Two dimensional quantum memories

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Theory Canada 8, Bishop’s University, May 2013
1. Check operators & local codes
2. Holographic Disentangling Lemma
3. Holographic Minimum Distance
4. Capacity-Stability Tradeoff
5. String-Like Logical Operators
Outline

1. Check operators & local codes
2. Holographic Disentangling Lemma
3. Holographic Minimum Distance
4. Capacity-Stability Tradeoff
5. String-Like Logical Operators
Classical codes

Noisy bit

At each time interval, the bit has a probability $p$ of being flipped.

$$0 \rightarrow 1 \quad \& \quad 1 \rightarrow 0$$

Encoding:

- $0 \rightarrow 000$
- $1 \rightarrow 111$

Receive $001 \rightarrow 000$

Error probability $p \rightarrow 3p^2$ improvement provided $p < \frac{1}{3}$.

Quantum encoding:

- $|0\rangle \rightarrow |000\rangle$
- $|1\rangle \rightarrow |111\rangle$

But we can't look at the bits to see if there was an error!

$$\alpha|000\rangle + \beta|111\rangle \rightarrow \begin{cases} |000\rangle \quad \text{with prob. } |\alpha|^2 \\ |111\rangle \quad \text{with prob. } |\beta|^2 \end{cases}$$
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The first two bits are the same, and the last two bits are different.

⇒ Flip the last one.

These are degenerate measurements: \{00, 11\} vs \{01, 10\}.

Quantum mechanics

\[
P_E = |00\rangle\langle 00| + |11\rangle\langle 11| \quad P_O = |01\rangle\langle 01| + |10\rangle\langle 10|
\]

⇒ Observable $\sigma_z \otimes \sigma_z$

Measure $\sigma_z \sigma_z = -1$ on first two qubits and $-1$ on last two qubits

⇒ apply $\sigma_x$ to middle qubit.

This type of measurement requires interactions between qubits.
Syndrome measurement

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Quantum codes

- Set of states that obey a bunch of check conditions
  \[ C = \{ |\psi\rangle : P_j |\psi\rangle = |\psi\rangle, \forall j \} \]
- There must be more than one state in \( C \) for the code to be interesting.
- We measure the check operators, eigenvalue \( \neq +1 \) indicates an error.

Locality

- Because coherent measurement of checks requires coupling the qubits, we restrict the \( P_j \) to couple only neighbouring qubits in some geometry.
- In 2D, this leads to topological codes.

\[ C = \text{degenerate ground space of Hamiltonian } H = - \sum_j P_j. \]
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Definitions

- $\Lambda$ is a 2D lattice.
- Each vertex occupied by $d$-level quantum particle.
- Hamiltonian $H = -\sum_{X \subseteq \Lambda} P_X$ with
  - $P_X = 0$ if $\text{radius}(X) \geq w$.
  - $[P_X, P_Y] = 0$.
  - $P_X$ are projectors (optional).
- Code $C = \{\psi : P_X |\psi\rangle = |\psi\rangle\}$
  = ground space of $H$
  = image of code projector $\Pi = \prod_X P_X$
- With proper coarse graining, we can assume that
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Well known examples

- Kitaev’s toric code
- Bombin’s topological color codes
- Levin & Wen’s string-net models
- Turaev-Viro models
- Kitaev’s quantum double models
- Most known models with topological quantum order
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Lattice

- Two-dimensional square lattice
- Periodic boundary conditions
Kitaev’s code

Site operator:
\[ A_s = \prod_{i \in v(s)} \sigma_x^i \]

Plaquette operator:
\[ B_p = \prod_{i \in v(p)} \sigma_z^i \]

Hamiltonian:
\[ H = - \left( \sum_s A_s + \sum_p B_p \right) \]
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**Threshold** $\approx 15\%$.

- Order-disorder phase transition.
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Order-disorder phase transition.
Desirable features

- Let $|\psi_1\rangle$ and $|\psi_2\rangle$ be two code states (ground states).
- Suppose there exists a local (e.g. single spin) measurement $\sigma$ that distinguishes them.
- Then the environment can also learn which state is encoded by “looking” at a single spin.

$$\alpha |\psi_1\rangle + \beta |\psi_2\rangle \rightarrow \begin{cases} 
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- So a code should not have such local “order parameter” : all codes states should look identical locally.
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Correctable region

A region $M \subset \Lambda$ is *correctable* if there exists a recovery operation $\mathcal{R}$ such that $\mathcal{R}(\text{Tr}_M \rho) = \rho$ for all code states $\rho$.

$M$ correctable $\iff$ No order parameter on $M$ $\iff$ $\Pi O_M \Pi \propto \Pi$.

Minimum distance

The minimum distance $d$ is the size of the smallest non-correctable region.

Logical operator

Operator $L$ such that $L|\psi\rangle$ is a code state for any code state $|\psi\rangle$. 

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1. Check operators & local codes
2. Holographic Disentangling Lemma
3. Holographic Minimum Distance
4. Capacity-Stability Tradeoff
5. String-Like Logical Operators
Holographic disentangling lemma (Bravyi, DP, Terhal)

Let \( M \subseteq \Lambda \) be a correctable region and suppose that its boundary \( \partial M \) is also correctable. Then, there exists a unitary operator \( U_{\partial M} \) acting only on the boundary of \( M \) such that, for any code state \( |\psi\rangle \),

\[
U_{\partial M}|\psi\rangle = |\phi_M\rangle \otimes |\psi'_{\overline{M}}\rangle
\]

for some fixed state \( |\phi_M\rangle \) on \( M \).
Let $M$ be correctable.

Assume $\partial M$ is correctable.

Let $M = A \cup B, \overline{M} = C \cup D$, and $\partial M = B \cup C$.

There exists a unitary transformation $U_{\partial M}$ such that, for any $|\psi\rangle \in C$,

$$U_{\partial M}|\psi\rangle = |\phi_M\rangle \otimes |\psi_M^{\prime}\rangle$$

where $|\phi_M\rangle$ is the same for all $|\psi\rangle$.

Remark

For a trivial code $\text{Tr}\Pi = 1$, every region is correctable, so we recover the area law $S(M) \leq |\partial M|$ for commuting Hamiltonians of Wolf, Verstraete, Hastings, and Cirac.
Let $M$ be correctable.

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Region $M \subset \Lambda$ is correctable if its boundary is smaller than the minimum distance $|\partial M| \leq cd$.

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Let $M \subset \Lambda$ be a correctable region.

- If $|\partial M| \leq d$, then $\partial M$ is also correctable.
- Thus, we can reconstruct any code state $\rho$ from $\rho_{AD} = \text{Tr}_{\partial M} \rho$.
- But from the Holographic disentangling lemma, $\rho_{AD} = \eta_A \otimes \rho_D$ with $\eta_A$ independent of the encoded state $\rho$.
- Thus, we can reconstruct $\rho$ from $\rho_D = \text{Tr}_{M \cup \partial M} \rho$, so $M \cup \partial M$ is correctable.
- We can continue to grow $M$ this way until $|\partial M| \geq d$. 
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David Poulin (Sherbrooke)
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1. Check operators & local codes
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Statement of the result

- $n =$ number of qubits
- $k =$ number of encoded qubits
- $d =$ minimum distance

Capacity-Stability Tradeoff

\[ k \leq c \frac{n}{d^2} \]

- Singleton’s bound: $k \leq n - 2(d - 1)$.
- Hamming bound: $k \leq n \left[ 1 - \frac{d}{2n} \log 3 - H\left( \frac{d}{2n} \right) \right]$.
- Kitaev’s codes (with punctures) saturate this bound, so it is tight.
- No “good codes” in 2D, i.e. $k \propto n$ and $d \propto n$.
- For 2D classical codes, $k \leq c \frac{n}{\sqrt{d}}$. 
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String-like logical operators (Haah, Preskill)

There exists a non-trivial logical operator supported on a string-like region.

- Exists $U_M$ such that $U_M|\psi\rangle = |\psi'\rangle$.
  - $|\psi\rangle \neq |\psi'\rangle$.
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- Well known for Kitaev’s toric code.
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Quantum error correction requires joint qubit measurements.
- Local check operators in 2D $\Rightarrow$ topological codes.

Natural relation between codes and quantum many-body physics.
- Large minimum distance $\Leftrightarrow$ Topological quantum order (order with no local order parameter).
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Take home messages

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