Two dimensional quantum memories

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1. Check operators & local codes
2. Holographic Disentangling Lemma
3. Holographic Minimum Distance
4. Capacity-Stability Tradeoff
5. String-Like Logical Operators
6. Thermal instability
Check operators & local codes

Holographic Disentangling Lemma

Holographic Minimum Distance

Capacity-Stability Tradeoff

String-Like Logical Operators

Thermal instability
Classical codes

Noisy bit

At each time interval, the bit has a probability $p$ of being flipped.

$0 \rightarrow 1 \quad \& \quad 1 \rightarrow 0$

Encoding: $0 \rightarrow 000$  
$1 \rightarrow 111$

Receive 001 $\rightarrow$ 000

Error probability $p \rightarrow 3p^2$ improvement provided $p < \frac{1}{3}$.

Quantum encoding:  
$|0\rangle \rightarrow |000\rangle$  
$|1\rangle \rightarrow |111\rangle$

But we can’t look at the bits to see if there was an error!

$\alpha|000\rangle + \beta|111\rangle \rightarrow \begin{cases} |000\rangle \text{ with prob. } |\alpha|^2 \\ |111\rangle \text{ with prob. } |\beta|^2 \end{cases}$
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Syndrome measurement

- We do not need to know the bit values for the classical code, only the parities.
- The first two bits are the same, and the last two bits are different. ⇒ Flip the last one.
- These are degenerate measurements: \{00, 11\} vs \{01, 10\}.
- Quantum mechanics

\[ P_E = |00\rangle\langle 00| + |11\rangle\langle 11| \quad P_O = |01\rangle\langle 01| + |10\rangle\langle 10| \]

⇒ Observable \(\sigma_z \otimes \sigma_z\)

- Measure \(\sigma_z \sigma_z = -1\) on first two qubits and \(-1\) on last two qubits ⇒ apply \(\sigma_x\) to middle qubit.

This type of measurement requires interactions between qubits.
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Quantum codes

- Set of states that obey a bunch of check conditions
  \[ C = \{ |\psi\rangle : P_j |\psi\rangle = |\psi\rangle, \forall j \} \]

- There must be more than one state in \( C \) for the code to be interesting.
- We measure the check operators, eigenvalue \( \neq +1 \) indicates an error.

Locality

- Because coherent measurement of checks requires coupling the qubits, we restrict the \( P_j \) to couple only neighbouring qubits in some geometry.
- In 2D, this leads to topological codes.

\[ C = \text{degenerate ground space of Hamiltonian } H = - \sum_j P_j. \]
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\[ C = \text{degenerate ground space of Hamiltonian } H = - \sum_j P_j. \]
Definitions

- \( \Lambda \) is a 2D lattice.
- Each vertex occupied by \( d \)-level quantum particle.
- Hamiltonian \( H = - \sum_{X \subset \Lambda} P_X \) with
  - \( P_X = 0 \) if radius\((X)\) \( \geq w \).
  - \([P_X, P_Y] = 0\).
  - \( P_X \) are projectors (optional).
- Code \( \mathcal{C} = \{ \psi : P_X|\psi\rangle = |\psi\rangle \} \)
  = ground space of \( H \)
  = image of code projector \( \Pi = \prod_X P_X \)
- With proper coarse graining, we can assume that
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Well known examples

- Kitaev’s toric code
- Bombin’s topological color codes
- Levin & Wen’s string-net models
- Turaev-Viro models
- Kitaev’s quantum double models
- Most known models with topological quantum order
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Lattice

- Two-dimensional square lattice
- Periodic boundary conditions
Kitaev’s code

- Site operator: \( A_s = \prod_{i \in \nu(s)} \sigma_x^i \)
- Plaquette operator: \( B_p = \prod_{i \in \nu(p)} \sigma_z^i \)
- Hamiltonian: \( H = -\left( \sum_s A_s + \sum_p B_p \right) \)
Check operators & local codes

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- Aharonov & Eldar ’11: Topological order requires 4-qubit commuting checks.
  - Low-weight non-commuting checks possible?
  - Less error-prone

- Bombin ’10, Topological subsystem colour codes
  - Weight = 2
  - Low threshold

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- Aharonov & Eldar ’11: Topological order requires 4-qubit commuting checks.
  - Low-weight non-commuting checks possible?
  - Less error-prone

- Bombin ’10, Topological subsystem colour codes
  - Weight=2.
  - Low threshold.

- Bravyi, Duclos-Cianci, DP, Suchara
  - Weight = 3.
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  - Surface with boundaries.
Check operators & local codes

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Let $|\psi_1\rangle$ and $|\psi_2\rangle$ be two code states (ground states).

Suppose there exists a local (e.g. single spin) measurement $\sigma$ that distinguishes them.

Then the environment can also learn which state is encoded by “looking” at a single spin.

\[\alpha|\psi_1\rangle + \beta|\psi_2\rangle \rightarrow \begin{cases} 
|\psi_1\rangle \text{ with prob. } |\alpha|^2 \\
|\psi_2\rangle \text{ with prob. } |\beta|^2 
\end{cases}\]

So a code should not have such local “order parameter” : all codes states should look identical locally.
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Desirable features

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So a code should not have such local “order parameter" : all codes states should look identical locally.
### Correctable region

A region $M \subset \Lambda$ is **correctable** if there exists a recovery operation $R$ such that $R(\text{Tr}_M \rho) = \rho$ for all code states $\rho$.

$M$ correctable $\iff$ No order parameter on $M \iff \Pi \Omega_M \Pi \propto \Pi$.

### Minimum distance

The minimum distance $d$ is the size of the smallest non-correctable region.

### Logical operator

Operator $L$ such that $L |\psi\rangle$ is a code state for any code state $|\psi\rangle$. 
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Outline

1. Check operators & local codes
2. Holographic Disentangling Lemma
3. Holographic Minimum Distance
4. Capacity-Stability Tradeoff
5. String-Like Logical Operators
6. Thermal instability
Holographic disentangling lemma (Bravyi, DP, Terhal)

Let $M \subset \Lambda$ be a correctable region and suppose that its boundary $\partial M$ is also correctable. Then, there exists a unitary operator $U_{\partial M}$ acting only on the boundary of $M$ such that, for any code state $|\psi\rangle$,

$$U_{\partial M} |\psi\rangle = |\phi_M\rangle \otimes |\psi'_M\rangle$$

for some fixed state $|\phi_M\rangle$ on $M$. 
Let $M$ be correctable.

Assume $\partial M$ is correctable.

Let $M = A \cup B$, $\overline{M} = C \cup D$, and $\partial M = B \cup C$.

There exists a unitary transformation $U_{\partial M}$ such that, for any $|\psi\rangle \in \mathcal{C}$

$$U_{\partial M}|\psi\rangle = |\phi_M\rangle \otimes |\psi'_M\rangle$$

where $|\phi_M\rangle$ is the same for all $|\psi\rangle$.

Remark

For a trivial code $\text{Tr} \Pi = 1$, every region is correctable, so we recover the area law $S(M) \leq |\partial M|$ for commuting Hamiltonians of Wolf, Verstraete, Hastings, and Cirac.
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With pictures

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Region $M \subset \Lambda$ is correctable if its boundary is smaller than the minimum distance $|\partial M| \leq cd$.

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Let $M \subseteq \Lambda$ be a correctable region.

- If $|\partial M| \leq d$, then $\partial M$ is also correctable.
- Thus, we can reconstruct any code state $\rho$ from $\rho_{AD} = \text{Tr}_{\partial M} \rho$.
- But from the Holographic disentangling lemma, $\rho_{AD} = \eta_A \otimes \rho_D$ with $\eta_A$ independent of the encoded state $\rho$.
- Thus, we can reconstruct $\rho$ from $\rho_D = \text{Tr}_{M \cup \partial M} \rho$, so $M \cup \partial M$ is correctable.
- We can continue to grow $M$ this way until $|\partial M| \geq d$. 
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Statement of the result

- $n =$ number of qubits
- $k =$ number of encoded qubits
- $d =$ minimum distance

**Capacity-Stability Tradeoff**

$$k \leq c \frac{n}{d^2}$$

- Singleton’s bound: $k \leq n - 2(d - 1)$.
- Hamming bound: $k \leq n \left[1 - \frac{d}{2^n} \log 3 - H(\frac{d}{2^n})\right]$.
- Kitaev’s codes (with punctures) saturate this bound, so it is tight.
- No “good codes” in 2D, i.e. $k \propto n$ and $d \propto n$.
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String-like logical operators (Haah, Preskill)

There exists a non-trivial logical operator supported on a string-like region.

- Exists $U_M$ such that $U_M|\psi\rangle = |\psi'\rangle$.
  - $|\psi\rangle \neq |\psi'\rangle$.
  - $|\psi\rangle, |\psi'\rangle \in \mathcal{C}$.
- Well known for Kitaev’s toric code.
- Intuitive for known models that support anyons:
  - The ground state can be changed by dragging an anyon around a topologically non-trivial loop.
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Classical memories are robust

- Energy barrier $\propto \sqrt{n}$ between logical states through local moves.
- Boltzmann: configuration $x$ has probability $\propto \exp(-E(x)/T)$.
- Probability of flipping the whole configuration by local moves decreases with $n$. 

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- (TQO1) System has no local order parameter.
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The system has a stable spectrum.
Long lived memory at zero temperature.

\[ H = - \sum_i \sigma_i^z \sigma_{i+1}^z + \sigma_{23}^z \]

The ground state manifold changes abruptly when including site 23.

Can we combine this spectral stability with the thermal stability of the 2D Ising model?
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Main result (Landon-Cardinal & DP)

The minimum set of conditions required to prove spectral stability imply the existence of a sequence of local maps that corrupt the system at an energy cost bounded by a constant.
Thermal instability

Noise model

1. Apply random unitary on sites 1 & 2.
2. Measure $P_{12}$
   - If $P_{12} = 0$ go to 1.
3. Apply random unitary on site 3.
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- Only a constant amount of energy at any given time.
- No need to backtrack.
- Number of steps $\propto$ lattice linear size.
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- Quantum error correction requires joint qubit measurements.
  - Local check operators in 2D $\Rightarrow$ topological codes.
- Natural relation between codes and quantum many-body physics.
  - Large minimum distance $\Leftrightarrow$ Topological quantum order (order with no local order parameter).
  - Disentangling lemma $\Leftrightarrow$ Area law.
  - Fault tolerant threshold $\Leftrightarrow$ phase transition.
- Impossible to combine spectral and thermal stability with existing tools.
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David Poulin (Sherbrooke)
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Open questions

- **String-like logical operators +TQO ⇒ constant energy barrier.**
  - This is not directly related to thermal instability.
  - 2D Ising model has an energy barrier $\propto \sqrt{n}$, but an energy $\propto n$ at finite temperature.
  - What matters is entropy (for a given energy, many more configurations many with small error droplets than with a large one).
  - Can we characterize all string-like logical operators?
  - We have shown information corruption in time $\propto \sqrt{n}$. Can it be parallelized? (Percolation)
  - Relation between commuting projector codes and anyon models.

- Can we engineer dead ends?
  - Memory that is stabilized by complexity.

- Extension to subsystem codes?
  - With local stabilizer (Bombin) and without (Bacon-Shor).

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