CRM Series in Mathematical Physics

Springer
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Theoretical Methods for Strongly Correlated Electrons

With 99 Illustrations

Springer

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PACS: 71.27

Library of Congress Cataloging-in-Publication Data
Theoretical methods for strongly correlated electrons / editors, David Sénéchal,
André-Marie Tremblay, Claude Bourbonnais.
p. cm. — (The CRM series in mathematical physics)
Includes bibliographical references and index.
ISBN 0-387-00895-0 (alk. paper)
1. Electron configuration. 2. Condensed matter. 3. Mathematical physics. I. Sénéchal,
David. II. Tremblay, André-Marie. III. Bourbonnais, Claude. IV. Series.
QC176.8.E4T448 2003
530.4′11—dc21 2003045457

ISBN 0-387-00895-0 Printed on acid-free paper.

Printed in the United States of America.

9 8 7 6 5 4 3 2 1 SPIN 10922040

Typesetting: Pages created by the author using a Springer TEX macro package.

www.springer-ny.com

Springer-Verlag New York Berlin Heidelberg
A member of BertelsmannSpringer Science+Business Media GmbH

Series Preface

The Centre de recherches mathématiques (CRM) was created in 1968 by the Université de Montréal to promote research in the mathematical sciences. It is now a national institute that hosts several groups and holds special theme years, summer schools, workshops, and a postdoctoral program. The focus of its scientific activities ranges from pure to applied mathematics and includes statistics, theoretical computer science, mathematical methods in biology and life sciences, and mathematical and theoretical physics. The CRM also promotes collaboration between mathematicians and industry. It is subsidized by the Natural Sciences and Engineering Research Council of Canada, the Fonds FCAR of the Province de Québec, and the Canadian Institute for Advanced Research and has private endowments. Current activities, fellowships, and annual reports can be found on the CRM Web page at www.CRM.UMontreal.CA.

The CRM Series in Mathematical Physics includes monographs, lecture notes, and proceedings based on research pursued and events held at the Centre de recherches mathématiques.

Yvan Saint-Aubin
Montreal, Quebec, Canada

Preface

The workshop *Méthodes théoriques pour les fermions fortement corrélés/ Theoretical methods for strongly correlated electrons* was held from Wednesday May 26 to Sunday May 30, 1999, at the Centre de recherches mathématiques, Université de Montréal, Québec, Canada.

Despite numerous conferences and workshops on strongly correlated electrons, one rarely finds workshops dedicated only to purely theoretical aspects of the problem. According to the speakers, this workshop was a first. It brought together experts on various approaches, focused around a unique problem, namely, the solution of models of the Hubbard type (problems where both localized and delocalized aspects are present) in low dimension. Relatively long pedagogical introductions (one and a half hours) during the morning, allowed everyone to appreciate the strengths and weaknesses of each approach as well as to understand open problems for each of the important methods. Short presentations in the afternoon provided a broad overview of current problems. One and a half hours a day were reserved for posters. Since these were exhibited during the whole workshop in a common area they also provided stimuli for informal discussions during coffee breaks and early in the morning. The workshop was open to theorists as well as to advanced students and postdocs.

These proceedings do not do full justice to this workshop, but they do strive to give the same mix of pedagogical review and outlook on current problems that was at the heart of this meeting. By avoiding exhaustive lists of short summaries to concentrate on only a few *in-depth* articles, we do hope that these proceedings will have lasting value for the student and the researcher alike. A list of talks and of participants follows this preface. They are provided both as a memento and as a reference for those who wish to contact an outstanding researcher on a topic not covered by these proceedings. The fact that a given talk or topic does *not* appear in these proceedings is uncorrelated with the quality of the presentation. Look for correlations only in electronic properties!

The contents of these proceedings can, roughly speaking, be divided into three parts that cover an impressive range of methods.

Part I, *Numerical Methods*, deals with two of the most widely used numerical methods in strongly correlated electrons. Both of these methods have found applications in areas spanning condensed-matter, high-energy, and sometimes nuclear or statistical physics. The first one, Density Matrix

Renormalization Group is, by now, one of the leading numerical methods for one-dimensional and quasi-one-dimensional problems. The paper by Karen Halberg covers the different areas where the method is applied and also gives the reader a general overview on the subject. Much activity has focused on extensions of the method to dynamical quantities, finite temperatures, disorder, phonons, etc. They are all discussed. The extensive list of references will be extremely valuable to the reader. The second method, Quantum Monte Carlo, is introduced in a pedagogical chapter by Shiwei Zhang. Most of the aspects of this methodology are covered, including primarily auxiliary-field (or determinantal) methods, but also configuration-space methods and Constrained Path Monte Carlo. There is a focus on the latter method that allows one to free oneself from the infamous *sign problem*. In this chapter, Shiwei has developed a formalism that unifies the different methods and allows for a systematic understanding of their strengths, weaknesses, and common features.

Problems in one dimension, or weakly coupled chains, have lended themselves to the development of a variety of analytical methods. These analytical methods have allowed detailed understanding of the fascinating physics that arises in the presence of interactions, such as spin-charge separation. And these methods continue to produce new results, some of which find extensions to higher dimension.

Part II, *Lagrangian, Functional Integral, Renormalization Group, Conformal and Bosonization Methods*, contains three reviews that cover the most widely used methods. The chapter by C. Bourbonnais, B. Guay, and R. Wortis reviews the renormalization group method and scaling concepts for interacting fermions. Peculiarities of the two-loop calculation are clarified for the first time. Dimensional crossover, multiple fixed points, and Kohn–Luttinger mechanisms in different channels are subjects of current interest that are also discussed. The chapter by D. Sénéchal contains a pedagogical review of bosonization methods and conformal invariance. These methods are applied, as an example, to the solution of the Tomonaga–Luttinger model, which embodies the physics of spin-charge separation. Non-Abelian bosonization as well as a variety of applications, such as edge states in the quantum Hall effect, are also discussed. The last chapter of Part II was written by T. Giamarchi and E. Orignac. Using one-dimensional spinless fermions as a pedagogical example, the authors explain various methods, in particular the replica method and the Gaussian Variational Method, to treat the elastic disordered theory that describes a large class of disordered fermionic systems. Extensions to higher dimension of problems such as Wigner crystal, Charge Density Waves, and Bose glass are also presented. These fall into the general class of disordered quantum solids. The authors also investigate in detail the interesting example of a disordered Mott insulator and argue that intermediate disorder can lead to a novel phase, the Mott glass, intermediate between a Mott and an Anderson insulator.

Part III, *Functional Derivatives, Mean-Field, Self-Consistent Methods, Slave-Bosons, and Extensions*, begins with a review by E. Bickers of Baym-Kadanoff, or Φ-derivable, approximations. Functional integrals results are also used to establish the connection between conventional mean-field theory and higher-order Baym-Kadanoff approximations. The Φ derivability criterion for thermodynamic consistency is discussed and contrasted with parquet, or crossing-symmetric, approximations. Instabilities of the electronic normal state and numerical techniques for the solution of self-consistent field approximations are reviewed, with particular emphasis on renormalization group methods for frequency and momentum space. This leads to the next chapter by J. Kroha and P. Wölfle. They review a new systematic many-body method capable of describing both Fermi liquid and non-Fermi liquid behavior of quantum impurity models at low temperatures on the same footing. The method covers the crossover to the high temperature local moment regime as well. In more technical terms, this chapter deals with the method of auxiliary particles introduced to effect the projection in Hilbert space while keeping most of the desirable features of renormalized perturbation theory. After a pedagogical introduction, approximations are derived from a generating Luttinger-Ward functional, Φ, in terms of renormalized perturbation theory in the hybridization V. The conserving T-matrix approximation (CTMA), discussed in the previous chapter, is used here again, but for the auxiliary particles. The results are compared with the non-crossing approximation (NCA) and with data obtained by the numerical renormalization group and the Bethe ansatz. Generalizations are discussed as well. The last chapter by S. Allen, A.-M. S. Tremblay, and Y. Vilk presents a formal derivation of a non-diagrammatic approach that was developed a few years ago. The derivation makes the analogies and differences with Φ-derivable approximations clearer. The two-particle self-consistent approach presented in this chapter has produced results that are more accurate, both quantitatively and qualitatively, than other methods when compared with Quantum Monte Carlo calculations, in particular for the so-called pseudogap problem. But its extensions to different problems are not as obvious as with Φ-derivable approximations. This is one of the many areas presented in this book where the reader will find challenges for the future.

In conclusion, we thank the Centre de recherches mathématiques and its skillful and courteous staff that made this event run smoothly and allowed the organizers to participate in the event instead of being caught up in logistics. We are indebted also to Luc Vinet and Yvan Saint-Aubin, who encouraged us to organize this workshop and applied for grants that made it possible. Yvan Saint-Aubin also gave valuable advice on the philosophy behind these proceedings. Finally, we acknowledge the financial support of the Natural Sciences and Engineering Research Council (NSERC) of Canada, the U.S. National Science Foundation (NSF), and the Fonds pour

la formation de Chercheurs et l'Aide à la Recherche (FCAR) of the Québec government.

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May 2003

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