The current era of research on superconductivity might be said to have started in 1986, with the discovery of superconductivity in the cuprates, oxides made of stacks of CuO$_2$ planes, in which the critical temperature $T_c$ can be as high as 164 K – seven times higher than the previous record and halfway to room temperature. The discovery sparked huge interest both for reasons of technology – a room-temperature superconductor would be a revolution – and reasons of fundamental science – what new microscopic mechanisms might be at play? It is widely believed that the force – or glue – that binds electrons into Cooper pairs is not the usual attraction mediated by the ionic lattice and its vibrations. The force would come from within the electron system itself.

However, it may be useful to view this major event not in isolation but as a landmark in an era that actually started earlier, with the discovery of heavy-fermion and organic superconductors in 1979. Although in both families of materials – the former, $f$-electron metals, and the latter, stacks of organic molecules – $T_c$ is rarely higher than 10 K, the case for a purely electronic pairing is just as strong as in the cuprates. In fact, here there is little doubt that the glue for pairing is magnetic in origin. The basic idea is that the pairing interaction which relies on the polarizability of the surrounding ionic lattice in conventional superconductors (like mercury, aluminium or lead) would now rely instead on the magnetic susceptibility of the surrounding electrons.[1]. And this susceptibility is enhanced near the quantum critical point (QCP) for spontaneous magnetic order. (A QCP is a quantum phase transition, a phase transition at $T = 0$ induced by tuning some parameter like pressure, composition, doping or magnetic field.) This makes quantum criticality a fertile ground for the emergence of novel states of matter quite generally, in particular unconventional superconductivity[2].

Because of its relative simplicity – a single-band quasi-one-dimensional metal, with neither Kondo effect nor quantum criticality. This makes quantum criticality a fertile ground for the emergence of novel states of matter quite generally, in particular unconventional superconductivity[2].

In several families of materials, including the recently discovered iron pnictides, superconductivity is found near the quantum critical point where a magnetic phase ends, pointing to a magnetic glue as the source of electron pairing, distinct from the usual phononic glue. In the copper oxides where superconductivity persists to the highest temperatures, the nature and role of an elusive quantum critical point are the subject of much debate. Is the glue again magnetic? If so, why is it so strong in this case?

**SUMMARY**

In several families of materials, including the recently discovered iron pnictides, superconductivity is found near the quantum critical point where a magnetic phase ends, pointing to a magnetic glue as the source of electron pairing, distinct from the usual phononic glue. In the copper oxides where superconductivity persists to the highest temperatures, the nature and role of an elusive quantum critical point are the subject of much debate. Is the glue again magnetic? If so, why is it so strong in this case?
Mott physics – the Bechgaard salt (TMTSF)$_2$PF$_6$ may be considered the archetype of magnetic pairing\cite{3}. Its pressure-temperature phase diagram is shown in Figure 1, where the superconducting phase is seen to reside on the boundary of an antiferromagnetic (or spin-density-wave; SDW) phase. Renormalisation group calculations reproduce this phase diagram, as well as the detailed evolution of the antiferromagnetic spin fluctuations with pressure and temperature. The calculations show that those fluctuations are the cause of superconductivity\cite{3}.

Each of the three phases in the phase diagram (Figure 1) has some key characteristic signatures. 1) Because interactions from antiferromagnetic correlations are intrinsically anisotropic, the resulting superconducting state is also anisotropic. In the Bechgaard salts, calculations predict a $d$-wave state, with a gap that changes sign along the Fermi surface\cite{3} – in contrast with conventional superconductors where the $s$-wave gap is isotropic. 2) Because it introduces a new periodicity, thereby breaking the translational symmetry of the crystal lattice, the SDW phase causes a reconstruction of the Fermi surface. This leads to a sharp drop in carrier density and a concomitant drop in conductivity, or rise in resistivity, as shown in Figure 2. 3) Near the QCP, the spin fluctuations that cause pairing below $T_c$ also cause scattering above $T_c$. This scattering leads to deviations from the standard Fermi-liquid behaviour of metals, and its typical signature is a linear (as opposed to quadratic) temperature dependence of the resistivity as $T \to 0$. A linear resistivity was recently observed in (TMTSF)$_2$PF$_6$ at the QCP (see Figure 2) and an interesting empirical correlation was discovered\cite{3,4}; the pressure needed to restore the Fermi-liquid behaviour (by moving away from the QCP) is the same pressure that is needed to suppress superconductivity. In other words, non-Fermi-liquid resistivity and $d$-wave superconductivity are intimately linked: superconductivity and quantum criticality coexist.

The magnetic pairing mechanism, which only causes a $T_c$ of order 1 K in the Bechgaard salts (and not much higher in most heavy-fermion metals), acquired new significance with the discovery of iron pnictides, whose $T_c$ can be as high as 55 K\cite{5}. The phase diagram of pnictides is strikingly similar to that of organics – see Figure 1 – except that temperature scales (magnetic and superconducting) are now up to two orders of magnitude higher. And here again there is evidence of linear resistivity at the antiferromagnetic QCP (see Figure 2) and of the same empirical correlation between scattering and pairing\cite{4}. An interesting difference is the multi-band Fermi surface, which offers the possibility of inter-band scattering (not present in the Bechgaard salts), believed to play a key role in the pairing, as the SDW wavevector now connects different sheets of the Fermi surface. That could offer more flexibility in the design and search for a stronger interaction and a higher $T_c$.

This brings us to the cuprates: in what ways are they similar to pnictide and organic superconductors? What are the important differences? Is quantum criticality relevant? Is the pairing glue magnetic? The doping-temperature phase diagram (Figure 3) reveals some of the differences: 1) the antiferromagnetic phase at low doping is an insulator, rather than a metal; 2) the QCP where it ends lies either before or at the onset of superconductivity, rather than at optimal doping (where $T_c$ is maximal); 3) there is an additional region called the “pseudogap phase” which sets in below a crossover temperature $T^*$, above $T_c$ in the underdoped regime\cite{6}. There is no consensus on the nature of the pseudogap phase. There are

![Fig. 2](image-url)
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cuprate, associated with antiferromagnetic (SDW) spin
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Comparison this cuprate to the Bechgaard salt (TMTSF)$_2$PF$_6$, even
La$_{1.6-x}$Sr$_x$Nd$_{0.4}$CuO$_4$ (Nd-LSCO) [4]. In the absence of
superconductivity, suppressed by application of a magnetic
field, there is a QCP at $p^* = 0.24$, below which the Fermi
surface undergoes a reconstruction, as revealed by a pronounced rise in the resistivity (see Figure 2). This QCP is
where “stripe order” ends. This is a unidirectional modulation of both the spin and charge densities, which break the rotational and translational symmetries of the lattice. So this is a kind of SDW order, with an associated charge-density-wave (CDW). The standard signature of quantum criticality is observed: the resistivity is perfectly linear as $T \to 0$. At high doping, far away from the Mott insulator, it is legitimate to compare this cuprate to the Bechgaard salt (TMTSF)$_2$PF$_6$, even though its $\Delta T_c$ is 20 times higher: both are good single-band metals with strong 2D character (at low temperature), both are $d$-wave superconductors, both have a QCP at which SDW order ends, and both show a linear resistivity at that QCP. It therefore seems reasonable to invoke a similar pairing mechanism for the cuprate, associated with antiferromagnetic (SDW) spin fluctuations.

There is one cuprate material where the picture is fairly clear: La$_{1.6-x}$Sr$_x$Nd$_{0.4}$CuO$_4$ (Nd-LSCO) [4]. In the absence of superconductivity, suppressed by application of a magnetic field, there is a QCP at $p^* = 0.24$, below which the Fermi surface undergoes a reconstruction, as revealed by a pronounced rise in the resistivity (see Figure 2). This QCP is where “stripe order” ends. This is a unidirectional modulation of both the spin and charge densities, which break the rotational and translational symmetries of the lattice. So this is a kind of SDW order, with an associated charge-density-wave (CDW). The standard signature of quantum criticality is observed: the resistivity is perfectly linear as $T \to 0$. At high doping, far away from the Mott insulator, it is legitimate to compare this cuprate to the Bechgaard salt (TMTSF)$_2$PF$_6$, even though its $\Delta T_c$ is 20 times higher: both are good single-band metals with strong 2D character (at low temperature), both are $d$-wave superconductors, both have a QCP at which SDW order ends, and both show a linear resistivity at that QCP. It therefore seems reasonable to invoke a similar pairing mechanism for the cuprate, associated with antiferromagnetic (SDW) spin fluctuations.

Two questions arise: 1) is Nd-LSCO representative of other
cuprates, in particular those with a higher $T_c$? 2) What is the
relation between pseudogap phase and stripe order? Let us take these in turn. There is growing evidence that the basic mechanisms at play in Nd-LSCO are also active in YBa$_2$Cu$_3$O$_y$ (YBCO), one of the archetypal cuprate superconductors, with a maximal $T_c = 94$ K. Since the discovery of quantum oscillations in 2007 [7], it is clear that YBCO also undergoes a Fermi-surface reconstruction, so that it too must have a QCP somewhere in the phase diagram. Recent studies show this reconstruction to be very similar to that of Nd-LSCO [4], such that stripe order is also the likely cause of broken translational symmetry in YBCO. Now all cuprates, including YBCO, show a linear resistivity near optimal doping, of universal slope. This means that the same scattering is responsible for the linear resistivity in YBCO and in Nd-LSCO, the latter being associated with the stripe QCP. Moreover, the onset of non-Fermi-liquid behaviour in overdoped cuprates is known to set in precisely at the onset of superconductivity, so as in the organic (TMTSF)$_2$PF$_6$ and the pnictide Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ (Figure 1), scattering and pairing are again intimately linked [4].

If stripe order sets in at low temperature, one expects, in a
strongly 2D metal, a precursor regime of stripe fluctuations at
temperatures above that onset [8], and this could be the pseudogap phase going up to $T^*$ – an example of scenario 2) above. Supporting evidence for this came recently with the discovery that rotational symmetry in YBCO is broken at $T^*$ [9], consistent with the onset of stripe correlations, well before the onset of stripe order at $T \sim T^*/2$. Interestingly, this same sequence of broken symmetries upon cooling – first rotational, then translational – is also found in the iron pnictides [10], associated with their stripe-like SDW order. The wider regime of “nematic order” (as a state of broken rotational symmetry may be called [11]) seen in the cuprates may come from their stronger 2D character.
In summary, if we start from the metallic state on the far right of the phase diagrams (Figures 1 and 3), sit at low temperature, and tune to the left, we find that in all three families of materials—organics, pnictides and cuprates—superconductivity appears precisely where quantum criticality is first being felt. \( T_c \) then increases steadily as the QCP is approached. So proximity to SDW or stripe order is good for superconductivity, the glue to be found in the fluctuations of those orders. If we go further, past the QCP, and enter the SDW or stripe phase then \( T_c \) drops. So coexistence with SDW or stripe order is bad for superconductivity: the two phases compete. This suggests two guiding principles for a high \( T_c \): 1) the magnetic energy scale should be high; 2) magnetic fluctuations should be strong, but magnetic order weak. The first principle is illustrated in Figure 1: even though \( T_c \) in the pnictide is 20 times higher than in the organic salt, the ratio of maximal \( T_{SDW} \) to maximal \( T_c \) is nevertheless comparable (6 in the former, 10 in the latter). In the cuprate Nd-LSCO, the maximal onset temperature for stripe order is 4 times larger (and the maximal \( T^* \) 10 times larger) than maximal \( T_c \), again comparable. The second principle is illustrated by comparing Nd-LSCO and YBCO. They have comparable \( T^* \) values (only 1.5 times larger in the latter), but the maximal \( T_c \) is 5 times lower in Nd-LSCO, a material whose crystal structure stabilizes stripe order, to the detriment of superconductivity. Our second principle argues for strong 2D character, which favours fluctuations over order. It also argues for frustration of the underlying magnetic state, and hole-doped cuprates may be an example of this, with the commensurate antiferromagnetic phase at low doping becoming unstable with respect to the incommensurate stripe phase (with spin and charge order) at intermediate doping. This instability in the magnetic ordering is absent in the electron-doped cuprates, where the commensurate antiferromagnetic phase persists up to higher doping, without any sign of incommensurate order or charge order. This might explain why \( T_c \) is lower in the electron-doped cuprates.

In the last 3-4 years, the current era of superconductivity research has come to a point of convergence: in a wide range of materials, superconductivity is found to live in the wake of quantum criticality. A major organizing principle is emerging. The task is now to elucidate its workings. And to find out how the quantum-critical environment can be optimized to enhance \( T_c \). An intriguing avenue is the role of a nematic tendency, found in both pnictides and hole-doped cuprates, the two families with the highest \( T_c \). Is the 1D character of spin fluctuations a good thing? What about a tendency towards charge-density-wave order – is that a booster for superconductivity? The second century is off to an exciting start.

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