is a direct consequence of Tellegen's theorem, shows that to linear order in the fluctuations [7]

$$\delta R I^2 = \sum_\alpha \delta r_\alpha i_\alpha^2 \, . \tag{3}$$

Normalizing the input current to unity, and using the model of eqs. (1a–c), one obtains

$$\langle \delta R(t)\, \delta R(0) \rangle = \langle \delta r(t)\, \delta r(0) \rangle \sum_\alpha i_\alpha^4 \, . \tag{4}$$

We then ask the following question: How does the *magnitude* of $1/f$ noise diverge near the percolation threshold, and is this divergence governed by a new exponent or by an exponent related to previously defined ones [7, 8]? Let us recall that the geometrical properties of percolating clusters [9] are usually determined by two exponents, β and ν: ν is the correlation length exponent and β/ν is related to the so-called fractal dimension D of the infinite cluster by $D = d - \beta/\nu$. The dc electrical resistance on the other hand diverges as $(p - p_c)^{-t}$ near the percolation threshold.

What was found [7, 8] is that the noise diverges as $(p - p_c)^{-\kappa}$ for $p \geq p_c$, with κ a new exponent different from any of the previously defined exponents for percolation [7, 8]. From numerical simulations, one obtains [7, 10–13], $\kappa \approx 1.12 \pm 0.03$, 1.56 ± 0.13, 1.57 ± 0.14, 1.8 ± 0.1, and 2 respectively for $d = 2, 3, 4, 5$ and $d \geq 6$. Above two dimensions, these exponents were obtained from bounds [10, 14]. More recent numerical simulations gave [15] $\kappa = 1.47 \pm 0.04$ on $d = 3$ lattices and [16] $\kappa = 1.57 \pm 0.08$ and 2.06 ± 0.08 in $d = 3$ and $d = 6$ respectively. Series expansion [17] and ε expansion [18] results have also been obtained for these numbers. (See refs. [17, 19] for a summary.) Less precise Migdal–Kadanoff [20–22] and effective medium [20, 8, 23, 24] results are also available. Note that effective medium results seem accurate far from threshold, even though the definition of effective medium theory seems more ambiguous in the present case [23] than in the case of resistance.

Before comparing with experiment, we should note an additional complication: In the mapping from the real system to a lattice, the best model leads to a power law distribution for the conductance of the bonds [25] and for the magnitude of their $1/f$ noise [10, 26]. More specifically, one can distinguish at least two simple models [25]: the "random-void," with insulating holes in a conducting matrix, and the "inverted random-void", with conducting holes and insulating matrix. The predictions for the exponents when these "continuum corrections" are taken into account are [10]: $w \equiv \kappa/t \approx 3.2$ and 2.1 in the random-void model, in two and three dimensions respectively, while in the inverted random-void model, $w = \kappa/t \approx 0.87$, and 2.4 in two and three dimensions. These predictions are for the "nodes-links-blobs" model of percolation clusters and hence only approximate [10]. They also differ from some of the predictions of ref. [27]. Recent numerical work [28] in three and six dimensions which also takes more carefully into account the mapping of the continuum system to the random resistor network, confirms the results of ref. [10]. Note however that numerical simulations for continuum corrections [29] to the *resistance* exponent are obscured by very small corrections to scaling exponents which amplify finite-size effects [30] so that theory and numerical simulations seem to disagree on this resistance problem [19], by factors which can be of the order of 20%. It is believed however, [30] that for sufficiently large systems there would be agreement. How these finite-size corrections to the resistance influence simulations for continuum corrections to the noise is not completely clear yet.

2.2. *Experiments*

Experimentally, the above general ideas have been tested on fractal networks of carbon resistors by Giraud et al. [31]. The thermal response of films is also related to the exponent κ and is being studied by Dubson et al. [32].

In composites, one measures the magnitude of the noise as a function of the resistance instead of as a function of $p - p_c$, which is not easily accessible experimentally. In other words, one obtains directly the exponent w. Given the uncertainties of the nodes-links-blobs model for continuum corrections as well as the experimental uncertainties, the following experiments may be interpreted as in agreement with theory [10, 23, 28]: Garfunkel and Weissman [26] find, for sand-blasted Al, Cr and In films $w \approx 3.4$ to 6. Given the small range of resistances measured, and the fact that inhomogeneities tend to increase the exponent, this is probably consistent with the random-void model in $d = 2$, $w \approx 3.2$. Rudman et al. [33] worked with mixtures of At–Pt alloy in insulating tetrafluoroethylene. Far from p_c they find $w \approx 1$, a result which can be explained by effective medium theory [8]. In the critical regime, close to the transition, they obtain $w \approx 3$. In this case, the appropriate model is the inverted random-void in $d = 3$, which gives $w \approx 2.4$. Finally, Octavio et al. [34] found $w = 0.9^{+0.2}_{-0.1}$ for evaporated Ag films, in agreement with the $d = 2$, inverted random-void model, $w = 0.87 \pm 0.03$.

Finally, the following experiments seem to disagree with theory: Koch et al. [35] find $w = 2.1 \pm 0.1$ for $d = 2$ ion milled gold films at room temperature, basically the same value $w = 2.1 \pm 0.2$ as Octavio et al. [34] for ion-milled Ag films at 77 K. It was however pointed out [36] that ion-milling may both remove material and change the noise mechanism in the conducting elements, an effect which should also be accounted for before comparing with experiment. Results similar to those of refs. [34, 35] were also obtained in 1969 by Williams and Burdett [37] for evaporated gold films. These authors interpreted their results as coming from noisy tunneling or hopping conduction. Chen and Chou [38] in $d = 3$ mixtures of carbon and wax find $w = 1.7 \pm 0.2$, a result which they also try to interpret as tunneling noise. The results of Mantese and Webb [39] for $Pt-Al_2O_3$ and $Mo-Al_2O_3$ composite $d = 2$ films were also shown [24] to arise from noisy tunneling conduction in the insulator, and hence are beyond the scope of the present theory. The problem of additional noise mechanisms coming from the insulator has been addressed before [37, 24, 40] but it deserves more attention. The work of Kusy et al. [41] for example shows the subtleties of modeling this type of system.

2.3. *Effect of tunneling*

In this section, we partially address the question of noisy insulators. This problem has been considered in the effective-medium limit in ref. [24]. This is expected to be valid far from the percolation threshold. Here we instead ask the question of how the noise exponent may be modified near the percolation threshold. Consider the limit where the leakage current in the insulators is small enough to be neglected in the resistance while the noise is totally dominated by the noise due to these not perfectly insulating bonds. That this is possible may be seen as follows. If we label the conducting bonds by α and the bonds with bad insulators by β, then

$$\langle \delta R^2 \rangle = \langle \delta r_c^2 \rangle \sum_\alpha i_\alpha^4 + \langle \delta r_i^2 \rangle \sum_\beta i_\beta^4 \,. \tag{5}$$

Even though the currents in the second term are much smaller than those of the first term, the magnitude of the resistance fluctuations of the insulators, $\langle \delta r_i^2 \rangle$ may be so much larger than those of the conductors $\langle \delta r_c^2 \rangle$ that they can dominate the noise. Estimating $r_i i_\beta \approx r_c i_\alpha$ we see that the second term of (5) may in principle dominate when $(\langle \delta r_i^2 \rangle / \langle \delta r_c^2 \rangle)(r_c/r_i)^4 \gg 1$, which is possible even if $(r_c/r_i) \ll 1$. The scaling with size of the two sums must however also be verified before a definite conclusion can be drawn. This is done below.

Two limiting cases come to mind. That where the insulators are ohmic and all contribute to the noise, and that where conduction in the insulators is due to tunneling conduction. The first case has been discussed briefly in ref. [42] and we come back to it briefly below. For the latter case, we consider the following model. When two points of the incipient infinite cluster (including dead ends) are separated by a single insulator, we assume that this insulator contributes to the noise. We neglect the cases where two or more insulators are needed to bridge the incipient infinite cluster to account for the exponential dependence of tunneling resistance on distance. In the limit where the noisy insulators dominate the noise but do not influence the conduction, we can obtain the relative resistance noise from

$$S_{\mathrm{R}} = \frac{\langle \delta R^2 \rangle}{R^2} \approx \frac{\langle \delta r_{\mathrm{i}}^2 \rangle}{r_{\mathrm{i}}^2} \, \frac{1}{(r_{\mathrm{c}}^2 r_{\mathrm{i}}^2)} \, \frac{\sum_{\beta} \Delta V_{\beta}^4}{\left(\sum_{\alpha} i_{\alpha}^2 \right)^2} \, . \tag{6}$$

The terms in front of the sums are constants which do not influence the scaling with size. For the calculation of the sums over currents in the conducting bonds α and the sums over potential differences ΔV_β in the insulators β, the resistance of the insulator is taken as infinite. The potential differences all along the incipient infinite cluster (including dead ends) are directly obtained from the solution of Kirchoff's laws and are all that is needed to obtain the ΔV_β. Working at the percolation threshold, on a system of linear size L and stetting $S_{\mathrm{R}} \sim L^{-b}$, we find in two-dimensions, on the square lattice $b = -2.7 \pm 0.1$, and on the triangular lattice $b = -2.8 \pm 0.2$. (The parameters of the simulation are as in the figure caption.) Using [8] $\kappa = \nu(d - b)$ we find $\kappa = 6.3 \pm 0.3$, which is very different from the usual case. Clearly however, our modeling of the tunneling conductances is very naïve. In a real material, one should expect large continuum corrections when there is a tunneling contribution [43]. Note that the fractal dimension 1.8 ± 0.2, for this set of bonds, which was not determined with high precision, is consistent with that of the incipient infinite cluster.

Going back to the case where all the insulators are included in the calculation, their fractal dimension is clearly equal to the Euclidean dimension but the noise exponent $b = -2.9 \pm 0.2$ is consistent with that found above, which indicates that the noise here is dominated by the type of bonds considered above, which in a sense play the role of "singly connected" bonds and hence dominate the noise for reasons similar to the usual case [10, 14].

3. Multifractals

The $1/f$ noise then may be considered as the second cumulant of the resistance fluctuations. More generally, if one is interested in higher order *cumulants* of the resistance fluctuations it was found [7, 8] that each of these cumulants diverges with a new exponent. Going back to the original problem of noiseless insulators and noisy conductors, by analogy with eq. (4), the cumulants are obtained from

$$\langle \delta R^n \rangle_c \sim \langle \delta r^n \rangle_c \sum_\alpha i_\alpha^{2n} , \tag{7}$$

where the subscript c indicates cumulant average. The actual numerical results are obtained from simulations on samples of size $L \ll \xi$ from which one extracts a power law dependence on L:

$$\sum_\alpha i_\alpha^{2n} \approx L^{-x_n} . \tag{8}$$

Eq. (8) leads to an infinite set of *measurable* [7, 8] exponents to which belong the fractal dimension of the conducting bonds ($n = 0$), the resistance exponent ($n = 1$) and the noise exponent ($n = 2$). The case $n = \infty$ gives the scaling of the singly connected bonds and hence by Coniglio's theorem [44] $-x_\infty$ is equal to the inverse of the correlation length exponent ν. The exponents describing the dependence on $p - p_c$ are obtained from the x_n through finite-size scaling. Note that even though all the exponents are *in principle* accessible experimentally, the assumption of resistors fluctuating independently means in practice that they must be large enough to contain a few of the independent microscopic sources of noise. This in turn means that the microscopic noise amplitudes entering the right hand-side of eq. (7) are extremely small (the noise is Gaussian to a very good approximation).

The existence of an infinite set of non-trivially related exponents was discovered in many different fields involving fractals [45]: From turbulence [46] to diffusion limited aggregation [47] and dynamical systems [48]. This general phenomenon has now come to be known under the name "multifractals" [49]. The present case is, to our knowledge, the only one where the infinite set of exponents has direct experimental relevance for *macroscopic* measurements. The exponents $-x_n$ were computed numerically by a number of groups [8, 11, 12] and analytically by the techniques mentioned in the case of the noise exponent κ. An approximate analytic formula for the $-x_n$ may also be obtained [17, 19]. An infinite set of exponents may also be calculated from the simulation for the model defined in section 2.3. Keeping only the insulators bridging the incipient infinite cluster and normalizing the voltage drops to the total resistance for convenience, we define the exponents $-x'_n$ by $\Sigma_\beta \, \Delta V_\beta^{2n} / (\Sigma_\alpha i_\alpha^2)^{2n} \sim L^{-x'_n}$. The results are illustrated in fig. 1. For large values of n, the differences between square and triangular lattice seem significant when com-

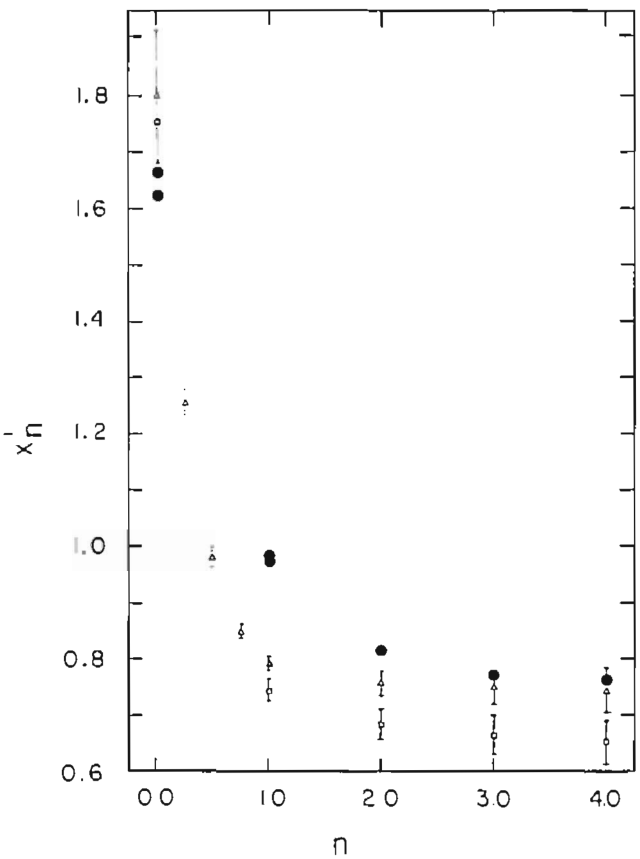

Fig. 1. Exponents $-x_n'$ as defined by $\Sigma_\mu \Delta V_\mu^{2n}/(\Sigma_\alpha i_\alpha^2)^{2n} \sim L^{-x_n'}$. Triangles with error bars indicate the results for 2000 conducting samples of two-dimensional triangular lattices of sizes $L = 6, 8, 10, 12, 14$. Squares with error bars are the corresponding results for 2000 conducting samples of two-dimensional square lattices of sizes $L = 7, 11, 15, 21, 31$. The exponents $-x_n$ as defined by eq. (8) are also shown as dots without error bars for comparison. The upper dots are the results for the triangular lattice, and the lower ones, the results for the square lattice. (For large values of n, the two results become indistinguishable on this scale.)

pared with the corresponding deviations for the $-x_n$. However, large values of n are more sensitive to statistical fluctuations, so we do think that this may come from finite size effects instead of breakdown from universality and/or scaling. The corresponding results when all insulators are included have also been obtained for the square lattice [50]. Note that we have a multifractal in the latter case also, even though the fractal dimension of the bonds is equal to the Euclidean dimension.

While non-integer values of n have been plotted in fig. 1, note that if the left hand-side of eq. (8) is known exactly for all values of n, this suffices to determine completely the current distribution and hence all the quantities of interest [51]. In other words, we do not need in principle to consider n non-integer. This is closer to the more usual situation in critical phenomena where one has only a discrete (albeit infinite) set of operators. In the multifractal literature, one also considers negative moments. These are not accessible from noise measurements but have analogs in the problem of hydrodynamic dispersion for example. These negative moments do not all have scaling behavior due to Lifshitz-type singularities [17]. This means that in practice negative moments of the current distribution are too difficult to obtain from positive ones [52] so that it is better to compute them directly if they are needed [51, 52].

While multifractals appear quite different from the usual situation in critical phenomena where one focuses on only a few relevant exponents, more detailed investigations seem to show many analogies. In particular, higher order susceptibilities of a given observable (cumulant) also obey gap scaling [51]. There is a generating function for these susceptibilities which is analogous to the free energy in critical phenomena [51]. Close to p_c, this function is universal. It appears further that each moment of the current distribution is associated with operators which break a continuous symmetry in replica-space [53, 54, 51]. The observability of the exponents seems then associated more with symmetry than with relevance. Indeed, in this and other "standard" multifractal cases, the exponents appear in a "second renormalization group" which contains an additional arbitrary scale factor [53, 54] which changes the exponents but not the observable quantities or the original fixed point properties. Observable exponents in this second renormalization group are "dominant" [54]. This arbitrary scale factor seems to be a true difference with standard critical phenomena [53, 55] except perhaps for the Gaussian model which exhibits a similar arbitrariness [56]. On the other hand the situation where an infinity of symmetry-related exponents is observable experimentally has a direct analog in critical phenomena. More specifically [53–55], operators which transform like higher dimensional representations of the group O(2) are observable in experimental realisations of the XY model [57–59]. These operators are associated with crossover exponents and those with less than cubic symmetry are irrelevant but nevertheless observable. We are also working on analogies between critical phenomena and multifractals in dynamical systems [54, 55, 60].

4. Conclusion

While the divergence of $1/f$ noise near the percolation threshold in metal-insulator mixtures may be understood from the theory presented in the first part of this talk, some experiments are still not fully explained by theory. Hopefully, going further beyond the nodes-links-blobs model for continuum corrections and more careful modeling of the microscopic details of the material will help explaining the results. On the theoretical side, the study of this problem has lead naturally to a physically simple example of multifractals. In this context higher order cumulants of the noise are interesting but probably hard to obtain experimentally. A problem which also deserves further attention, especially in more than two dimensions, is that of the crossover behavior of the multifractal moments [51, 61] when the conductivity of the insulators is not perfectly zero (the analog of an applied magnetic field). This problem is deeply related to the existence of a single correlation length.

Related topics which have not been discussed here include the dual problem of superconductor–normal metal mixtures [14, 21], and percolating nonlinear circuits [62] as well as their noise and corresponding multifractal properties [63, 62]. Finally, no experiment has been done yet, to our knowledge, on diffusion noise in percolating networks [4].

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