Exploring Quantum Control with Quantum Information Processors

David Poulin

Institute for Quantum Computing
Perimeter Institute for Theoretical Physics
Two aspects of quantum information

Practical

- Build a quantum computer.
- Design new quantum algorithms.
- Invent useful quantum protocols.
- Design quantum error corrections techniques.

Fundamental

- Use the byproducts to study fundamental physics.
- Foundation of quantum mechanics: what gives quantum mechanics extra computational power?
- Explain renormalization groups as error correction.
- Study the "holographic principle" in information theoretic language.
- Black hole formation = quantum data compression?
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Concrete example

- Design useful task for a present days quantum information processors: Can we do something useful in DQC1?
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- How can we improve our control over quantum systems? (QECC, NSS, bang-bang)
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Identify good candidates for QIP.

Quantum algorithm to find symmetries of a quantum system?
- Relation between dynamical stability and decoherence.
- Relation between decoherence and entanglement.
Outline

Simulating quantum systems: the readout problem.
The QDC1 model of computation.
The elementary scattering circuit.
Looking for symmetries in complex quantum systems.
  - Random matrix conjecture.
  - Estimating the form factors.
Hypersensitivity to perturbations: fidelity decay.
  - DQC1 algorithm.
  - Quantum information processors as quantum probes.
  - Decoherence and dynamical instability.
  - Entanglement, decoherence and quantum-computational speed-up.
Conclusion.
In a physical experiment, a system evolves from its initial state and is finally measured in some basis.
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We know that quantum computer can be used to simulate the evolution of some quantum systems (Zalka, Lloyd, ...).

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U(t) = U_n \ldots U_2 U_1
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Universal set
Readout problem

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The ability to efficiently simulate the dynamics \( U(t) \) alone can be used to extract interesting physical quantities.
The initial state is thermal: \[ \rho = \frac{1}{Z} e^{-\beta H}. \]
Liquid state NMR QIP

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$H \simeq \sum_i \vec{\mu}_i \cdot \vec{B} + \sum_{i \neq j} J_{ij} \vec{\mu}_i \vec{\mu}_j$, $\omega \gg J$ and $\beta \ll 1$, so

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- Apply an inhomogeneous magnetic field to induce a random phase:

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Pseudo pure state, the identity part does not contribute to the computation.

This exponential decay can be avoided using algorithmic cooling, but the overhead is $\sim 10^{10}$. 

Can we do something non trivial with this?

E. Knill and R. Laflamme, 1998
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Add restrictions to the “standard” model of quantum computation.
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Add restrictions to the “standard” model of quantum computation

1. Only a single pseudo-pure qubit.

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\rho = \left( \frac{1 - \epsilon}{2} \mathbb{1} + \epsilon \lvert 0 \rangle \langle 0 \rvert \right) \otimes \frac{1}{2^n} \mathbb{1}
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4. One pure qubit $\Leftrightarrow \log(n)$ pure qubits: cost $poly(n)$.
   - From a computational complexity point of view:
Scattering circuit

\[ |0\rangle\langle 0| \xrightarrow{\text{H}} |0\rangle \langle 0| \xrightarrow{\text{U}} |\sigma_k\rangle \]

\[ \gamma = Tr\{U \rho\} \]

If \( U \) can be implemented efficiently and \( \rho \) prepared efficiently, we can evaluate \( Tr\{U \rho\} \) within accuracy \( \delta \) in time \( poly(n)/\delta^2 \).
Scattering circuit

\[ \langle 0|0 \rangle \xrightarrow{\text{H}} \langle \sigma_k \rangle \xrightarrow{\text{H}} \langle \sigma_k \rangle \]

\[ \rho \xrightarrow{U} \rho' \]

\[ \rho'_1 = \frac{1}{2} \left( \begin{array}{cc} 1 + \text{Re}\{\gamma\} & i\text{Im}\{\gamma\} \\ -i\text{Im}\{\gamma\} & 1 - \text{Re}\{\gamma\} \end{array} \right) \]

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\[
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1 + \text{Re}\{\gamma\} & i\text{Im}\{\gamma\} \\
-i\text{Im}\{\gamma\} & 1 - \text{Re}\{\gamma\}
\end{pmatrix}
\]

$k = z$ : $\langle \sigma_z \rangle = \text{Tr}\{\rho'_1 \sigma_z\} = \text{Re}\{\gamma\} = \text{Re} [\text{Tr}\{\rho U\}]$

$k = y$ : $\langle \sigma_y \rangle = \text{Tr}\{\rho'_1 \sigma_y\} = \text{Im}\{\gamma\} = \text{Im} [\text{Tr}\{\rho U\}]$
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\rho
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\end{pmatrix}
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We can evaluate $Tr\{\rho U\}$
Scattering circuit

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Control $U$ and learn about $\rho$: quantum state tomography.
Scattering circuit

We can evaluate $\text{Tr}\{\rho U\}$

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- Control $\rho$ and learn about $U$: quantum process tomography.

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- These algorithms require an exponential repetition of the scattering circuit.
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- Can we get useful partial information about $U$?
Scattering circuit

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- Control $U$ and learn about $\rho$: quantum state tomography.
- Control $\rho$ and learn about $U$: quantum process tomography.
  

- These algorithms require an exponential repetition of the scattering circuit.
- Can we get useful partial information about $U$?
- DQC1 imposes restrictions on $\rho$. We will focus on $\rho \propto 1_l$.

$$\gamma = \frac{1}{N} \text{Tr}\{U\} = \frac{1}{N} \sum_j \lambda_j$$
Quantum control

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Systems with as many symmetries as degrees of freedom are integrable.

- These are good candidates for quantum information processors.
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Quantum information processing is the art of manipulating complex quantum systems.

Control a quantum system = exploit its symmetries (QECC, NSS, bang-bang)

Does my system possess any symmetries?

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These are good candidates for quantum information processors.

Chaotic systems possess no or very few symmetries.

They are hard to control.

They are dynamically unstable (sensitive to perturbations).
Looking for symmetries

Resonances of Thorium 232
Looking for symmetries

List of resonances

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<tr>
<td>$154.20$</td>
<td>$154.21$</td>
<td>$0.2$</td>
<td>$0.6$</td>
<td>$154.21$</td>
<td>$0.2$</td>
<td>$0.6$</td>
<td>$154.21$</td>
<td>$0.2$</td>
</tr>
<tr>
<td>$165.00$</td>
<td>$165.01$</td>
<td>$0.2$</td>
<td>$0.6$</td>
<td>$165.01$</td>
<td>$0.2$</td>
<td>$0.6$</td>
<td>$165.01$</td>
<td>$0.2$</td>
</tr>
<tr>
<td>$168.20$</td>
<td>$168.21$</td>
<td>$0.2$</td>
<td>$0.6$</td>
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<td>$168.21$</td>
<td>$0.2$</td>
</tr>
</tbody>
</table>

* $E^*$ and $L'$ are in MeV, while $L$ and $I'$ are in $10^{-1}$ ev. All calculations were done using the code of Ellis and Jones (Ref. 2).

Level spacing

![Graph showing level spacing](attachment:image.png)

**Fig. 10.** Histogram of the observed distribution of nearest-neighbor level spacings $x = D/(2D)$ for Th. The three theoretical curves correspond respectively to random, orthogonal ($\beta = 1$) and unitary ($\beta = 2$). The orthogonal ($\beta = 1$) curve is the favored theoretical distribution.
Looking for symmetries

**Wigner's intuition**: The main characteristics of the spectrum of can be reproduced by random matrices which possess the same symmetries as the system.
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- \( P(V) \neq 0 \) only for those \( V \) with the same symmetries as \( U \).
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Symmetries = Block diagonal

\[
U = \exp \left\{ -\frac{i H t}{\hbar} \right\} = \begin{pmatrix}
U_1 & & \\
& U_2 & \\
& & \ddots \\
& & & U_d
\end{pmatrix}
\]

Mixture of i.i.d variables → Poisson distribution.
Looking for symmetries

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With symmetries (regular):

\[ \frac{1}{N} \text{Tr}\{U\} = \frac{1}{N} \sqrt{N} = \frac{1}{\sqrt{N}} \]

No symmetries (chaotic):

\[ \frac{1}{N} \text{Tr}\{U\} = \frac{1}{N} O(1) \]

Requires accuracy \( \frac{1}{\sqrt{N}} \) so overhead is \( N \) square-root improvement over brute force classical computation.

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\[
\frac{1}{N} Tr \{U\} = \frac{1}{N} \sum_j e^{i\phi_j} \\
= \frac{1}{N} \sum_j \vec{v}_j
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Fidelity decay

Quantum map $U$, e.g. $U = \exp\{-iHt\}$. How sensitive is $U$ to perturbation?
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Fidelity after $n$ steps (Lodschmidt echo):

$$F_n(\psi) = \left| \langle \psi | (U^n)^\dagger U^n_p | \psi \rangle \right|^2$$
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State independent signature (average over $\psi$):

$$\overline{F_n} = \int F_n(\psi) d\psi = \frac{|Tr\{(U^n)_\dagger U^n_p\}|^2}{N^2 + N} + N$$
Fidelity decay

- Peres’ conjecture: $F_n(\psi)$ is a signature of quantum chaos (dynamical instability).
  - Chaotic systems have an exponential decay $\exp(-\Gamma n)$.
  - Regular systems have a polynomial decay $1/poly(n)$.
Fidelity decay

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Fidelity decay measures the relative randomness of $U$ and $K$.
- Universal response to perturbation.
- “Simple” perturbation produce good signature of quantum chaos.
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  - “Simple” perturbation produce good signature of quantum chaos.

J. Emerson, Y.S. Weinstein, S. Lloyd, and D.G. Cory, 2002

\[ |0⟩⟨0| \quad \text{H} \quad \frac{1}{2^n} \quad U \quad \text{H} \quad ⟨σ_k⟩ \]

\[ k = y : ⟨σ_y⟩ = \frac{1}{N} \text{Im} [Tr\{U\}] \]

\[ k = z : ⟨σ_z⟩ = \frac{1}{N} \text{Re} [Tr\{U\}] \]

\[ F_n = \frac{|Tr\{(U^n)^\dagger U_p^n\}|^2 + N}{N^2 + N}, \quad U_p = UK \]

\[ |0\rangle\langle 0| \quad \begin{array}{c} \text{H} \\ \frac{1}{2^n} \text{U} \\
\end{array} \quad \begin{array}{c} \text{H} \\
\end{array} \langle \sigma_k \rangle \]

\[ k = y : \langle \sigma_y \rangle = \frac{1}{N} Im [Tr \{U\}] \]
\[ k = z : \langle \sigma_z \rangle = \frac{1}{N} Re [Tr \{U\}] \]
\[ F_n = \frac{|Tr\{(U^n)^\dagger U^n_p\}|^2 + N}{N^2 + N}, \quad U_p = UK \]

All we have to do it replace \( U \) in the above circuit by \( (U^n)^\dagger(UK)^n \).
Quantum circuit

Since the $U$'s and $U^\dagger$'s annihilate each other when the $K$'s are absent.

Measure $F_N$ within accuracy $\epsilon$ with error probability $p$ in time $nO(\log(1/p)/\epsilon^2)$.
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Since the $U^\dagger$ are made after the final measurement.

Measure $\overline{F_n}$ within accuracy $\epsilon$ with error probability $p$ in time

$$nO\left(\frac{\log(1/p)}{\epsilon^2}\right) T$$
Quantum probe

Use a QIP to simulate a system with Hamiltonian $H$. Use a quantum probe to investigate the dynamical stability of a system whose Hamiltonian is unknown.

$H = \sum_{j>1} J_{1j} Z_1 Z_j \rho_0$ perturbation $+ \text{Unknown}$ $H(i,j) \neq 1)$

Small QIP can be used to perform experimental measurements (POVM).
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$\rho_0$
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Use a quantum probe to investigate the dynamical stability of a system whose Hamiltonian is unknown.

$$H = \sum_{j>1} J_{1j} Z_1 Z_j + \left\{ \begin{array}{c} \text{Unknown } U \\ \text{perturbation} \end{array} \right\} + H(i, j \neq 1)$$
Use a QIP to simulate a system with Hamiltonian $H$.

Use a quantum probe to investigate the dynamical stability of a system whose Hamiltonian is unknown.

Small QIP can be used to perform experimental measurements which were unconceivable before (POVM).

$$H = \sum_{j>1} J_{1j} Z_1 Z_j + \underbrace{H(i, j \neq 1)}_{\text{perturbation}}$$
We use the circuit to model a system interacting with an environment, the system is initially in a pure state $\alpha |0\rangle + \beta |1\rangle$: 

\[
\rho_0 \longrightarrow \cdots \longrightarrow \rho_n
\]

\[
\frac{1}{N} \begin{array}{cccc}
U & K & U & K \\
\hline
U & K & \cdots & U & K
\end{array}
\]

The decoherence rate is governed by the average fidelity decay rate of the environment! The decoherence rate depends on the strength of the coupling to the environment but also on the dynamics of the environment.
We use the circuit to model a system interacting with an environment, the system is initially in a pure state $\alpha|0\rangle + \beta|1\rangle$:

\[
\begin{pmatrix}
|\alpha|^2 & \alpha\beta^* \\
\alpha^*\beta & |\beta|^2
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
|\alpha|^2 & \alpha\beta^*\gamma(n) \\
\alpha^*\beta\gamma(n)^* & |\beta|^2
\end{pmatrix}
\]
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$$\rho_0 = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix} \Rightarrow \rho_n = \begin{pmatrix} |\alpha|^2 & \alpha\beta^*\gamma(n) \\ \alpha^*\beta\gamma(n)^* & |\beta|^2 \end{pmatrix}$$

$|\gamma(n)| = \sqrt{\frac{F_n}{n}}$: The decoherence rate is governed by the average fidelity decay rate of the environment!
Decoherence

We use the circuit to model a system interacting with an environment, the system is initially in a pure state \( \alpha |0\rangle + \beta |1\rangle \):

\[
\begin{align*}
\rho_0 &= \begin{pmatrix} |\alpha|^2 & \alpha \beta^* \\ \alpha^* \beta & |\beta|^2 \end{pmatrix} \\
\rho_n &= \begin{pmatrix} |\alpha|^2 & \alpha \beta^* \gamma(n) \\ \alpha^* \beta \gamma(n)^* & |\beta|^2 \end{pmatrix}
\end{align*}
\]

| \( \gamma(n) \) | = \( \left\{ \overline{F_n} \right\}^{1/2} \): The decoherence rate is governed by the average fidelity decay rate of the environment!

The decoherence rate depends on the strength of the coupling to the environment but also on the dynamics of the environment.
Using the "phase kick-back", we see that the final state is
\[ \rho_k = \sum_j \alpha_j \langle \alpha_j | \otimes | \phi_j \rangle \langle \phi_j | \]
where \( | \alpha_j \rangle = \alpha | 0 \rangle + \beta_{e_j} | 1 \rangle \) and the \( e_{\pm j} \) are the eigenvalues of \( S = U_k \prod_k U_2 \prod U_1 \prod U_1^\dagger \prod U_2^\dagger \prod U_k^\dagger \).

This circuit does not produce any entanglement between the probe qubit and the rest, throughout the computation.
Entanglement

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- Pure states: YES.
- Mixed states: ?
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Is there entanglement between other parts of the information processor? (Depends on the system of interest, i.e. $U$.)
- If no, is there a classical dynamics which can provide the sequence of "classical" states?
Conclusion

- DQC1 is interesting.
  - Measure the form factors to learn about symmetries.
  - Measure fidelity decay to learn about dynamical stability.

- Simple quantum information processors can help to perform experiments.

- Decoherence rate is influenced by the dynamical properties of the environment.
  - Decoherence does not require entanglement.

- Entanglement is not the key ingredient to quantum computational speed-up.

- We can learn about fundamental physics by working on quantum computation, and vice versa.