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Supporting Online Material for

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Anisotropic Violation of the Wiedemann-Franz Law at a Quantum Critical Point

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Methods

Electrical and thermal resistivities were both measured using four-contact geometry in a dilution refrigerator down to 25 mK. Thermal measurements use one-heater-two-thermometer technique with *in situ* thermometer calibration in magnetic field [1]. In order to produce a measurable thermal gradient, the sample must be heated and so the base temperature in thermal measurements is inevitably higher, and is determined mainly by the ratio of the sample resistance to cold current contact resistance. Due to a more favourable sample geometry, this was slightly better for *a*-axis samples, resulting in a base temperature of 62 mK, compared to 67 mK for *c*-axis samples.

Quantum criticality and superconductivity

The fact that H_c in $CeCoIn_5$ coincides with the superconducting upper critical field $H_{c2}(0)$ should not be taken to mean that the quantum critical fluctuations are superconducting in nature. The transition to the superconducting state is strongly first order, and H_c and H_{c2} can be split by the application of pressure [2]. Of course it may be that critical fluctuations cause superconductivity to emerge, as they do in other quantum critical systems [3–5].

Low-temperature resistivity

The Wiedemann-Franz (WF) law is strictly exact only at $T = 0$, where it was shown to remain valid for arbitrarily strong scattering [6], disorder [7] and interactions [8]. To test this law, we therefore need to have a reliable way of extrapolating our data to $T = 0$. The extrapolated (residual) values of resistivity plotted in Fig. 2 of the paper for both heat and charge transport are obtained by linear extrapolation to $T = 0$ of raw data, shown here in Fig. S1 (and other similar data not shown for fields intermediate between 5.3 and 10 T). Solid and dashed lines are linear fits to w_c (5.3 T), respectively below 0.2 K and 0.6 K. This procedure works quite well as long as the $\rho(T)$ and $w(T)$ curves are linear, as is the case for c-axis transport at H close to H_c. For the case of $J \parallel a$ at 5.3 T, we use a fit to $T^{3/2}$, given that this is the limiting power law [9]. Because the data goes down to 25 mK (65 mK) for ρ (w), a simple extrapolation of this kind is quite accurate.

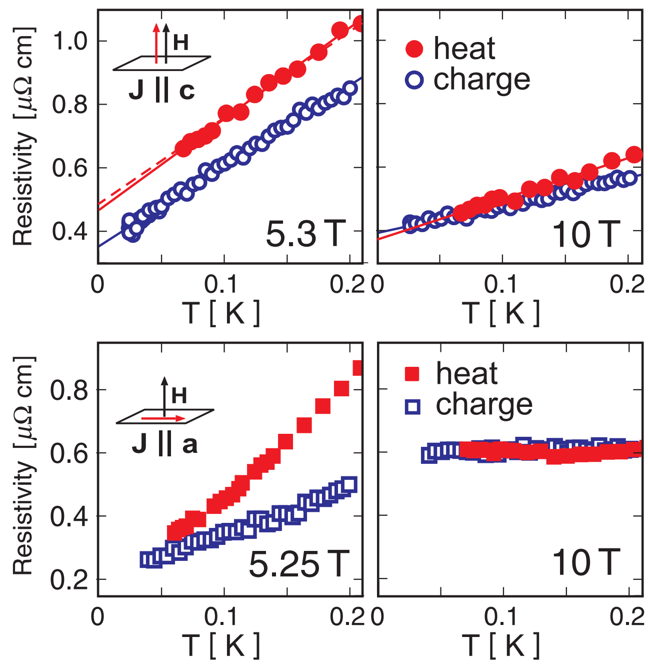

FIG. S1: **Transport of heat and charge in the $T = 0$ limit.** Temperature dependence of heat ($w \equiv L_0T/\kappa$, red) and charge (ρ, blue) resistivities at the QCP ($H = 5.3$ T $\simeq H_c$; left) and away from it ($H = 10$ T; right) for inter-plane (top) and in-plane (bottom) current directions. The solid (dashed) lines are linear fits to the $J \parallel c$ data below 0.2 K (0.6 K). As $T \to 0$, the Wiedemann-Franz (WF) law, $w = \rho$, is obeyed away from H_c, for both current directions. However, near H_c, it is anisotropically violated: still obeyed in-plane, no longer obeyed inter-plane. The T dependence of the resistivities at H_c exhibits a different asymptotic power law depending on current direction: T for inter-plane and $T^{3/2}$ for in-plane transport [9].

In order to ascertain the robustness of the results, we compare them with those obtained using two other extrapolation procedures: one based on the difference in

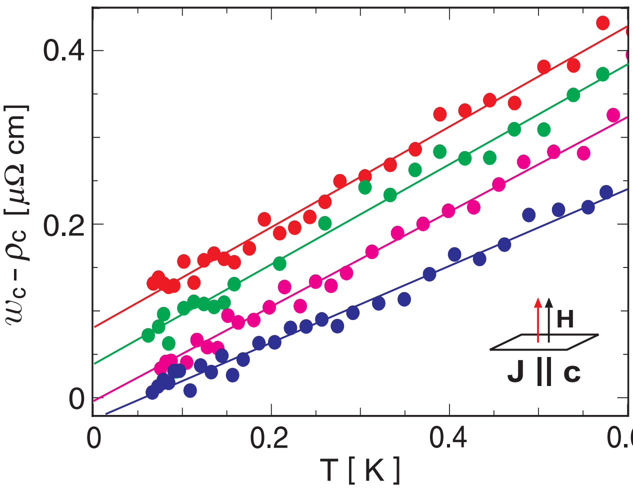

FIG. S2: **Difference data**. Temperature dependence of the difference in resistivities, $\delta(T) \equiv w_c - \rho_c$, with linear fits below 0.6 K (solid lines), for $H = 5.3, 6, 8$ and 10 T (top to bottom).

the raw data, the other based on the ratio. The difference data, $\delta(T) \equiv w - \rho$, is plotted as a function of T in Fig. S2 for different fields. It can be seen by inspection that the curves shift upwards *rigidly* as the field approaches H_c. Such a rigid shift immediately implies a violation of the WF law, independent of any extrapolation procedure. Linear extrapolations of $\delta(T)$ to $T = 0$ yield values that start close to zero at 10 T and gradually increase with decreasing field.

The other way of comparing heat and charge conductivities is to look at their ratio, called the Lorenz ratio, $L \equiv \kappa/\sigma T \equiv L_0\rho/w$. A plot of $L(T)$ (normalized to L_0) vs T is shown in Fig. S3, for fields gradually going away from H_c. It is clear by inspection that the 5.3 T curve does not extrapolate to $L/L_0 = 1$ as $T \to 0$, unlike the 10 T data, which does. Circles on the $T = 0$ axis in Fig. S3 are the L/L_0 values calculated using $w_c(T \to 0)$ and $\delta_c(T \to 0)$, the two sets of data least sensitive to extrapolation procedures (because they stay essentially linear all the way up to 0.6 K). They are seen to be perfectly in line with the $L(T)$ curves.

Extrapolation to $T = 0$

In Fig. S4 we compare quantitatively these different ways of extrapolating the data to $T = 0$. (i) Squares show the difference in the separate extrapolations of $\rho_c(T)$ and $w_c(T)$ (shown in the top panel of Fig. 2 of the main text), namely $\rho(T \to 0) - w(T \to 0)$. These were determined from linear fits below 0.2 K. (ii) The linear extrapolations of the difference, $\delta(T \to 0)$. These were made over the range below 0.2 K (diamonds), or below 0.6 K (circles). (iii) Pentagons show $\delta_c(0)$ values determined from Lorenz plots for two different samples, as discussed in the section "Absolute accuracy" and Fig. S7. For both samples this gives $L(T \to 0)/L_0$=0.8, as seen by inspection of Fig. S7. Using w_c extrapolations at 5.3 T and assuming $L = L_0$ at

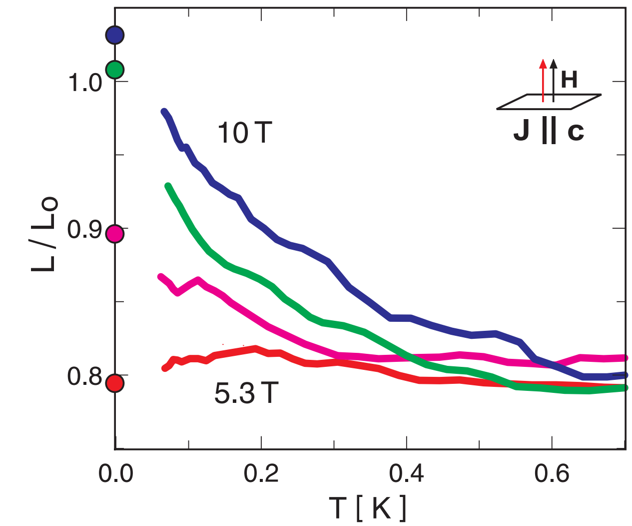

FIG. S3: **Lorenz ratio.** Temperature dependence of the normalized Lorenz ratio, $\kappa\rho/(L_0T) \equiv \rho/w$ for a current along the c-axis (top to bottom 10 T, 8 T, 6 T, 5.3 T). Away from the critical field H_c, i.e. at H=10 T, the Lorenz ratio aims towards 1.0 as $T \to 0$, thus satisfying the WF law. As $H \to H_c$, L gradually deviates from L_0 more and more strongly until eventually $L/L_0 \to 0.8$ as $T \to 0$ close to H_c. Circles show values of $L/L_0 \equiv (w - \delta)/w$ at $T = 0$ obtained from linear extrapolations to $T = 0$ of $\delta(T)$ and $w(T)$ below 0.6 K.

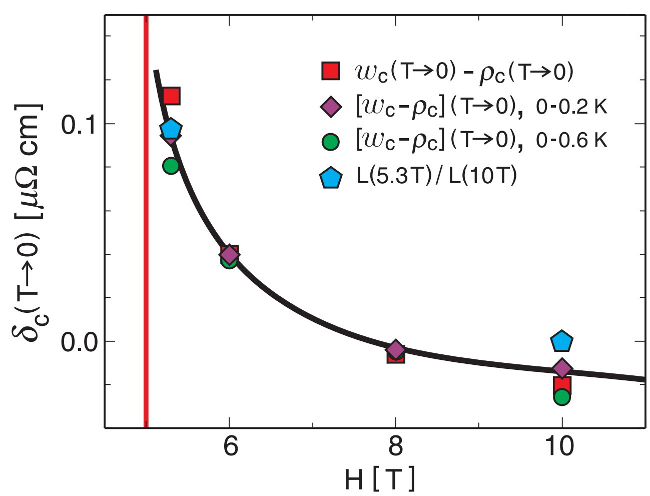

FIG. S4: **Extrapolations**. Field dependence of the $T = 0$ extrapolation of $\delta \equiv w - \rho$, using different extrapolation procedures. Squares: from separate linear extrapolations of $w_c(T)$ and $\rho_c(T)$ (data as shown in Fig. S1) below 0.2 K. Diamonds (circles): from linear fits of $\delta(T)$ shown in Fig. S2 below 0.2 K (0.6 K). Pentagons: δ_0 determined from Lorenz ratios $L(T)$ at 5.3 T and 10 T (see text and Fig. S6), assuming $L(T \to 0) = L_0$ at 10 T. The black line is a guide to the eye. The red line marks H_c=5.0 T.

H=10 T (see below), we get $\delta(0) = w(0)(1 - L(0)/L_0) \simeq$ 0.1 $\mu\Omega cm$.

In conclusion, three separate ways of extrapolating the $J \parallel c$ data to $T = 0$ show the WF law to be obeyed at

H=10 T and then gradually violated as $H \to H_c$.

Recovery of WF law below base temperature.

Here we examine the possibility that below 67 mK, the lowest temperature of our thermal measurements, the transport properties deviate from the asymptotic behavior observed above 67 mK in such a way that the thermal and electrical resistivities converge and the WF law is recovered.

The thermal resistivity $w_c(T)$ is perfectly linear in T over a decade, from 0.6 K down to 67 mK (see inset of Fig. 3). Extrapolating this linear dependence all the way to $T = 0$ yields an intercept of 0.5 $\mu\Omega$ cm (see Fig. S1). The electrical resistivity $\rho_c(T)$ reaches a measured value of 0.4 $\mu\Omega\ cm$ at the lowest temperature of 25 mK (see Fig. S1). In other words, ρ_c(25 mK) $< w_c(T \to 0)$. This implies that the WF law can never be satisfied if $w_c(T)$ continues to be linear down to $T = 0$, or indeed if it deviates upwards from the linear extrapolation, for it would require $\rho_c(T)$ to have an insulating-like upturn as $T \to 0$, an unphysical evolution in these extremely clean metallic crystals.

Note that the same analysis applied to the a-axis transport (linear fit to $w_a(T)$ from 400 to 120 mK) yields the opposite inequality: ρ_c(25 mK)$> w_c(T \to 0)$. This is illustrated in Fig. S5. This leaves room for a metallic-like convergence of both resistivities, which is seen to proceed via an upward curvature at the lowest temperatures. The same kind of curvature can never bring about a convergence for the c-axis transport.

Therefore, the only way for the c-axis transport to eventually satisfy the WF law is if $w_c(T)$ were to deviate downwards from its linear trend. A downward deviation is what is seen in $\rho_a(T)$ at much higher temperatures (see Fig. 3), whereby ρ_a drops from its linear dependence below the characteristic temperature $T_{SF} \simeq 4$ K. So perhaps $0 < T_{SF} < 60$ mK for the c-axis current direction?

While it is in principle possible that T_{SF} is finite but very small for $J \parallel c$, available evidence argues against this. First, we would expect a downturn not only in $w_c(T)$ but also in $\rho_c(T)$. No such downturn is seen, at least down to 25 mK. More importantly, we would expect T_{SF} to grow as the field is tuned away from H_c, as it does for $J \parallel a$. This is not seen in the c-axis data (see for example Fig. S2, where curves for 5.3, 6 and 8 T are all equally linear and featureless over the entire temperature range).

We conclude that, barring some entirely unexpected development at extremely low temperature, the quantum critical regime of $CeCoIn_5$ is characterized by inter-plane transport properties whose extrapolation to $T = 0$ violate the WF law.

Reproducibility

In order to solidly confirm our findings for $J \parallel c$, the entire experiment was re-done on a second sample, cut with different dimensions from a different crystal obtained from a separate growth. Both sets of data are

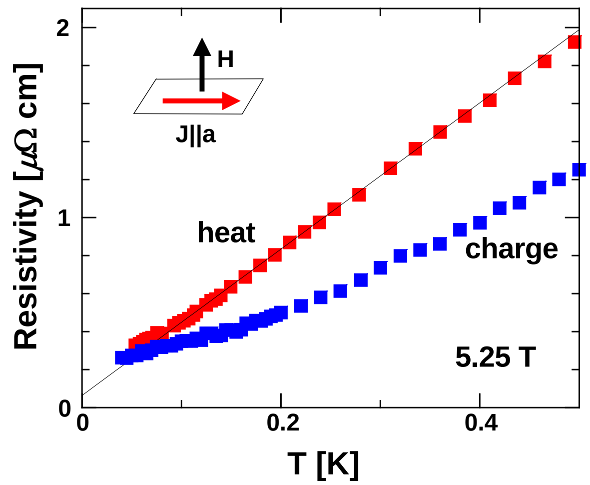

FIG. S5: **Convergence of heat and charge resistivities for $J \parallel a$.** Temperature dependence of heat (red) and charge (blue) resistivities at H_c for in-plane currents ($J \parallel a$). The line is a linear fit to $w_a(T)$ above 120 mK. Extending it to $T = 0$ shows that at the lowest temperatures w_a deviates upwards from this linear extrapolation. Note that $w_a(T \to 0)$, the intercept of the linear extrapolation, lies below ρ_a(50 mK), the lowest measured data point for the electrical resistivity. The reverse is true for c-axis transport (see Fig. S1, top left panel). This shows that no matter how low the temperature a convergence of $w(T)$ and $\rho(T)$ as $T \to 0$ can never happen for $J \parallel c$ in the way that it does for $J \parallel a$, namely via upward deviations from linearity.

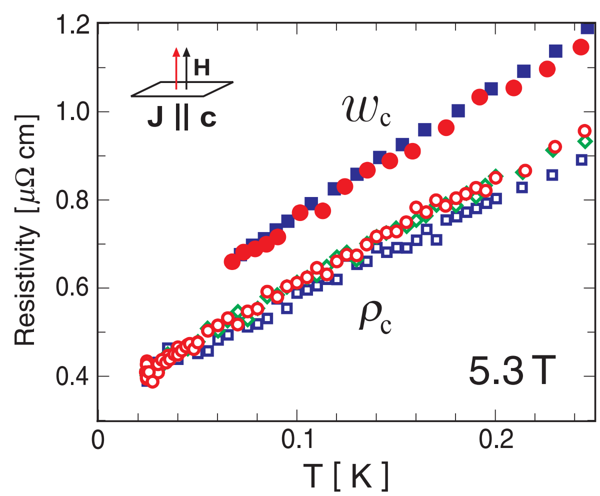

FIG. S6: **Reproducibility.** Heat and charge resistivity for three different samples with $J \parallel c$, at $H = 5.3$ T $\simeq H_c$. Resistivity values were normalized at room temperature in order to remove the $\pm 10\%$ relative error coming from the uncertainty in estimating the geometric factors.

shown in Fig. S6, with red circles for sample no. 1, whose data is reported in the main article, and blue squares for sample no. 2. In order to remove the ±10% uncertainty from geometric factor determination, the resistivity of sample no. 2 was normalized to that of sample no. 1 at room temperature. The electrical resistivity for yet a third sample is also shown (green triangles; same normalization of geometric factor). The degree of reproducibility is seen to be excellent. We conclude that the reported violation of the WF law has been thoroughly reproduced.

Absolute accuracy

The accuracy with which one can test the WF law is limited by the size of voltage contacts. Indeed, even if the separate measurements of heat and charge transport are performed in identical conditions, which in our case they were, namely on the same sample, using the same contacts (in a four-terminal geometry), under exactly the same applied magnetic field strength and orientation, the two points at which temperature is measured along the length of the sample will in general be slightly different from the two points at which electrical voltage is measured, given that the "voltage" contacts have a finite width and a finite resistance. In other words, the "thermal geometric factor" may not be strictly identical to the "electrical geometric factor". The discrepancy is limited by contact size, and the maximum error is given by the ratio of contact width over contact separation, which is approximately ±10% in our samples. Measurements on a total of five different samples (three with in-plane and two with inter-plane transport) all give $L(T \to 0) = L_0$ at 10 T to within 6% or better, *i.e.* $L/L_0 = 1.00 \pm 0.06$, consistent with the uncertainty in the geometric factor.

The advantage of being able to tune to a critical point is that a test of the WF law can be performed away from the QCP on the same sample and contacts, thereby providing an *in situ* reference. Assuming that the WF law is exactly obeyed away from the QCP, we get an accurate measurement of the violation at H_c free from geometric factor uncertainty. In Fig. S7 we show the Lorenz ratio for c-axis samples no. 1 and no. 2 close to the critical field (at 5.3 T) and away from it (at 10 T). At 10 T, $L(T \to 0)/L_0$ for no. 1 slightly overshoots 1.0 (see also Fig. S3), while it slightly undershoots 1.0 for no. 2, in both cases within 6 % absolute accuracy. $L(T \to 0)/L_0$ at 5.3 T is correspondingly lower in no. 2, by some 5 %. In the bottom panel of Fig. S7, we show the ratio of the 5.3 T and 10 T curves for both samples. By inspection it is clear that as $T \to 0$, this ratio converges somewhere close to 0.8 for both samples, as marked by the grey dot. This agrees very well with the value one obtains by other extrapolation procedures.

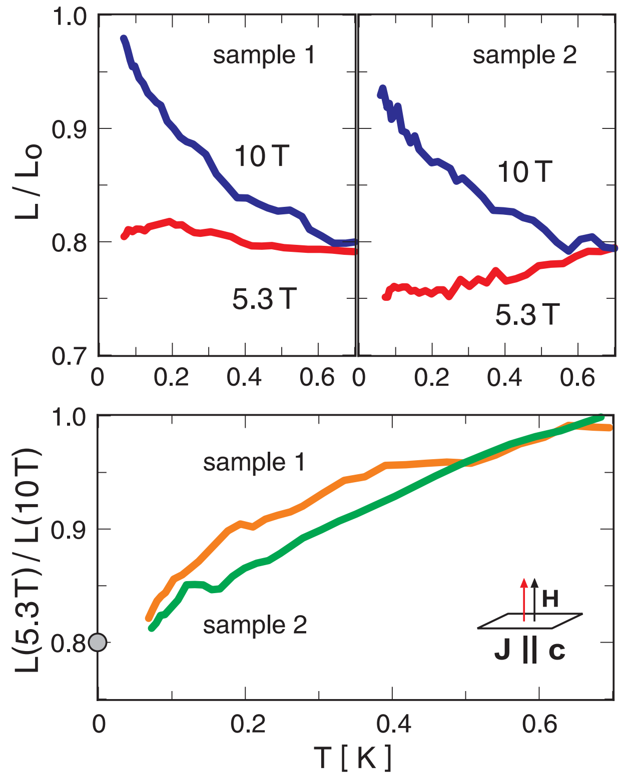

FIG. S7: **Absolute accuracy.** Top: Temperature dependence of the normalized Lorenz ratio, $L/L_0 \equiv \kappa\rho/(L_0 T) \equiv \rho/w$ for a current along the c-axis close to H_c (5.3 T) and away from it (10 T) for two c-axis samples. At 10 T, L/L_0 extrapolates slightly above (below) 1.0 for sample no. 1 (no. 2), satisfying the WF law within the uncertainty in thermal vs electrical geometric factors (see text). The 5.3 T data are shifted accordingly. Bottom panel: the ratio of the 5.3 T and 10 T curves, giving geometric-factor independent normalized Lorenz ratio at 5.3 T relative to that at 10 T. In the $T = 0$ limit, it yields $L/L_0 \simeq 0.8$ at 5.3 T if one assumes $L/L_0 = 1.0$ at 10 T, as marked by grey dot.

Electron-phonon decoupling

The electrical resistance of the interface between heater and sample plays a key role in whether a thermal conductivity experiment can access the intrinsic conduction of heat by electrons in the sample as $T \to 0$. In the limit of an infinite contact resistance R_c, *i.e.* an electrically insulating contact, the heat is brought into the sample entirely by phonons and the electrons in the sample can contribute to the heat flow only if they come into thermal equilibrium with the phonons. This thermalization process occurs through the electron-phonon coupling, which goes to zero as $T \to 0$, typically as T^4 or T^5. As a result, the electronic contribution to κ/T inevitably vanishes at low T, and one finds that κ_e/T apparently goes to zero as $T \to 0$. This decoupling effect shows up as a precipitous drop in the measured κ/T below some temperature characteristic of the material.

In copper oxide materials, electron-phonon decoupling was first observed in $Pr_{1.85}Ce_{0.15}CuO_{4-\delta}$ below $T = 0.3$ K [10], causing an apparent violation of the WF law as κ_e/T dropped far below L_0/ρ as $T \to 0$. (Note that beyond this spurious effect there is still a real violation of the WF law in this material, extracted from data *above* 0.3 K, where heat conduction *exceeds* charge conduction (by a factor 2 or so), *i.e.* $\kappa_e/T > L_0/\rho$, or $w < \rho$.) This anomaly, not well understood at the time, has since been given a thorough theoretical treatment [11], and an excellent description of the downturn in κ_e/T is obtained in terms of the ratio of contact resistance to sample resistance, $r \equiv R_c/R_s$. The theory shows how in the limit of large r κ/T becomes anomalously small as $T \to 0$. On the other hand, in the limit of a low contact resistance or small r the measurement yields the correct value, because electrons in the sample do not depend on phonons to be thermalized, but rather on the electrons in the metallic contact.

We have carefully investigated the issue of electron-phonon decoupling in $CeCoIn_5$, by measuring samples with deliberately high contact resistances. In Figure S8, we show the in-plane thermal conductivity of two nominally identical samples with different contacts, respectively made with silver epoxy on gold-evaporated pads, giving $R_c \simeq 500$ mΩ, and indium, giving $R_c \simeq 5$ mΩ. For the former, a downturn is immediately evident, starting below $\simeq 0.2$ K. For the latter, used in all measurements reported in this article, there is no trace of a downturn.

In the end, the infallible test of whether thermal conductivity data is contaminated by electron-phonon decoupling is the WF law itself. The fact that the law is accurately satisfied for a magnetic field of 10 T allows us to definitively rule out electron-phonon decoupling effects in samples with indium contacts. Then, the observation of a deviation from the WF law in the same sample when the field is simply tuned towards H_c (for $J \parallel c$) in otherwise *identical* experimental conditions (in particular with the very same contacts) is compelling. (Note that the electron-phonon coupling is not expected to vary at all with magnetic field.) In addition, a number of other features independently confirm that there is no contamination: 1) $w(T)$ is perfectly linear in T all the way from 1 K down to 70 mK, without a hint of deviation below the known downturn temperature of 0.2 K (see inset of Figure 3); 2) the high degree of reproducibility for five different samples, each with different contacts (see discussion on "Reproducibility" above and Figure S6); 3) nominally identical samples and contacts never yield a violation of the WF law when $J \parallel a$.

In conclusion, the measurements presented here are demonstrably free of any contamination from the electron-phonon decoupling effects that were encountered in other studies [10, 12], and the reported violation of the WF law, tunable and anisotropic, is undoubtedly intrinsic.

Contamination from superconductivity

Because the QCP at $H_c = 5.0 \pm 0.1$ T coincides with the onset of superconductivity at $H_{c2}(0) = 5.0 \pm 0.1$ T, one must carefully consider the possibility of a contamination of the transport data by traces of superconductivity just above $H_{c2}(0)$ (due to either small superconducting regions in the sample or paraconducting fluctuations). Detailed measurements show that there is no contamination for fields of 5.3 T and above. This can be seen from the data in Fig. S9. The field sweep of the electrical resistivity $\rho_c(H)$ at fixed $T = 25$ mK, displayed in the inset, shows that the superconducting downturn starts below 5.3 T. In the main panel, three T-sweeps of $\rho_c(T)$ are displayed, at $H = 5.1, 5.3$ and 5.75 T. The sample is partially superconducting at $H = 5.1$ T. An increase in the field by $\delta H = 0.2$ T, to $H = 5.3$ T, causes a significant increase in ρ_c, of order $\delta\rho_c \simeq 0.2$ $\mu\Omega$ cm. However, a further increase by $\delta H = 0.45$ T, to 5.75 T, produces no detectable change, with $\delta\rho_c < 0.02$ $\mu\Omega$ cm or so. In other words, if we assume that no trace of superconductivity is present at 5.75 T, then the 5.3 T data must have a level of contamination below 4% or so. Now

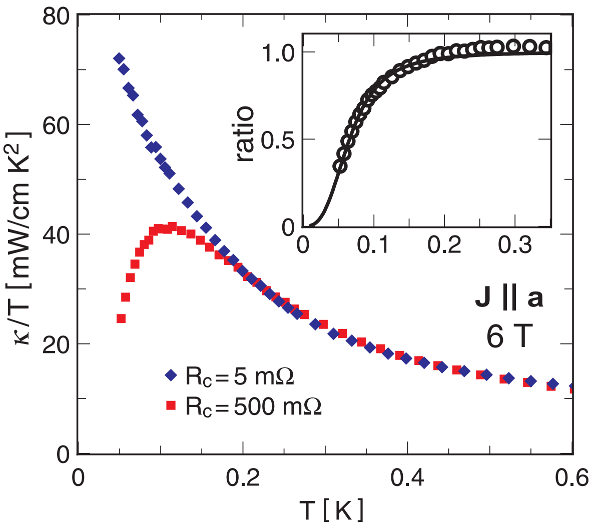

FIG. S8: **Electron-phonon decoupling.** Thermal conductivity data obtained on two nominally identical single crystals of $CeCoIn_5$ with different contact resistances in otherwise identical experimental conditions, plotted as κ/T vs T. The current is in-plane and the magnetic field is 6 T. In the first case (red squares), contacts were made by evaporating gold and using silver epoxy to attach the silver wires, leading to a contact resistance $R_c \simeq 500$ mΩ. In the second case (blue circles), the silver wires were soldered in place by melting indium directly on the sample surface, which gave a contact resistance two orders of magnitude lower, $R_c \simeq 5$ mΩ. A pronounced downturn is observed below 0.2 K in the sample with high contact resistance. This downturn is the characteristic signature of electron-phonon decoupling [11], whereby as temperature is reduced electrons in the sample rapidly fail to thermalize with the phonons which, in this case, are predominantly responsible for carrying the heat into the sample, through the interface between heater and sample. Inset: ratio of heat conductivities for the two samples. The solid line is a fit to the theory of electron-phonon decoupling by Smith *et al.* [11].

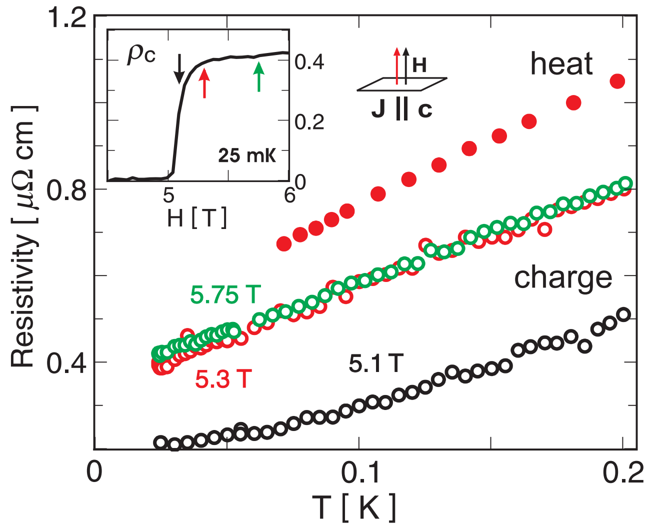

FIG. S9: **Superconductivity.** Temperature dependence of the c-axis resistivity ρ_c (open circles) for three values of the magnetic field close to the superconducting upper critical field $H_{c2}(0) = 5.0$ T. The thermal resistivity w_c at 5.3 T is also shown for comparison (closed circles). From $H = 5.1$ T to 5.3 T, ρ_c rises as the sample leaves the superconducting region. By 5.3 T, there is no further increase, confirming that data at 5.3 T and above are free of contamination by traces of superconductivity. Inset: field dependence of ρ_c at $T = 25$ mK. Arrows indicate the three fixed field values at which data in the main panel was obtained.

even if there was a slight contamination of ρ_c at the level of 4-5%, it would have an entirely negligible impact on our test of the WF law because thermal conductivity is much less sensitive to traces of superconductivity. Indeed, it is well-known that superconducting regions in a sample (very unlikely in metallic single crystals with such extremely low residual resistivities) can short-circuit the normal state resistivity entirely and still have negligible impact on bulk properties such as specific heat or thermal conductivity. As for paraconductivity, it is expected to be much smaller in the heat channel (and has in fact never been observed in any superconductor).

We emphasize that the WF law is violated because of a rise in w, not because of a superconducting-like drop in ρ (see Figure 2 of the article).

Finally, if the WF law was violated due to traces of superconductivity, this spurious violation would show up in both current directions, not only for inter-plane transport.

Phonon conduction

The thermal conductivity of a metal is the sum of two contributions, respectively from electrons and phonons, so that $\kappa = \kappa_e + \kappa_p$. In order to estimate the phonon contribution in our samples of pure $CeCoIn_5$, we measured the in-plane conductivity κ of a $CeCoIn_5$ sample doped with La impurities, namely $Ce_{1-x}La_xCoIn_5$ with $x = 0.1$. This level of doping increases the residual resistivity ρ_0 by a factor of 15. With such a huge increase in elastic scattering, the inelastic scattering becomes negligible below 1 K, being only a few % at 0.5 K, for example. It is then reasonable to assume that the *electronic* conductivity satisfies the WF law to within a few % up to 0.5 K. The in-plane data for this La-doped sample is shown in Fig. S10 (top panel), plotted as κ/T vs T and compared to the electrical conductivity L_0/ρ. The WF law is seen to hold at $H = 4.6$ and 10 T as $T \to 0$. Making the assumption that it remains true up to $T = 1$ K, so that $\kappa_e/T \simeq L_0/\rho_0$, we can extract an estimate of the phonon term by taking the difference $\kappa_p/T = \kappa/T - L_0/\rho$. The result is shown for both fields and the excellent agreement between the two curves confirms the soundness of the approach. Note that κ_p/T varies linearly with temperature,

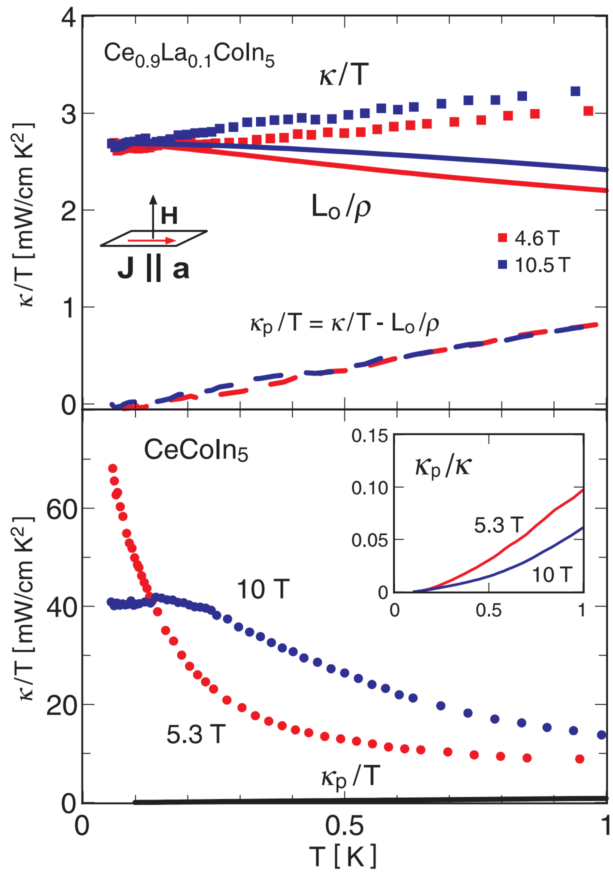

FIG. S10: **Phonon conductivity.** Top: In-plane thermal conductivity of $Ce_{0.9}La_{0.1}CoIn_5$ (squares), plotted as κ/T vs T, and the corresponding electrical conductivity L_0/ρ, for two values of the c-axis magnetic field. In an impurity-doped sample such as this, where elastic scattering dominates entirely at low temperature, the difference between thermal and electrical conductivities gives the phonon conductivity κ_p, plotted as dashed lines. Appropriately, the phonon contribution is insensitive to magnetic field. Bottom: the same in-plane phonon conductivity κ_p, shown in comparison to the thermal conductivity of *pure* $CeCoIn_5$ samples. Inset: ratio of phonon to measured thermal conductivities.

so that $\kappa_p \propto T^2$, as expected and found for the phonon conductivity of metals at low temperature, where electron scattering is the dominant scattering process [13].

Taking $\kappa_p(T)$ thus estimated as valid for pure samples (since the scattering of phonons by the extra La impurities should be negligible at such low temperatures [13]), we compare it to the total measured conductivity in the pure samples, in the lower panel of Fig. S10. The inset displays the ratio of κ_p/κ_a vs T, which shows that κ_p is on the order of 5-10% (1-2%) of the measured in-plane conductivity at 1 K (0.3 K). Preliminary estimates suggest that the phonon term κ_p is even smaller for inter-plane transport, by a factor 2 or so.

In conclusion, the phonon contribution to the measured thermal conductivity in pure $CeCoIn_5$ samples is negligible below 1 K. Consequently, in the article we simply plot the raw data in all cases. We emphasize that these estimates are entirely irrelevant when it comes to testing the WF law in the $T = 0$ limit, because the phonon contribution to κ/T goes to zero as $T \to 0$ (since $\kappa_p/T \propto T$).

Current alignment

The precise measurement of the different components of the resistivity tensor is a delicate procedure. In the widely used Montgomery version [14] of the van der Pauw technique [15], non-negligible contact size and the resulting uncertainty in geometry inevitably lead to a contamination of the data by minority components. To avoid this problem, and to achieve the most identical conditions for electrical and thermal resistivity measurements, we used the standard four-probe technique with long samples cut along the principal directions of the conductivity tensor. The sample geometry always satisfied the criterion $l > 5w, 5t$, where l is the distance between potential contacts, and w and t are the width and thickness of the sample. Since samples of $CeCoIn_5$ normally grow in platelets with the c-axis perpendicular to plane, it is important to control the alignment of cutting and polishing tools with high precision. It is essential to have a way to verify *in-situ* that, once cut and polished, the sample has the correct alignment. Fortunately, this can be done via the field dependence of the resistance: in the case of a perfect alignment of the current to flow along the c-axis, the resistance above T_c shows negligible field dependence at low fields, as ρ_c shows no (longitudinal) magneto-resistance, in sharp contrast with the large (transverse) magneto-resistance of ρ_a, as seen in Fig. S11.

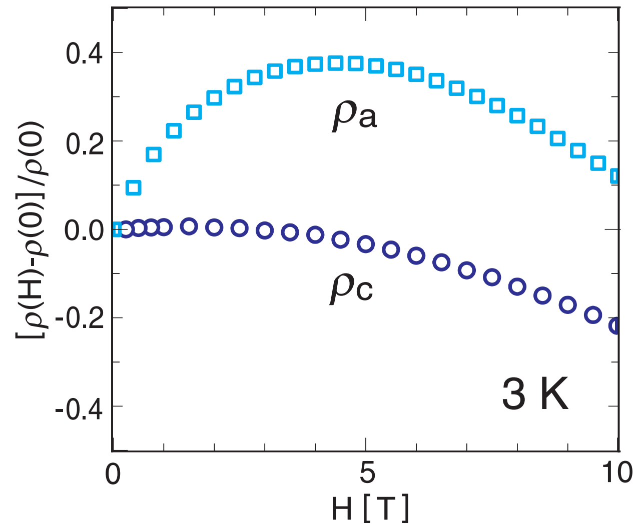

FIG. S11: **Current alignment.** At temperatures above $T_c = 2.4$ K, ρ_c does not show any increase with field at low fields ($H \parallel c$), in stark contrast with ρ_a. This strong anisotropy in the magneto-resistance was used to establish that samples used for our study of c-axis transport were properly aligned after cutting, thereby ensuring that the current flowed precisely along the c-axis.

Previous studies

Theoretical studies predict a violation of the WF law in cases of strong modification of Fermi-Dirac statistics, like in Luttinger and Laughlin liquids [16–19]. In the case of a marginal Fermi liquid [20], a minor violation was predicted for a special case of out-of-plane impurities, with $L < L_0$ [21].

Experimental tests of the Wiedemann-Franz law were undertaken in the normal state of cuprate superconductors. The phase diagram of these compounds shows evolution from Mott insulator for undoped materials towards Fermi liquid for overdoped compositions. Theoretical treatments of the metal-insulator transition yield either no violation [8] or a very small one [22] as $\sigma \to 0$. Upward violations by as much as $L/L_0 \simeq 2 - 3$ were measured in underdoped cuprates at the lowest temperatures, always in the presence of incipient charge localization (upturn in $\rho(T)$ as $T \to 0$) [10, 23, 24]. In the most detailed study [24], heat transport was found to remain constant with decreasing carrier concentration while resistivity grew. Moreover, as $T \to 0$, $\kappa/T \propto$ constant, while $\rho \propto \log(1/T)$, so that $L/L_0 \to \infty$. In a strongly overdoped cuprate, the WF law was found to be perfectly obeyed [25].

Two previous studies have tested the WF law at or near a QCP. Both found the law to be well obeyed. The first was on $CeNi_2Ge_2$ [26], a $4f$ heavy-fermion metal believed to be naturally close to a magnetic QCP of the SDW type at ambient pressure [27]. In this system, the temperature dependence of the resistivity is close to $T^{3/2}$ for both high-symmetry directions of the lattice, similar to our in-plane transport measurements. The WF law as $T \to 0$ was verified for both current directions. This is consistent with the phenomenology of in-plane transport in $CeCoIn_5$.

The second study was on $Sr_3Ru_2O_7$ [28], a layered oxide with a field-tuned quantum critical end point [29]. In the $T = 0$ limit, the WF law was found to be obeyed.

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