CONFORMAL FIELD THEORY

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Errata on second printing

These errata (and eventual additional ones) are listed on the following Web page:
`http://www.physique.usherb.ca/~dsenech/cft.htm`

CHAPTER 2

p. 36 : Eq. (2.111)
The sign before the last term should be $+$, not $-$.

CHAPTER 4

p. 102 : Eq. (4.41)
The third term within braces on the first line should read $-\eta^{\lambda\nu}\sigma_+^{\mu\rho}$ (μ and ν should be interchanged). In addition, the term on the last line of the equation should change sign.
The full equation is then

$$\begin{aligned} \sigma_+^{\mu\nu} &= \frac{1}{2}(\sigma^{\mu\nu} + \sigma^{\nu\mu}) \\ X^{\lambda\rho\mu\nu} &= \frac{2}{d-2}\Big\{\eta^{\lambda\rho}\sigma_+^{\mu\nu} - \eta^{\lambda\mu}\sigma_+^{\rho\nu} - \eta^{\lambda\nu}\sigma_+^{\mu\rho} + \eta^{\mu\nu}\sigma_+^{\lambda\rho} \\ &\qquad - \frac{1}{d-1}(\eta^{\lambda\rho}\eta^{\mu\nu} - \eta^{\lambda\mu}\eta^{\rho\nu})\sigma^{\alpha}_{+\alpha}\Big\} \end{aligned} \tag{4.41}$$

CHAPTER 5

p. 118 : line 1
$\eta = -z_4$ should read $\eta = z_4$.

p. 123 : 2 lines before Eq. (5.52)
"$h = \bar{h} = 2$" should read "$h = 2$ and $\bar{h} = 0$".

p. 123 : line before Eq. (5.53)
"special conformal transformation" should read "global conformal transformation"

p. 147 : Exercise 5.5
The normalization $2\pi g = 1$ is implicit here.

CHAPTER 6

p. 163 : Eq. (6.69)
Normal ordering is missing in the first line.

p. 169 : Eq. (6.101)
$2\pi L$ should be replaced by L.

p. 169 : Eq. (6.105)
The equation is less confusing if it reads as follows:

$$\begin{aligned} \psi_{\text{cyl.}}(w) &= \left(\frac{dz}{dw}\right)^{1/2}\psi_{\text{pl.}}(z) \\ &= \sqrt{\frac{2\pi z}{L}}\psi_{\text{pl.}}(z) \end{aligned} \tag{6.105}$$

p. 177 : Eq. (6.151)
Summation over i must be in *front* of the integral as the contour depends on i.

p. 179 : Third line of Eq. (6.158)
In the integral, a factor $\frac{1}{2\pi i}$ is missing.

p. 181 : Eq. (6.166)
The factor $w^{2\bar{h}_r}$ should read $\bar{w}^{2\bar{h}_r}$.

p. 182 : line following Eq. (6.173)
We now apply the operator $L_n \ldots$ (with $n > 0$).

p. 182 : Eq. (6.174)
The second line has a z^n missing. It should read

$$\left(z^{n+1}\partial_z + (n+1)hz^n\right)\phi_1(z,\bar{z})|h,\bar{h}\rangle$$

p. 182 : Eq. (6.179)
The last term of the first line should be $-2L_{-1}$, not $-2L_{-2}$:

$$L_1 L_{-1}^2 = L_{-1}^2 L_1 + 4L_{-1}L_0 - 2L_{-1}$$

p. 185 : Eq. (6.191)
This equation should read

$$\begin{aligned}
&\mathscr{F}_2 = \frac{A}{B} + \frac{C}{B} \quad \text{where} \\
&A = (h_p + h_2 - h_1)(h_p + h_2 - h_1 + 1) \\
&\qquad \times \left[(h_p + h_3 - h_4)(h_p + h_3 - h_4 + 1)(4h_p + c/2) - 6h_p(h_p + 2h_3 - h_4)\right] \\
&B = 4h_p(2h_p + 1)(4h_p + c/2) - 36h_p^2 \\
&C = (h_p + 2h_2 - h_1)\left[4h_p(2h_p + 1)(h_p + 2h_3 - h_4) - 6h_p(h_p + h_3 - h_4)(h_p + h_3 - h_4 + 1)\right]
\end{aligned} \tag{6.191}$$

p. 191 : Eq. (6.222)
On the l.h.s. the argument w is missing.

p. 191 : Eq. (6.226)
Third line, third term: one bracket] too many.

p. 194 : Ex. 6.7 a)
In the second equation, one normal ordering symbol : is missing on the l.h.s.

p. 194 : Ex 6.7, part b)
The argument of A should be z and that of B should be w:

$$\begin{aligned}
e^{\overbrace{A(z)}} e^{\overbrace{B}(w)} &= \sum_{m,n,k} \frac{k!}{m!n!} \binom{m}{k} \binom{n}{k} [\overbrace{A(z)B}(w)]^k : A^{m-k}(z) B^{n-k}(w) : \\
&= \exp\left\{\overbrace{A(z)B(w)}\right\} : e^{A(z)} e^{B(w)} :
\end{aligned}$$

p. 194 : Ex 6.8, last line)
Reference to Eq. (6.213) should be to Eq. (6.144)

p. 196 : Eq. (6.246)
On the second line, the power $1/(z-w)^5$ should be $1/(z-w)^4$.

CHAPTER 7

p. 206 : line before Eq. (7.22)
$b = Ua$ should read $b = U^\dagger a$.

p. 213 : Eq. (7.49)
The powers of z have the wrong sign. This should read

$$\langle \phi(z)\phi_1(z_1)\phi_2(z_2)\rangle = \frac{g(h,h_1,h_2)}{(z-z_1)^{-h_2+h+h_1}(z_1-z_2)^{-h+h_1+h_2}(z-z_2)^{-h_1+h+h_2}}$$

p. 217 : Eq. (7.70)
In the second sum, the modulo condition is $l+s+n=1 \mod 2$, not $k+s+n=1 \mod 2$.

p. 219 : Eq. (7.78)
h should read H (capitals). The same remark applies to the paragraph following Eq. (7.81).

p. 235 : Ex. 7.2
The matrix given was calculated by using $L_{-2}L_{-1}|0\rangle$ instead of $L_{-1}L_{-2}|0\rangle$ as one of the basis states. The correct matrix reads

$$M^{(3)} = \begin{pmatrix} 24h(1+h)(1+2h) & 36h(1+h) & 24h \\ 36h(1+h) & 2c+34h+ch+8h^2 & 2(c+8h) \\ 24h & 2(c+8h) & 2(c+3h) \end{pmatrix}$$

p. 236 : Ex. 7.9b)
$\Phi^{(2)}$ should read $\partial\Phi^{(2)}$ in the last line of the four-line equation. Also, in the expression for $\Phi^{(3)}$, $2/(h+1)$ should read $2/(h+2)$:

$$\Phi^{(3)} = \left(L_{-3} - \frac{2}{h+2}L_{-1}L_{-2} + \frac{1}{(h+1)(h+2)}L_{-1}^3\right)\Phi$$

CHAPTER 8

p. 249 : Eq. (8.43)
The r.h.s. should be $-2h_i$, not $2h_i$.

p. 253 : Eq. (8.66)
The first equation should have a minus sign on the r.h.s., in front of the ratio.

p. 253 : Eq. (8.68)
In the third equation, $z\partial_z$ should be $-z\partial_z$.

p. 255 : Eq. (8.72)
First line: Sum over h should be in *front* of the r.h.s. as h already appears in the exponent of $(z-w)$

p. 263 : Eq. (8.129)
The upper bound of the sum should be

$$\min(m+r, 2p'-4-m-r) \quad \text{instead of} \quad \min(m+r, 2p'-2-m-r)$$

p. 286 : Ex.8.9 a) eq. (8.230)
In the result for F a factor $n!$ is missing in the denominator.

CHAPTER 9

p. 297 : line following Eq. (9.16)
The reference should be to Eq. (9.8) instead of (9.11).

p. 302 : Eq. (9.42)
One should read ϕ and not φ in the exponent.

p. 305 : Eq. (9.61)
In the second line of this equation,

$$z_{24}^{h_{r,s}} \quad \text{should read} \quad z_{24}^{2h_{r,s}}$$

p. 308 : Eq. (9.72)
In the last two lines of this equation, the prefactor $z^{1+a+b+c}$ should be z^{1+a+c}. In the fourth line, the exponents b and c should be interchanged. The corrected lines read

$$\begin{aligned} &= z^{1+a+c} \int_0^1 dw\; w^a (1-zw)^b (1-w)^c \\ &= z^{1+a+c} \frac{\Gamma(a+1)\Gamma(c+1)}{\Gamma(a+c+2)} F(-b, a+1; a+c+2; z) \end{aligned}$$

p. 310 : Fig. 9.3
In the last line of the figure, b should be replaced by c : $e^{i\pi c} - e^{-i\pi c}$

p. 310 : Eq. (9.79)
The sign of the last term should be changed. Alternately, on could change the integration domain from $[1, z]$ to $[z, 1]$.

p. 311 : Eq. (9.88)
The correlation function has holomorphic and antiholomorphic parts, and so the left-hand side should read

$$\langle \Phi_{(r_1,s_1)}(0,0)\Phi_{(2,1)}(z,\bar{z})\Phi_{(r_3,s_3)}(1,1)\Phi_{(r_4,s_4)}(\infty,\infty)\rangle$$

p. 313 : Eq. (9.97)
On the right-hand side, z should be replaced by $1-z$. Indeed, $\tilde{I}$ differs from I by the interchange $a \leftrightarrow b$.

p. 325 : Eq. (9.154) and (9.156)
s', not s, should appear in the upper indices of V in these two equations. Also, the lower-right term in Eq. (9.156) should be $F_{r+r'-2i-1,2j+1-s-s'}$ (no $2p$ in the lower index).

CHAPTER 10

p. 337 : last line
There is no i in the definition of P: It should read $P = (2\pi/L)(L_0 - \bar{L}_0)$.

p. 339 : Eq. (10.8)
The matrix T is of course

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Also, the choice one should make here for the matrix S is

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(a sign change) in order to recover $(ST)^3 = S^2$. Eq. (10.9) should then read

$$(\mathcal{S}\mathcal{T})^3 \;=\; \mathcal{S}^2 \;=\; -1$$

p. 343 : last line of eq. (10.22)
τ should be replaced by t, the exponent of e should read

$$-[tn^2\tau_2^2 + \pi^2 p^2/t - 2\pi i p n \tau_1]$$

p. 344 : eq. (10.26)
There is a $\tau_2^{-1/2}$ missing on the r.h.s.

p. 351 : Eq. (10.60)
The r.h.s. on the second line should read $Z_{m',-m}$, not $Z_{-m',m}$.

p. 361 : Eq. (10.125)
In the last line, $1/\sqrt{2N}$ should read $1/\sqrt{N}$.

p. 362 : line before Eq. (10.129)
χ_λ should read χ_μ.

p. 367 : Table 10.2
All the $2 - i$ in lower indices should be replaced by $3 - i$.

p. 372 : Eq. (10.164)
The sum should run over $(r, s) \in E'_{p,p'}$ instead of $(r, s) \in F_{p,p'}$.

p. 379 : Unnumbered equation preceding Eq. (10.189)
$z + 1$ should read $z + \omega_1$.

p. 379 : Eq. (10.190)
$z + \tau$ should read $z + \omega_2$.

p. 389 : Eq. (10.242)
$C^*_{\Pi(\lambda)}$ should read $\bar{C}_{\Pi(\lambda)}$.

p. 393 : Eq. (10.255)
q should read $q^{1/2}$ in the top two equations of the r.h.s., i.e., the same as the bottom two equations.

p. 401 : Ex. 10.17 c)
Strike out "and for the three-state Potts model", since this is also asked in part (d) of the same problem.

CHAPTER 13

p. 489 : Line 8
so instead of son.

p. 493 : Eq. (13.16)
On the l.h.s. should appear the matrix element of the inverse matrix given by K, but it looks like the inverse of a matrix element.
Error in the indices. It should read

$$f_{abc} = \sum_d f^{ab}{}_d \, [K(J^d, J^c)]^{-1}$$

p. 495 : Eq. (13.28)
It should be mentioned that the n_i are integers.

p. 497 : Eq. (13.38)
The Serre relations are subject to the condition $i \neq j$.

p. 499 : Line after eq. (13.49)
It should read: ...the product of this equation with ω_j reproduces...

p. 500 : Eq. (13.59)
$m_{ij} = 1$, not 2, if $i = j$.

p. 506 : Fig. 13.2
ω_1 et ω_2 should be interchanged in the figure.

p. 508 : four lines below Eq. (13.112)
$\lambda'_i - p_i > 0$ should read $\lambda'_i + p_i > 0$.

p. 511 : Eq. (13.120)
$so(4r + 2)$ should read $so(2r + 2)$.

p. 515 : Eq. (13.146)
It would be nicer if the index i would be different from the summation index i.

p. 517 : Eq. (13.158)
The equation should read

$$\beta_i^{(j+1)} \geq \beta_i^{(j)} \geq \beta_{i+1}^{(j+1)}$$

p. 521 : Eq. (13.188)
A factor $\epsilon(w)$ is missing in the first sum :

$$D_\lambda = \sum_{w \in W} \epsilon(w) e^{w\lambda} = \sum_{\sigma \in S_N} \epsilon(\sigma) \prod_{i=1}^{N} q^{\ell_{\sigma(i)}} = \det\, q_j^{\ell_i}$$

p. 524 : line 14 in 13.5.2
It should read $s_i(w\xi) = w\xi$ instead of $s_i(w\xi) = 0$.

p. 534 : Eq. (13.244) and following lines
$D_{\lambda^*+\rho}(X)$ is only up to a sign $\epsilon(w_0)$ the complex conjugate of $D_{\lambda+\rho}(X)$. Therefore, also the orthogonality relation is changed by a sign $\epsilon(w_0)$. In eq. (13.245) the signs cancel, so this result is unchanged.

p. 535 : Figure 13.8
The meaning of the open circles in the figure is not explained.

p. 538 : 8 lines before Eq. (13.268)
Replace $su(4) \oplus so(11)$ by $su(5) \oplus su(5)$.

CHAPTER 14

p. 558 : Paragraph after eq. (14.10)
$\{H_0^1, \ldots, H_0^r, \hat{k}\}$ seems not to be maximal Abelian subalgebra, one could add e.g. all H_n^i with $n > 0$.

p. 561 : Eq. (14.33)
The positive roots $n\delta$ with $n > 0$ are missing.

p. 562 : Eq. (14.40)
The second sum runs over j, not i.

p. 569 : Eq. (14.82)
The condition $(n, m) \neq (0, 0)$ is missing.

p. 574 : Eq. (14.107)
There should not be a factor of 2 on the first line. It should read

$$[J_m^0, J_n^0] = km\delta_{m+n,0}$$

p. 574 : Eq. (14.109)
The r.h.s. should be divided by two, as follows:

$$J_m^0 \to \frac{k}{2}\delta_{m,0} - J_m^0$$

p. 582 : on the line following Eq. (14.151)
The roots should have hat signs: $x = e^{-\hat{\alpha}_0}$ and $y = e^{-\hat{\alpha}_1}$.

p. 583 : Eq. (14.164)
Replace $z_i\lambda_i$ by $z_i\alpha_i^\vee$.

p. 584 : Eq. (14.165)
Add a summation over i, i.e., replace $z_i\lambda_i$ by $\sum_i z_i\lambda_i$.

p. 585 : 2 lines after Eq. (14.176)
there is one k too many: it should read $e^{2\pi in\lambda_1\tau}$.

p. 586 : Eq. (14.181)
One should read $q^{n(2n+1)}$ instead of $q^{2n(n+1)}$.

p. 588 : four lines before Eq. (14.200)
The condition is of course $|\alpha|^2/2 \in \mathbb{Z}_+$, not $\mathbb{Z}$.

p. 590 : Eq. (14.208)
A product sign $\prod_{n>0}$ is missing on the r.h.s.:

$$e^{\hat{\rho}} \prod_{\hat{\alpha}>0} (1 - e^{-\hat{\alpha}})^{\mathrm{mult}(\hat{\alpha})} = \prod_{n>0} (1 - e^{-n\delta})$$

p. 594 : Eq. (14.240)
σ (as in ξ_σ) should read λ, and likewise in the sentence before.

p. 594 : Eq. (14.242)
Not wrong, but for the next conclusion one needs the stronger inequalities

$$0 < (\alpha, \lambda + \rho) < k + g$$

p. 595 : Line after Eq. (14.245)
ν should be λ

p. 600 : Eq. (14.280)
The last two results should be interchanged:

$$(\hat{\omega}_0, \lambda) = 0 \qquad (a\hat{\omega}_0, \lambda) = \frac{1}{3}(2\lambda_1 + \lambda_2) \qquad (a^2\hat{\omega}_0, \lambda) = \frac{1}{3}(\lambda_1 + 2\lambda_2)$$

p. 613 : Ex. 14.11
The expansion of c_3 should read

$$c_3 \equiv \cdots = (1 + 2q + 4q^2 + 8q^3 + 14q^4 + \cdots)$$

CHAPTER 15

p. 629 : Eq. (15.76)
The second line should be expressed in terms of antiholomorphic variables.

$$\bar{J}^a(\bar{z})\; g(w, \bar{w}) \sim \frac{g(w, \bar{w})\; t^a}{\bar{z} - \bar{w}}$$

p. 647 : Eq. (15.191)
The correct expression for the structure constants is

$$f_{abc} \equiv f_{(rs)(pq)(mn)} = (\delta_{rm}\delta_{nq}\delta_{sp} - \delta_{ms}\delta_{nq}\delta_{rp}) + (\delta_{mp}\delta_{sq}\delta_{nr} - \delta_{np}\delta_{sq}\delta_{rm}) + (\delta_{pr}\delta_{ns}\delta_{mq} - \delta_{rq}\delta_{ns}\delta_{mp}) \quad (15.191)$$

p. 649 : Eq. (15.200)
Third summand on r.h.s. : One bracket] is missing.

p. 656 : Eq. (15.241)
$m^2 - m$ should read $m^2 + m$ on the r.h.s. Two lines below, m can be 0 or -1 instead of 0 or 1.

p. 657 : Table 15.2
The values of m in the heading should be shifted right by one, as follows :

p. 659 : Second line before the end
"For these" should read "For these (except for G_2)"

p. 667 : Eq. (15.306)
γ_α should read $\partial\gamma_\alpha$.

Table 1: States in the lowest grades of the $\widehat{su}(2)_1$ module $\mathsf{L}_{[0,1]}$.

L_0	m						$su(2)$
	-3	-2	-1	0	1	2	decomposition
$\frac{1}{4}$			1	1			(1)
$\frac{5}{4}$			1	1			(1)
$\frac{9}{4}$		1	2	2	1		(3)+(1)
$\frac{13}{4}$		1	3	3	1		(3)+2(1)
$\frac{17}{4}$		2	5	5	2		2(3)+3(1)
$\frac{21}{4}$		3	7	7	3		3(3)+4(1)
$\frac{25}{4}$	1	5	11	11	5	1	(5)+4(3)+6(1)

CHAPTER 16

p. 688 : 2 lines after Eq. (16.72)
"...the action of the affine Weyl group..." should read "...the action of one affine reflection..."

p. 704 : Fig. 16.3
Interchange μ_0+1 and μ_2+1.

CHAPTER 17

p. 775 : Eq. (17.261)
The second line, r.h.s., should be $Z_{\mathsf{b},\mathsf{a}^{-1}}$, not $Z_{\mathsf{b},\mathsf{a}}$.

p. 785 : unnumbered equation following Eq. (17.307)
$2-i$ appearing as a subscript should be replaced by $3-i$.

CHAPTER 18

p. 797 : Eq. (18.1)
A factor 6 is missing. It should read

$$c = 1 - 6\frac{(p'-p)^2}{pp'} < 1$$

p. 814 : Eq. (18.105)
The correct notation for this equation should be

$$\begin{aligned} \chi_{\{\hat\omega_0,\,\hat\omega_0;\,2\hat\omega_0\}} &= \chi^{\rm Vir}_{(1,1)} \\ \chi_{\{\hat\omega_0,\,\hat\omega_0;\,\hat\omega_1\}} &= \chi^{\rm Vir}_{(1,2)} \\ \chi_{\{\hat\omega_0,\,\hat\omega_0;\,\hat\omega_7\}} &= \chi^{\rm Vir}_{(2,1)} \end{aligned}$$

p. 815 : Eq. (18.113)
The second term of the r.h.s. of the first line has incorrect indices. The corrected line is

$$\chi_{[1,0,0]}\ \chi_{[1,0,0]} = [\chi^{\rm Vir}_{(1,1)} + \chi^{\rm Vir}_{(4,1)}]\ \chi_{[2,0,0]} + [\chi^{\rm Vir}_{(2,1)} + \chi^{\rm Vir}_{(3,1)}]\ \chi_{[0,1,1]}$$

p. 817 : Line after Eq. (18.117)
"l.h.s." should read "r.h.s."

p. 821 : Eq. (18.148)
In the last line, a factor 2 is missing, it should read

$$\chi_{(1,2)}(q) = \sqrt{\frac{\theta_2(q)}{2\eta(q)}} = \eta(q) c_1^1(q)$$

p. 822 : Eq. (18.153)
In the last line a factor 2 is missing, it should read

$$\chi_1^{(2)} = 2 \left[\frac{\theta_2}{2\eta} \right]^{\frac{3}{2}}$$

p. 825 : Eq. (18.170)
$2(k+l+2)+1$ should read $2(k+l+2)$.

p. 825 : Eq. (18.171)
upper summation index of s-summation should read $2(k+l+2)$ instead of $2(k+l+1)+1$

p. 831 : Two lines before Eq. (18.194)
$\lambda \to \lambda - 2\rho$ should read $\lambda \to -\lambda - 2\rho$.

p. 834 : Eq. (18.213)
A term is missing. The equation should read

$$\lambda_1^I + \mu_1 - \nu_1^I + \nu_1^F = 0 \mod 2$$

p. 835 : Eq. (18.222)
The r.h.s. of the last line should be $\{0, 1; 3\}$. The esthetically correct equation is then

$$\begin{aligned} a\{0,0;0\} &= \{-\tfrac{4}{3}, 1; -\tfrac{1}{3}\} \\ \mathsf{c}\{-\tfrac{4}{3}, 1; -\tfrac{1}{3}\} &= \{-\tfrac{2}{3}, 1; -\tfrac{5}{3}\} \\ a\{-\tfrac{2}{3}, 1; -\tfrac{5}{3}\} &= \{-\tfrac{2}{3}, 0; \tfrac{4}{3}\} \\ \mathsf{c}\{-\tfrac{2}{3}, 0; \tfrac{4}{3}\} &= \{-\tfrac{4}{3}, 0; -\tfrac{10}{3}\} \\ a\{-\tfrac{4}{3}, 0; -\tfrac{10}{3}\} &= \{0, 1; 3\} \end{aligned} \tag{18.222}$$

p. 836 : Eq. (18.227)
Same correction as the one applied to Eq. (18.213)

$$\lambda_1^I + \mu_1 - \nu_1^I + \nu_1^F = 0 \mod 2$$

$$\begin{aligned} & \lambda_1^I + \mu_1 - \nu_1^I + \nu_1^F = 0 \mod 2 \\ \Rightarrow \quad & k^I - \lambda_1^I + u(1-\mu_1) - k^I - u + \nu_1^I = 0 \mod 2 \end{aligned} \tag{18.227}$$

p. 841 : Fig. 18.1
In the second group of indices at the left, $(3p' - r, p + s)$ should read $(3p' - r, p - s)$.

p. 850 : First equation of problem 18.9
γ_α should read $\partial\gamma_\alpha$.

REFERENCES

p. 872 : Ref. 275
"Penda" should read "Panda".

p. 873 : Reference [291]
Wrong journal page: should be 397, not 1925.